An Introduction
TO
ALGEBRA,

TRANSLATED out of the
HIGH-DUTCH
INTO
ENGLISH,

By *THOMAS BRANCKER*. M. A.

Much *Altered* and *Augmented* by D. P.

Also

A TABLE of Odd Numbers
less than One Hundred Thousand,
SHEWING
Those that are *INCOMPOSIT*,
AND
Resolving the rest into their *FACTORS*
or *COEFFICIENTS*, *&c.*

SUPPUTATED by the same *THO. BRANCKER*.

London, Printed by *W. G.* for *Moses Pitt* at the *White-Hart*
in *Little Britain*. 1668.

Rahn (J. H.)

THe Title-Page *says that this Book was a* Translation, *but hath been much* altered. *If any man desire to know* what *the* alterations *are, and* why *they were made; he may do well to compare it with the Original: A Printed Copy whereof may be had at* Francfort *in* Germany, *by any that inquires there for it by this Title*, Algebra Rhonii Germanice; Tiguri apud Bodmerum, 1659. in quarto. *The Copy which I have, was given me* anno 1662. *by a good Friend, who* then *told me he much desired to read it in some Language that he understood; I then promised him to* English *it. As soon as my leisure permitted, I corrected it according to the Printed Catalogue of Errata, and then began the Translation. When it was finished, I desired to see it Printed, and got it Licensed* May 18. 1665. *with the name of* An Introduction to Algebra. *And so* without any alteration *either in the Precepts or Examples, save only the correction of many Mistakes: It was sent to the Press, with order to Re-print the six leaves of* His Table of Incomposits *precisely as they stand there.* M. F. T.

A little after, I heard that there was at that time in London, *a Person of Note very worthy to be made acquainted with my design, before I made any farther progress in the Impression. Being admitted to speak with him, I found him not only* able *to direct me, but also very* willing *so to do, so far as his leisure would permit. He gave me divers cautions concerning the Work. He shewed me the way of making the* Table of Incomposits, *of examining it, and of continuing as far as I would. He encouraged me to extend it to* 100 *thousand: Telling me that by that time that I had Calculated and Printed that Table, he hoped to be at leisure to review some of* Mon- D. I. P.

 sieur

sieur Rhonius *his Problemes, and to work them anew; and that he would send them to me, with leave to publish them or to keep them by me.*

I had finished and Printed that Table, as also Twelve sheets of the Book it self, *before he sent me his* Alterations. *They begin with* Probl. 24. pag. 100. *All from thence to the end is his Work: As also* pag. 79. 80. 81. 82. *which he sent last of all: So that instead of the first* 124 pages *of* Rhonius, this *hath just twice as many: Instead of those* 8 *or* 9 *sheets remaining in* Rhonius, *how much shall be hereafter published, I will not adventure to foretell, because of the uncertainty of life, health, leisure, and of the acceptance which* this *shall find amongst the lovers of these Studies, to whom this might have been more acceptable, if it had been wholly void of Press-faults.*

As for the Table of Incomposits, *I was very sensible of the bad effects of perfunctoriness in Supputating, Transcribing or Printing of it. My care therefore was not small, yet* pag. 198. *is almost filled with* Errata, *and I dare not warrant that none have escaped unseen: But seeing so few are fit to undertake to Supputate it anew, whosoever shall happen to discover any other fault in that Table, shall do well to signify it to the* Book-seller, *or to any other likely to be concerned in the next Impression.*

The Errata *in* the rest of the Book *are many, notwithstanding my care, and the diligence of a good friend, who Corrected part of it, after my removal to an abode so far from* London. *Most of them cannot trouble the more exercised sort of Readers. But fear of leaving any stumbling-block in the way of Beginners hath caused this larger Enumeration of them in the three next following* pages.

White-gate *in* Cheshire,
April, 22. 1658

T. B.

Preſs-faults with their Amendments.

PAg. 2 and 4. Subſtraction: Subtrahend *for* Subtra... Pag. 8, *In the broad margin over againſt* 3, *write* 1 ÷ 2 *three times,* Pag. 9. lin. 13. 14. 15. 19 *you have* 2 *inſtead of* z. Pag. 10. lin. 27, + 36 *for* — 36. lin. 28, — 22 *for* — 2z. Pag. 11. lin. 5. + b^3 *for* — b^3. Pag. 13. lin. 13. ce *for* c + e. lin. 14. c ÷ e, *for* c — e. lin. 17. aaa — aab *for* — abb. lin. 18. aaa — abb *for* — aab. Pag. 15. lin. 6. 99 rr *for* 9rr. Pag. 16. lin. 6. *write* $\frac{121}{4}$. lin. 18. z *for* 2. Pag. 18. lin. 8. + d *for* + dd. lin. 17. 2cc. *for* 2ce. Pag. 19. lin. 12. *you have* 3aabe *for* 3aab: *As alſo* — 144ddf *for* — 144def. lin. 21. 36ef *for* 36eef. Pag. 23, lin. 16. quantifies *for* quantities. lin. 24. *read* Denominator. lin. 30. *you have* Denominator *for* Denomination. Pag. 25. *In the narrow margin write* 1.2.3. *one ſpace lower: and after* 3 *ſet* 1 — 2 *in the broader margin.* Pag. 26. lin. 21. b + e *for* b + c. Pag. 32. lin. 17. √ of aa is *for* √ of a is... Pag. 37. lin. 23. ſer *for* ſet. Pag. 40. lin. 4. √ — + dd *for* √ — dd. Pag. 42. lin. 1. Quantity *for* Quantities. Pag. 45. lin. 14. B⊖3 *for* B⊖2. Pag. 48. lin. 12. + *for* x. Pag. 49. lin. 17. 1 ✱ 6 *for* 2 ✱ 6. Pag. 51. lin. 11. 1 — 13 *for* 1 + 13. Pag. 55. lin. 7. — 3a aazz *for* — 3aa azz. Equ. 5. *In the margin* 1 *for* 4. Pag. 58. Equ. 4. 3,,2 *for* 3 ÷ 2. Pag. 59. lin. 6. *put out* away. lin. 11. *read* quantities, as an Equation is a... Equ. 29. 28 ÷ 4 *for* $\overline{4}$. Equ. 31. BE *for* DE. Pag. 60. lin. 3. *put out* D — E. lin. 4. *write* $\frac{D-E}{2}$ = B. Pag. 65. lin. 18, ad, e, cf *for* a + d. e. c + f. Pag. 66. lin. 9. 6√2 *for* 9√2. Pag. 68. Equ. 11. *after* √5 *ſet* — aa. Pag. 69. Equ. 4. *In the margin ſet* 1 — b. Pag. 70. lin. 2. *read* c ✱ h — b = 15. what are h and b^2. See Figure (1.) Equat. 1. — c. *for* — bc. Pag. 71. lin. 5. The other two *for* of the other two. Pag. 72. lin. 6. ſecond *for* third. lin. 7. third *for* ſecond. lin. 10. a + b — c *for* a + c — b. Pag. 73. lin. 1. 8 — 9 *for* 8 — 10. lin. 2. 12ac *for* a — c. Pag. 74. Probl. X. *after* 14560 *ſet* What are thoſe numbers? Equ. 2. margin h = ? *for* b = ? Pag. 75. lin. 6. Divident *for* Dividend. lin. penult. — 2a — 6b — 2c *for* + 6b... Pag. 76. lin. 4. + c *for* — c. Probl. XII. lin. 2. or. c *for* or. between h and c. Equ. 5. *read.* o = cc. Pag. 83. lin. 5. 8 — g^6 *for* 8 — $2g^6$. Equ. 34. 8 — $2z^6$ *for* 8 — $2g^6$. Equ. 39. √c. *for* √c 2. Pag. 84. Equ. 45. DD *for* 2DD. Pag 85. Equ. 19. *read* $\frac{cc-DD}{2D}$ = 3. Pag. 86. Equ. 2. Letters *for* Denomination. Equ. 5. *write* 4dd *under the line* — Pag. 87. Collat. 4. c > c *for* c > d. Pag. 89. lin. 2. margin, *read* + 2bg. Equ. 37. — ffgg *for* fgg. Equ. 38. *write* $\frac{ffg-fgg}{g-f}$ > o. *After* Equ. 39. *read,* provided that f be greater than g. and c be greater than f. For then c will be greater than g. Equ. 42. *put out* 4 *after* (VIII.) Equ. 55. — ffg *for* — fgg. Pag. 90. Equ. 66. 143 *for* 123. 2

Caſe

[illegible] in the margin [illegible] *read* [illegible]. Pag. [illegible] lin. 17. These [illegible] 171 *for* 17. Pag. 95. [illegible] Proposition *for* Theorem. Equ. 4. [illegible] Pag. 95. lin. 2. *read* to find AB, BC. Equ. 11. *read* bb:ce:: Pag. 99. Equ. 4. margin $\frac{3}{5}$ *for* $\frac{4}{5}$. Pag. 100. Equ. 16. 715 *for* 7, 15. Equ. 18. 617 *for* 6, 17. Pag. 106. 4= *for* o=. Pag. 108. Equ. 72. 3365 *for* 3375. Pag. 110. Equ. 92. DDD—C *for* +C. Pag. 111. Equ. 30. 1> *for* Let 1>bbb. Equ. 39. *read* 37. 6 (*) Pag. 114. lin. 22. *read* greater than these [illegible]. Pag. 115. lin. 10. uuu ttt *for* uuu + ttt. lin. 11. *read* [illegible] =R. lin. last. *put in* F± [illegible]. Pag. 117. Equ. 23. a= *for* b=. Pag. 120. Equ. lowest, 70 *for* 73. Pag. 121. lin. 8. alled *for* called. Pag. 122. Equ. 92. margin —11 *for* —$\overline{11}$. Pag. 124. lin. 5. *In the margin write* Borrowed from pag. 116 and 117. Pag. 126. lin. 14. 4 Q *for* q Q. Pag. 127. lin. 6. do do *for* doe. Pag. 128. lin. 29. 2304 *for* 2034. Pag. 129. lin. 17. aditions *for* additions. Pag. 131. lin. 12. X_3^3 *for* X^3. Pag. 132. lin. 5. each other *for* each of the other. Pag. 133. Equ. 48. = *for* a=. Pag. 137. lin. 18. hight *for* height. lin. 20. *read* and in Perimeter. Pag. 138. lin. 3 . then *for* . Then, lin. 10. Isoscels *for* Isosceles. Equ. 15. Let =b *for* Let b=. Pag. 139. lin. 4. 1129 *for* 1229. Pag. 141. lin. 24. *read* that had been found. Pag. 142. lin. 10. sought *for* found. lin. 28. say *for* lay. lin. 29. *read* of those 24 of the. Pag. 143. lin. 7. —z^4 *for* +z^4. Pag. 144. lin. last, 10yyzz *for* —10yyzz. Pag. 145. lin. 7. 79 *for* 70. habitues *for* habitudes. Equ. 85. *read* 4yz= $\frac{V-T}{2}$. Pag. 146. lin. 3 and 5. Repetion *for* Repetition. lin. 18. *read* 5. 28. lin. 15. *put out* are. Pag. 147. lin. 1. *read* ...tiones of $\frac{y}{z}$ together. lin. 7. *read* Divisor Which ... lin. 9. —2yx *for* —2yz. lin. 21. *read* of the following. Equ. 109. *read* AD=BB. Pag. 148. lin. 2. *read* :: y. z, or $\frac{y}{z}$. Pag. 149. lin. 3. *put out* y>o. lin. 4. *write* y>o. z>2y. z=2y+1. Pag. 152. lin. 22. depend *for* depends. Pag. 154. lin. 19. a=15yy *for* 5yy. Pag. 156. lin. 11. *read* L>200,000, that is, L is *not greater than* 200,000. lin. 13. to all six *for* to all the six. lin. 16. DD=0 *for* DD>0. lin. 26. $143^{uu}2$ *for* 140uu2. Pag. 157. Equ. 195. *read* y>7 [illegible]† Equa. 207. *read* y>4 [illegible]† lin. penult. *read* and say. Pag. 158. lin. 9. 173250 *for* 173205. lin. 17. *read* Divisor is 1. lin. 19. required *for* rejected. lin. 23. cost *for* cast. Pag. 162. lin. 1. 39 *for* 38. lin. 27. 9348 *for* 6348. lin. 29. 1268 *for* 1368. Pag. 164. lin. 4. 1438 *for* 1488. Pag. 166. lin. last. 82 *for* 88. Pag. 167. lin. 7. 261 *for* 264. Pag. 168 lin. 18

and

and 19. *read* an answer: But giving no signe that he had thought of the Adherent. Pag. 169. lin. 23. *read* $s=\frac{n}{m}$ lin. 24. *read* $k+d=\frac{n}{s}=\frac{g}{b}$ Pag. 171. lin. 32. ratlo *for* ratio. lin. 35. it should be $e=\frac{4yz}{24}=\frac{1}{6}yz$. Pag. 172. lin. 23. p q *for* p-q. lin. 24. *read* therefore this Radius is. Pag. 174. lin. 14. 36 other *for* 37 other, lin. 36. 2|4.11| *for* 1|4.11| Pag. 176. Equ. 54. zz—2zy *for* zz>2zy. Pag. 177. lin. 19. divided by z *for* divided by 2. lin. 23. 21 ⊖ 2 *for* 91 ⊖ 2. Pag. 178. lin. 7. 3=a *for* 3a. lin. 19. al *for* all. Pag. 179. lin. 8. *read* 4.15|2. lin. 13 *read* You have them all in ... lin. 20. 80. 27. *for* 10. 27. Pag. 181. lin. 1. *read* pag. 178. Pag. 181. lin. 17. Equicrura *for* crural. Equat. 114. b+D *for* B+b. lin 31. transferred *for* transferred. Pag. 183. lin. 12. 34025 *for* 24025. lin. 24. 73 *for* 37. lin. 30. 79729 *for* 76729. Pag. 185. lin. 1. + *for* ÷. lin. 19. which b to be less *for* which makes b to be less. lin. 20. inverted *for* inverting. lin. 27. *read* 7.3.5.8. Pag. 186. lin. 28. *read* 2.13. 169. lin. 35. 177 *for* 117. Pag. 187. lin 2. y. z *for* *y. z.* [Roman *for* Italick.] lin. 28 and 29. *2t. 2a. for* 2t, 2a. [Italick *for* English] lin. 31. t—a *for* t—a. Pag. 189. Equ. 27. 16Sff *for* 16ff. Pag. 191. Equ. 21. 21÷4 *for* 20÷4. Equat. 19. margin. 13+187 *for* 18+187. Pag. 192. lin. 5 ∵=SS401 *for* SS=2401. Pag. 193. lin. 11. runing *for* running. lin. 19. 1767 *for* 1667. lin. 20. 1667 *for* 1767. Pag. 194. lin. 1. *Put out the full point after* Composit. Pag. 195. lin. 6. 2.27 *for* 2.72. lin. 19. 9.12 *for* 8.12. lin. 21. 14.|aabc *for* 10|aabc. lin. last, 3.66 *for* 6.36. Pag. 196. lin. 3. Whefore substract *for* Wherefore subtract. lin. 10. Divident *for* Dividend. Pag. 197. *On the Top write* Tariffs. lin. 2. √10000 *for* √ 100000. lin. 15. 256 *for* 356. lin. 29. 1061 *for* 2061. Pag. 198. lin. 1. Guildin for Guldin. lin. 2. *Put out the full point after* Catalog. lin. 24. 91793.23.17. *for* 23.13.

Pag. 1. of the Table, Make a stroke over 13 in column 3, and under 19 in column 5. to shew that 169 and 361 are there squares; according to pag. 196. lin. 8.

A N

AN INTRODUCTION To ALGEBRA.

First Part.

Algebraick Arithmetick.

CHAP. I.

ALGEBRA Consisteth of Two Parts, { *Arithmetical Operations*, & *Their Application to Particular Use*.

§ I.

ACcording to *Vieta*'s Way, we represent the concerned Quantities by Letters. As [*a*, *b*, *c*, *d*,] stand for four several Quantities; whereof we call one [*a*,] another [*b*,] &c. But when any Quantity is taken more than once, then we prefix its Number; as [2 *a*, 5 *b*, 7 *c*, $2\frac{1}{2}$ *d*,] stand for (*a*) taken twice, (*b*) five times, &c. And these Prefixed Numbers *Vieta* cals *Coëfficients*. But if no Number accompany the Quantity, then such Quantity stands for its self once taken, or supposing an *Unity* prefixed, as [a is 1 a.]

Signs.

Concerning the *Signs* wherewith quantities be connected, Note that they ever belong to the quantities which follow them. Also that when no Sign is prefixed to a quantity, it is as if + *i. e. plus* were expressed, as [*a*|is+ *a.*] But if unlike and several quantities be engaged, they may stand in any order at pleasure, as [$a+b-c$, or $a-c+b$, or $-c+b+a$ &c.

There are in *Arithmetick* six distinct single *Operations*, all which are performed in *Algebra*, viz. *Addition*, *Substraction*, *Multiplication*, *Division*, *Involution*, *Evolution*; which six Works arise from six several waies of comparing quantities with quantities.

Addition is putting unlike to unlike,

$$\begin{array}{ccccc} 2 & & 2a & & a \\ 3 & \cdot & 3a & \cdot & b \\ \hline 5 & \cdot & 5a & \cdot & a+b \end{array}$$

Substraction is taking unlike from unlike.

$$\begin{array}{ccccc} 3 & & 3a & & a \\ 2 & \cdot & 2a & \cdot & b \\ \hline 1 & \cdot & a & \cdot & a-b \end{array}$$

Multiplication is putting like to like once, or oftner.

$$\begin{array}{ccc} 3 & & 3a \\ 3 & \cdot & 3a \\ & & 3a \\ \hline 6 & & 9a \end{array}$$

Division is to take the same quantity from another as oft as may be; as

$$\begin{array}{cccccc} 6 & & 4 & & 2 & \\ 2 & \cdot & 2 & & 2 & (3 \\ - & & - & & - & \\ 4 & & 2 & & 0 & \end{array}$$

Involution is multiplying any quantity into its self, and that Product into the first quantity, and this last into the first, *&c.*

$$\begin{array}{cccccc} 2 & & 4 & & 8 & \\ 2 & \cdot & 2 & \cdot & 2 & \&c. \\ - & & - & & - & \\ 4 & & 8 & & 16 & \end{array}$$

Evolution is the discovery of the Ingredients of the involved quantity.

$$\begin{array}{cccc} 8 & 6 & 4 & 2 \\ 2 & 2 & 2 & 2 \\ - & - & - & - \\ 6 & 4 & 2 & 0 \end{array}$$

But we shall handle them distinctly.

§ 2.

Addition in Single whole Quantities.

The *Sign* for *Addition* is + *i.e. Plus*, more.

Rule I.

If the ingredient Quantities be like, add the Number prefixed, and adjoyn the Quantity.

a	$-2b$	$5a$	$-5f$	$6ch$	$+7ax$	1	
a	$-b$	$3a$	$-2f$	$12ch$	$+12ax$	2	
$2a$	$-3b$	$8a$	$-7f$	$18ch$	$+19ax$	3	1+2

N. The *Margin's* use is, that the Numbers set just under each other in the *Small Column* shew what quantities are to be added; (as here 1 and 2;) But those in the *Larger Column* shew what shall arise (*viz.* their *Sum.*) So [1+2] signifies the *Sum* of the quantities in the first place, added to the quantities of the second. *Which Sum* stands in the third place; as above, [$2a$, $-3b$, &c.]

Also when the Numbers in the *Broad Column* are without a *Line* over the Head, it notes their relation to the Numbers of the *Small Column*: but if they have a Line thus ($\bar{2}$) they are absolute Numbers (such as were not engaged before in the work) which must now be Added, Substracted, *&c.* according to their Sign.

Rule II.

When like quantities with unlike Signs are to be added, substract the prefixed Numbers from each other, and adjoyn the Sign of the greater.

$+3a$	$-3b$	$+5f$	$-6ch$	$+6abcd$	1	
$-2a$	$+b$	$-7f$	$+7ch$	$-4abcd$	2	
$+a$	$-2b$	$-2f$	$+ch$	$+2abcd$	3	1+2

Rule III.

When the quantities be unlike, set them down without altering their Signs, and thence comes a *Binomial.*

	1	a	a	$+d$	$+5f$
	2	b	b	$-r$	$-7fr$
	3	o	x	$-t$	$+3dc$
1+2+3	4	$a+b$	$a+b+x$	$d-r-t$	$5f-7fr+3dc$

	1	$-6z$	$-3x$	
	2	$-8az$	$+xx$	
	3	$+fx$	o	
1+2+3	4	$fx-6z-8az$	$xx-3x$	

$+$ to $+$ } are to be added, and the sum is { $+$ } *viz.* the same Sign.
$-$ to $-$ } { $-$ }

$+$ to $-$ } are to be substracted & leave a remainder, which is here called their sum. { G } the Sign of the greater.
$-$ to $+$ } { G }

§ 3.

Substraction in Single whole Quantities.

The Sign for *Substraction* is $-$ *i. e. Minus*, or the Negative Sign.

Rule I.

When the Quantities be like, and the Substrahend less, then substract the prefixed numbers, and to the Remainder annex both the upper Quantity and its Sign.

	1	a	$-2b$	$5a$	$-6chm$	$7axr$	$-bmp$
	2	a	$-b$	$3a$	$-chm$	axr	$-bmp$
1 − 2	3	o	$-b$	$2a$	$-5chm$	$6axr$	o

Rule II.

Rule II.

If, in this case, the Subtrahend be greater, substract (as before) the prefixed Numbers, but to the Residue prefix the *contrary* Sign.

$3a$	0	$-8c$	$-6chm$	1	
$5a$	$-2a$	$-10c$	$-20chm$	2	
$-2a$	$+2a$	$+2c$	$+14chm$	3	1 — 2

$7arx$	0	1	
$50arx$	$-18brs$	2	
$-43arx$	$+18brs$	3	1 — 2

Rule III.

If, in the same case, the Ingredients have *different* Signs, then add the prefixed Numbers and prefix the Sign of the upper.

$+8x$	$-4ar$	$+2ab$	$-2ab$	$-4abc$	1	
$-4x$	$+7ar$	$-2ab$	$+2ab$	$+ abc$	2	
$+12x$	$-11ar$	$+4ab$	$-4ab$	$-5abc$	3	1 — 2

Rule IV.

If the Quantities be unlike, let the upper stand with its Sign, but subjoyn to it the Subtrahend with its contrary Sign, whence comes a *Residue*.

a	$-2b$	$+2kd$	1	
b	$-3a$	$-4R$	2	
$a-b$	$3a-2b$	$2kd+4R$	3	1 — 2

$+6bbr$	$+3zzx$	1	
$+7cc$	$-7yy$	2	
$6bbr-7cc$	$3zzx+7yy$	3	1 — 2

+ from

+ from + / — from —	the Subtrahend less, subtr. and in the Residue	+ / —	Like Sign.
+ from + / — from —	the Substrahend greater, substract, and in Residue	— / +	the Contrary Sign.
— from + / + from —	adde, and the Sum is the Residue with	up / up	the Sign of the upper.

§ 4. *Multiplication in Single whole Quantities.*

The *Sign* of Multiplication is [*] *i. e.* multiplied with.

R. I. If the quantity have no annexed Numbers, set them immediately together, whether like or unlike.

	1	*a*	*ac*	*afg*
	2	*b*	*d*	*gh*
1 * 2	3	*ab*	*acd*	*afggh*

	1	*aaxy*	*rstu*
	2	*az*	*xyz*
1 * 2	3	*aaaxyx*	*rstuxyz*

R. II. If there be annexed Numbers, set the Quantities immediately together, and prefix the *Product* of the Numbers.

	1	2*a*	3*ac*	*afg*	*aaxy*	135*rstu*
	2	*b*	2*d*	5*gh*	2*az*	3*xyz*
1 * 2	3	2*ab*	6*acd*	5*afggh*	2*aaaxyz*	405*rstuxyz*

For

For the Sign, $\left\{\begin{array}{l}\text{like with like}\\ \text{unlike with unlike}\end{array}\right\}$ give Product $\left\{\begin{array}{c}+\\ -\end{array}\right\}$

$2b$	-5	$-8g$	*	1	
c	$+4f$	$-10h$		2	
$2bc$	$-20f$	$+80gh$		3	1 * 2

*	$+14xy$	$-135rs^3$	1	
	$-z$	$-4r^2s$	2	
	$-14xyz$	$+540r^3s^4$	3	1 * 2

N. Numbers *prefixed* to any Quantity ſhew how oft it is to be taken; but ſet at the *Head* of a Quantity, they ſhew a *Power* or *Degree* of that quantity, *viz.* *Quadratick*, *Cubick*, *Biquadratick*, &c.

As let [a] be a Line, [a^2] is the ſecond *Poteſtas*, or the *Square*, [a^3] is the third *Power* or *Cube*, &c.

So $a^7b^5c^8 = aaaaaaa\,bbbbb\,cccccccc$. And therefore uſed by Modern *Analyſts* for *Compendium*.

Diviſion in Single whole Quantities. § 5.

Its *Sign* is [$\div$] *i. e.* divided by.

R. I. When the Quantities have no prefixed Numbers, caſt away that in the Dividend which is like the Diviſor; as,

ab	$cdef$	xyz	hmn	$pqrſt$	1	
a	ef	z	hn	$pqrſt$	2	
b	cd	xy	m	1	3	$1 \div 2$

When the Dividend and Diviſor are equal, the Quotient is 1.

R. II. When

R. II. When the Divisor is not found in the Dividend, draw a Line under the Dividend, and subscribe the Divisor; and such Quotient is called a *Fraction*, or broken Number.

	1	ab	de	hm	yz	efg	n
	2	c	fg	n	yx	edg	hm
1 ÷ 2	3	$\frac{ab}{c}$	$\frac{de}{fg}$	$\frac{hm}{n}$	$\frac{z}{x}$	$\frac{f}{d}$	$\frac{n}{hm}$

R. III. If there be Numbers prefixed to the Ingredients, divide the Number, as in *Vulgar Arithmetick*, and set the Quotient before the Quantities.

	1	$4g$	$4gh$	$4gh$	$3gh$	$30b^5c^4dd$	fq
	2	$2h$	$2gh$	$2h$	2	$5bbccd$	$3bq$
1 ÷ 2	3	$\frac{2g}{h}$	2	$2g$	$\frac{3gh}{2}$	$6b^3ccd$	$\frac{f}{3b}$

R. IV. For the Signs. Like Signs give +. Unlike give — in Quotient.

	1	$+4ff$	$-9hik$	$+6gh$	$+6gh$	fq
	2	$+2fg$	$-3hi$	$-3gh$	-2	$-3bfq$
1 ÷ 2	3	$\frac{+2f}{g}$	$+3k$	-2	$-3gh$	$\frac{1}{-3b}$

Involution

Involution in Single whole Quantities. §. 6.

Its *Sign* is [⊛]

R. If a Quantity be involved or drawn into it self, or afterward into that Product, or again thirdly, into the last Product, *&c.* as manifold as is the *Power*, so great must the *number* be that is used to express it, being set after the Sign of Involution in the broader Column of the Margin.

$-a$	$+ab$	bcd	yx	$2yy$	*	1	
$+aa$	$+aabb$	$bbccdd$	$yyxx$	$4y^4$		2	1⊛2.
$-a^3$	$+a^3b^3$	$bbbcccd^3$	y^3x^3	$8y^6$		3	1⊛2.
$+a^4$	$+a^4b^4$	$b^4c^4d^4$	y^4x^4	$16y^8$		4	1⊛4.

	$3yz$	$abcd$		
z	$9y^2z^2$	$aabbccdd$	2	1⊛2
z	$27y^3z^3$	$a^3b^3c^3d^3$	3	1⊛3
z	$81y^4z^4$	$a^4b^4c^4d^4$	4	1⊛4

The Meaning is, let [$3yz$] be a *Line*, this twice involved gives [$9yyzz$] and is called a *Quadratick* Power or *Square*: Three times involved, it gives [$27yyyzzz$] which is a *Cubick*
z Power, or *Cube*: Four times *involved*, it gives [$81y^4z^4$] which is a *Biquadratick*, &c.

Evolution, or Extraction of Roots out of Single whole Quantities. § 7.

The *Sign* of this Operation is [ω]

R. I. If the Quantity have a Root, divide the Number of it's Power by 2, if you desire the Sqnare Root, or by 3, if you seek the Cubick Root, *&c.* and the Quotient will give you the desired Root; and deal with the prefixed Numbers as in *Vulgar Arithmetick*. But if either out of the

adjoyning Number, or the Numbers of its Power no exact Root can be extracted; or if the Quantity is on any other Account unfit for Evolution, then prefix before it the Sign of the desired Root, and thence ariseth a *Surd Quantity*. As,

	1	a^4	$216d^3$	abc	$36kk$
1 ω 2	2	aa	$\surd 216d^3$	$\surd abc$	$6k$
1 ω 3	3	$\surd c.a^4$	$6d$	$\surd c.abc$	$\surd c.36kk$
1 ω 4	4	a	$\surd\surd 216d^3$	$\surd\surd abc$	$\surd 6k$

	1	$81ff$	$8ggg$
1 ω 2	2	$9f$	$\surd 8ggg$
1 ω 3	3	$\surd c.81ff$	$2g$
1 ω 4	4	$\surd 9f$	$\surd\surd 8ggg$

The Sign $\left\{\begin{array}{l}\surd\\ \surd c\\ \surd\surd\\ \surd cc\end{array}\right\}$ denoteth the $\left\{\begin{array}{l}\text{Square}\\ \text{Cubick}\\ \text{Biquadratick}\\ \text{Bicubick}\end{array}\right\}$ Root.

N. Evolution in quadratick, biquadratick, and all the even powers supposeth the *Evolvend* to be affirmed, or to have + prefixed.

CHAP. II.

§ 1.

Addition in Compound Integers.

N. Forasmuch as in *Addition*, *Subtraction*, *Multiplication* and *Division* of compound Quantities, there chance many peculiar Difficulties, which cannot so readily be expressed in Rules, I shall make but one work of all by exhibiting Examples in the several Cases.

	1	$a+b$	$a+b+c$	$4d-f+3$	$6h-x+z\cancel{+}36$	—
	2	$a-b$	$b+c$	$-4d+2f-2$	$-h-x-2\cancel{z}+36$	z
1+2	3	$2a$	$a+2b+2c$	$f+1$	$5h-2x-z$	

Again.

Again.

$aa-2ab+bb$	$a+b-c+8$	$a^3-3aab+3ab^2+b^3$	1	
$4ab$	$-a+b+c-1$	$+6aab \quad -2b^3$	2	
0	$2a-b-2c+3$	0	3	
$aa+2ab+bb$	$2a+b-2c+10$	$a^3+3aab+2abb\cancel{+}b^3$	4	$1+2+3$

$pqq+r^2s+240$	1	
$-rrs+566$	2	
$g+b$	3	
$pq^2+g+b+806$	4	$1+2+3$

Subtraction in Compound Integers. § 2.

$a+b$	$aa+2ab+bb$	$a^3+3aab+3abb+bbb$	1	
$a-b$	$+4ab$	$+6aab \quad +2bbb$	2	
$2b$	$aa-2ab+bb$	$a^3-3aab+3abb-bbb$	3	$1-2$

$2a+3c-5x-860$	1	
$a+4c-5x+55-f$	2	
$a-c-915+f$	3	$1-2$

Multiplication in Compound Integers. § 3.

$2a+3f$	$2a-3f$	$a-b$	$aa+ff$	1	
d	d	cd	$-de$	2	
$2ad+3df$	$2ad-3df$	$acd-bcd$	$-aade-deff$	3	$1*2$

$-a+b-c+3$	1	
$-2f$	2	
$2af-2bf+2cf-6f$	3	$1*2$

Again.

	1	$a+b$	$aa+ba$
	2	$a-b$	$a-b$
1 $\ast$ 2	3	$aa(\ast)-bb$	$a^3(\ast)-abb$

	1	$aa-b-c$	$a+c+e$
	2	$b+c$	$a+c-e$
1 $\ast$ 2	3	$aab+aac-2bc-cc-bb$	$aa+2ac+cc-ee$

Such Operations stand more at length, thus;

$a+b$	$aa+ab$
$a-b$	$a-b$
$aa+ab$	$aaa+aab$
$-ab-bb$	$-aab-abb$
$aa\ (\ast)-bb$	$aaa\ (\ast)\ -abb$

$aa-b-c$	$a+c+e$
$b+c$	$a+c-e$
$aab-bb-bc$	$aa+ac+ae$
$+aac\quad -bc-cc$	$+ac\quad +cc+ce$
	$-ae\quad -ce-ee$
$aab+aac-2bc-cc-bb$	$aa+2ac+cc-ee$

N. 1. Here comes in the Sign ($\ast$) noting the Defect of a Degree or Place of Power, as in the second Example between [a^3 and $-abb$] there is wanting the Quadratick Power [aab] which in the Addition of contrary Signs was expunged.

N. 2. Sometimes to avoid the tediousness of Operation, one may omit the Process, and only set down thus,

$$\left.\begin{array}{l} a+b \\ a-b \end{array}\right| \text{ Or, } a+b \ast a-b.$$

Division in Compound Integers. § 4.

$2ad+3df$ / d	$2ad-3df$ / d	$acd-bcd$ / cd	$aade-deff$ / de	1 / 2	
$2a+3f$	$2a-3f$	$a-b$	$aa-ff$	3	$1 \div 2$

$2af-2bf+2cf-6f$ / $2f$	1 / 2	
$a-b+c-3$	3	$1 \div 2$

Again.

$aa-bb$ / $a-b$	$aaa-abb$ / $a-b$	$aab+aac-2bc-cc-bb$ / $c+b$	1 / 2	
$a+b$	$aa+ab$	$aa-b-c$	3	$1 \div 2$

$aa+2ac+cc-ee$ / $a+\cancel{c}$	$c+e$	1 / 2	
$a+c\cancel{+}e$	$-$	3	$1 \div 2$

$c+e$
$-$

More at large thus.

$$\begin{array}{l} aa-bb \\ \underline{aa-ab} \\ ab-bb \\ \underline{ab-bb} \\ \quad 0 \end{array} \Big\} \begin{array}{l} a-b \\ a+b \end{array} \qquad \begin{array}{l} \cancel{aaa-aab} \\ \underline{\cancel{aaa-abb}} \\ aab-abb \\ \underline{aab-abb} \\ \quad 0 \quad 0 \end{array} \Big\} \begin{array}{l} (a-b) \\ aa+ab \end{array} \qquad \begin{array}{l} aab+aac-2bc-cc-bb \\ \underline{aab+aac} \\ b+c) \\ (aa-b-c \end{array} \Big\} \begin{array}{l} 0-2bc-cc-bb \\ \underline{-bc \quad -bb} \\ -bc-cc-0 \\ \underline{-bc-cc} \\ \quad 0 \end{array}$$

N. 1. Here also one may omit the whole process, and only set down thus, [$aa-bb \div a-b$] &c.

N. 2. When

N. 2. When the Dividend will not be just wrought off without any Remainder, then only set all Fraction-wise.

	1	$fg-ef$	$fg-ef$	$aa+2ab+dd$	$5x+6z-30$
	2	$y+x$	fh	$a+2b$	6
1 ÷ 2	3	$\frac{fg-ef}{y+x}$	$\frac{g-e}{h}$	$a+\frac{dd}{a+2b}$	$\frac{5}{6}x+z-5$

§ 5. *Involution in Compound Integers.*

	1	$a+b$	$a-b$
1 ⊙ 2	2	$aa+2ab+bb$	$aa-2ab+bb$
1 ⊙ 3	3	$aaa+3aab+3abb+bbb$	$aaa-3aab+3abb-bbb$

N. In the Examples [$aa+2ab+bb$] and [$aa---2ab+bb$] we may note that [aa] and [bb] arise out of the Involution of ***Half the Quantity*** [$2a$] and [$2b$] of the middle place, [$2ab$] (the Square of $\frac{1}{2}2a$ is aa, and the Square of $\frac{1}{2}2b=bb$.) Whence it apppears, how (when the first and middle place be given) the third place may be found also, and annected to fill up the Square, that thence a Root may be extracted. As let [$bb--12b$] be given. Here [b] is compleat already, and the Defect belongs to the Number [12.] The half of which is 6, and the Square of 6 is 36; which adde to [$bb--12b$] thus [$bb--12b+36$] which is a compleat Quadrate, and its Root [$b--6$]

Let the Sign of such Completion be [C□] which is as much as to say, ***Compleat the Square.***

As

As for Example.

$aa+2c$	$bb+bf$	$mm+9m$	1	
$aa+2c+cc$	$bb+bf+\frac{1}{4}ff$	$mm+9m+\frac{81}{4}$	2	C □

$qq+4pq-6rq$		1	
$qq+4pq+4pp$ $--6rq--12pr+\cancel{9}9rr$	Its Root, $q+2p--3r$	2	C □

Evolution of Compound Integers. ∂ 6

FRom *Involution* we learn, that let $[a+b]$ be a Line, then the Square of it is $[aa+2ab+bb]$ and its Cube $a^3+3aab+3abb+b^3]$. These things give us a direction how to do in *Evolution* (according to the Rule, *viz. Genesis and Analysis are the Converse of each other*, and consequently performed by converse Operations.)

But it would make for our clearer apprehending the work, to set before the Eye the manner of Evolution in Numbers, and then just so is Evolution in *Algebra* performed.

Let the Number to be evolved be 50176.

1 Quotient	1	a	02	22	
$1 * 2$	2	$2a$	04	44	
2 Quotient	0	b	02	4	
$3 * 2$	4	$2ab$	8	17	6
3 ☉ 2	4	bb	04	..	16
$4+5$	6	$2ab+bb$	84	17	76

The first Subducend is $aa=4$.

Beginning at the right hand *distinguish*, or part the Number, allowing to each part two Figures, (as 5,01, 76,) according to the manner of *Quadratick* Powers. (But allow 3 to a part if the *Cube Root* be required, as 11,239,424,) Then take

take the Root of the greateſt Square that is contained in the firſt part (toward the left hand) as $2=a$ in the Table. *Invo've* it twice, it gives $4=aa$; which *Subtract* from the firſt Part, or Diviſion, there remains 10176. *Double the firſt Root*, and ſet it under the ſecond Part, ſo as to leave ſtill one Figure of that Part. as 1,01 And with that divide, the Quotient is $2=b$ in the Table. Now [a] the firſt Quotient was 2, and $2a=4$. Then $2ab=8$ and $bb=4$ which muſt be ſet in the Table one place further towards the *Right Hand* (*i. e.* next to 8 thus 84.) Wherefore $2ab+bb=84$, which ſubſtract from 10176, reſts 1776. ——Laſtly, *double* the whole Quotient (22) and proceed with this in all points, as with the *Former*, and ſo at laſt there will remain [0] and the Root will be 224.

101
4

The Work ſtands thus.

5	01	76	2
4)			
1	01	76	~~2~~
	4)		
	84		
	17	76	4
	4	4)	
	17	76	
	0	0	

2

On this Occaſion I have brought in firſt the uſe of the Sign of Equality = ſo $2a=4$. *i.e.* $2a$ be equal to 4.

Note that in this Table the firſt Operation takes in one Column of two Numbers, the Second goes to two Columnes, and ſo if more Works were needful, more Columnes would be employed.

Example.

Example in *Cubick Evolution.*

Extract the Cube Root of 11239424.

a	02	022		
aa	004	004	84	The first
$3aa$	012	14	52	Subduct
$3a$	06	066		is $aaa=8$
b	2	4		
$3aab$	24	580	8	
$3abb$	24	10	56	
bbb	8		64	
$3aab+3abb+b^3$	2648	591	424	

1	1 Quotient
2	1 ⊙ 2
3	2 * $\overline{3}$
4	1 * 3
5	2 Quotient
6	3 * 5
7	□ 5 * 4
8	5 ⊙ 3
9	6 + 7 + 8

The Work stands thus.

```
1 1 | 2 3 9 | 4 2 4 |
  8 |       |       | 2
----------------------
  3 | 2 3 9 | 4 2 4 |
  1 | 2 )   |       | 2
  2 | 6 4 8 |       |
----------------------
    | 5 9 1 | 4 2 4 |
    | 1 4 5 | 2 )   | 4
    | 5 9 1 | 4 2 4 |
    |     0 |     0 |
----------------------
```

The Root then is 224.

Examples in *Algebra.*

1 Quotient.
1 * 2
2 Quotient.
2 * 3
3 ⊖ 2
4 + 5

Extract the square root out of $cc+2cd+dd+2ce+2de+ee.$

1	a	c	$c+d$
2	$2a$	$2c$	$2c+2d$
3	b	d	e
4	$2ab$	$2cd$	$2ce+2de$
5	bb	dd	$+ee$
6	$2ab+bb$	$2cd+dd$	$2ce+2de+ee$

The first Subduct is $aa=cc$ and the Work stands thus,

$cc+2cd+dd+2ce+2de+ee$ cc		c
$2cd+dd$ $2c)$ $2cd+dd$	$+2ce+2de+ee$	$+d$
	$2ce+2de+ee$ $2c+2d)$ 2ce $\cancel{2ce}+2de+ee$ 0	$+e$

So the Root desired is $c+d+e$

1 Quotient.
1 * 2
2 Quotient.
2 * 3
3 ⊖ 2
4 + 5

Extract the Square Root out of $cc+2ce+ee-2cd-2de+dd.$

1	a	c	$c+e$
2	$2a$	$2c$	$2c+2e$
3	b	e	$-d$
4	$2ab$	$2ce$	$-2dc-2de$
5	bb	ee	$+dd$
6	$2ab+bb$	$2ce+ee$	$-2dc-2de+dd$

The first Subduct is $aa=cc$, and the work stands thus.

$cc+2ce+ee-2cd-2de+dd$ cc	c
$2ce+ee-2cd-2de+dd$ $2c)$ $2ce+ee$	$+e$
$-2cd-2de+dd$ $+2c+2e)$ $-2cd-2de+dd$	$-d$
$0\ 0\ 0\ 0$	

The Root hence extracted is $c+e-d$

So

So for the $\sqrt{c}$.of $64d^3-96dde+48de^2-8e^3+144d^2f-144def+36e^2f+108df^2-54ef^2+27f^3$

The first Subduct is $64ddd=aaa$.

1 Quot.	1	a	$4d$	$4d-2e$
1 ⊙ 2	2	aa	$16dd$	$16dd-16de+4ee$
2 * 1	3	$3aa$	$48dd$	$48dd-48de+12ee$
1 * 1	4	$3a$	$12d$	$12d-6e$
2 Quot.	5	b	$-2e$	$+3f$
5 ⊙ 2	6	bb	$4ee$	$9ff$
3 * 5	7	$3aab$	$-96dde$	$144ddf-144def+36eef$
4 * 6	8	$3abb$	$+48dee$	$+108df^2-54eff$
5 ⊙ 3	9	bbb	$-8eee$	$+27f^3$
7+8+9	10	$3aab-3abb+b^3$	$-96d^2e+48de^2-8e^3$	$144ddf-144ddf+36eef+108dff$ $(-54eff+27fff$

The Work stands thus,

$64ddd-96dde+48dee-8eee+144ddf-144def+36eef+108dff-54eff+27f^3$ | $4d$

$64ddd$

o $-96dde+48dee-8eee$ | $-2e$

$+48dd)$

$-96dde+48dee-8eee$

o o $+144ddf-144def+36eef+108dff-54eff+27f^3$ | $+3f$

$+48dd-48de+12ee)$

eef $+144ddf-144def+36e^2f+108df^2-54ef^2+27f^3$

oo oo

So the Cubick Root sought is $4d-2e+3f$.

And thus may the Roots be extracted out of all Quantities that are of equally multiplied Powers (that be a *just square Cube*, &c.) sc. Let [$a+b$] be so often involved as is the Number of the Power of the given Quantity, according to which also the *Margin* is to be supplied. And this way will be very useful in resolving Equations that are affected, as we shall see in its due place.

CHAP. III.

Of Fractions or Broken Quantities.

§ 1. WHat is done about Fractions in Vulgar Arithmetick, all that is also here wont to be performed.

R. I. If the *Numerator* and *Denominator* be equal, the Quotient is an *Unity*. $\left\{ \frac{aa}{aa} = 1 \quad \frac{abc+dde}{abc+dde} = 1. \right.$

R. II. When an *Integer* is to be expressed Fraction-wise, give it an *Unity* for its Denominator: As,

$$\frac{aa}{1} = aa \quad \text{Also} \frac{abc+dde}{1.} = abc+dde$$

R. III. To bring an *Integer* to a *Fraction* of a given Denominator, multiply both together for the Numerator, and set the given quantity for the Denominator: As,

$$\frac{ad}{d} = a \quad \frac{bcd}{bc} = d \;\Big|\; \frac{bc+dc}{b+d} = c \;\Big|\; \frac{bc+bb}{c} = b + \frac{bb}{c}.$$

R. IV. The

R. IV. The whole Fraction is multiplied by so much as is cast away out of the Denominator: As,

$$\frac{abc}{de} * de = abc \quad \frac{3cf}{2de} * 2d = \frac{3cf}{e} \quad \frac{3cf}{2de} * 2def = 3cff.$$

Reducing Fractions to a Lower Denomination.

§ 2.

R. I. WHen the same Quantities be found in Num. and Denominat. let them be cast away in both, and the Fraction will still remain of the *same Value*.

$$\frac{abc}{abd} = \frac{c}{d} \qquad \frac{abc}{ade} = + \frac{bc}{de} \qquad a + \frac{def}{ef} = a + d$$

R. II. The greatest *Common Divisor* brings the Fraction to the lowest Denomination. And that Divisor is thus found: Divide the Denominator by the Numerator, and the Numerator by the Residue, and so on, still divide the Last Divisor by its Remainder, till the Division *work off*. By the Last Divisor divide the Numerat. and Denominator of the Fraction severally, and thus take down the Fraction to the lowest Denomination.

As thus. Let the Fraction be $\frac{ac+bc}{aa+2ab+bb}$

Num.

Numerat. / *Denominat.*	1	$ac+bc$
	2	$aa+2ab+bb$
$2 \div 1$	3	$\frac{a}{c}+\frac{b}{c}$
Rest.	4	0
$3 * c$	5	$a+b$ the greatest common Measure.
$1 \div 5$	6	c *Num.* } of the Fraction depressed.
$2 \div 5$	7	$a+b$ *Den.* }

The Fraction reduced stands thus $\frac{c}{a+b}$ So

$$\frac{dn-fn+dm-fm}{am+an+bm+bn+cm+cn}=\frac{d-f}{a+b+c} \text{ Com. Meas. } n+m$$

Also,

$$\frac{bdd+bee-cdd-cee}{bb-cc}=\frac{dd+ee}{b+c} \text{ Com. Meas. } b-c$$

But since this way is sometimes tedious, another way, in the reduction of *Surd Quantities*, hath been invented, whereby all the *Partes aliquota* (or such as being Divisors leave no Remainder) are discovered.

When after all means used, the Division will not work off, then were the Num. and Denominator *Prime* to each other, *i. e.* they have no *common measure*.

§ 3. *To find the smallest Quantitie*, which by 2 or more others given is just divided.

Let

Let the given Quantities be [abc and ad.] Set them Fracti on-wise, and (if that can be done) reduce to less Denomina tion: as,

$$\frac{abc}{ad}=\frac{bc}{d}$$

Multiply these Fractions *Cross-wise*, and the Product is $ab\,cd$, which is the smallest Quantity, which will be divided by [abc and ad,] so as to leave no remainder. This Work is noted with a Cross, thus [*X.]

Let the Quantities given be $\frac{bdd+bee-cdd-cee}{bb-cc} * \mathsf{X} \frac{dd+ee}{b+c}$

The smallest Quantity desired is $bbdd-ddcc-ccee+bbee$.

But if the Quantities will not be brought to lower Terms, then multiply them into each other, and their Product is the Quantity desired.

So $bb+cc*dd+ee$ gives $bbdd+ccdd+bbee+ccee$.

When three or more quantities are given, they must first be reduced to their lowest terms, and then multiplied as is aforesaid.

How to reduce Fractions of diverse, into one Denomination. § 4.

R. I. IF the Fractions will be reduced to lower Terms, then seek, as aforesaid, the smallest quantity, which will be just divided by the Denominators of the Fractions, this is your *common Denominat.* Then for your Numerators, divide the common Denominator by the Denominator of the first Fraction, and multiply that Quotient by the Numerator of the first Fraction, which is your new Numerator; and so for the other Numerators.

$\frac{bbfff}{abc}$ and $\frac{ghhmm}{ad}$ stand under one Denominator thus; $\frac{bbdfff}{abcd}$ & $\frac{bcghhmm}{abcd}$.

X

R. II. If

R. II. If the Fractions cannot be brought to lower Terms, multiply the Denominators continually for a new Denominator, and the Numerators cross-wise for new Numerators, as in Vulgar Fractions.

So $\frac{a}{b}$, $\frac{c}{d}$, $\frac{2ef}{g}$, stand thus reduced $\frac{adg}{bdg}$, $\frac{cbg}{bdg}$, $\frac{2bdef}{bdg}$.

§ 5

Addition and Subtraction of Fractions.

R. I. IF the Fractions be of one Denomination, add or subtract their Numerators.

1	Examples in Addition	$\frac{5d}{c}$	$\frac{5d-7x}{a+b}$	$b+\frac{c+d}{e+f}$	$\frac{a-b+c-18}{a+7}$
2		$\frac{x}{c}$	$\frac{-d+x}{a+b}$	$\frac{c+d+g}{e+f}$	$\frac{a+b-c+18}{a+7}$
3 (1 + 2)		$\frac{5d+x}{c}$	$\frac{4d-6x}{a+b}$	$b+\frac{2c+2d+g}{e+f}$	$\frac{2a}{a+7}$

In

In Subtraction.

$\frac{5d}{c}$	$\frac{5d-7x}{a+b}$	$a-\frac{c+d}{c+f}$	$\frac{ac+ad}{c+d}$	1
$\frac{x}{c}$	$\frac{-d+x-r}{a+b}$	$b-\frac{c+d}{c+f}$	$\frac{aa+b}{c+d}$	2
$\frac{5d-x}{c}$	$\frac{6c-8x+r}{a+b}$	$a-b$	$\frac{ac+ad-aa-b}{c+d}=a+\frac{-aa-b}{c+d}$	3 1—2

R. II. If the Fractions be of *diverse Denominations*, first *reduce* them to one, and do as before, *R.* I.

Multiplication in Fractions.

R. I. Multiply (as in Vulgar Fractions) the Numerators and Denominators respectively into each other, for the *Product-Fraction*.

$\frac{2b}{d}$	$\frac{2a+b}{d-e}$	$\frac{3a+4}{b}$	$\frac{a+8}{rr}$	$\frac{a-8}{xy}$
$\frac{3c}{e}$	$\frac{de}{d+e}$	$\frac{3a-4}{x}$	$\frac{8-a}{ss}$	$\frac{8+a}{z}$
$\frac{6bc}{de}$	$\frac{2ade+bde}{dd-ee}$	$\frac{9aa-16}{bx}$	$\frac{64-aa}{rr\,ss}$	$\frac{aa-64}{xyz}$

R. II. Oftentimes there may be this *Compendium, viz.* To divide the Numerator of the one and the Denominator of the other by a common meaſure, when they are capable hereof.

	1	$\frac{bdd+bee-cdd-cee}{aa+2ab+bb}$	Contracted ſtand thus, $\frac{dd+ee}{a+b}$
	2	$\frac{ac+bc}{bb-cc}$	$\frac{c}{b+c}$
1 * 2	3	$\frac{cdd-cee}{ab+bb+ac+bc}$	

R. III. When an *Integer* is to be multiplied by a *Fraction* expreſs it *Fraction-wiſe* with an unity for its Denominator.

			Stand thus		
	1	$a+b$	$\frac{a+b}{1}$	$\frac{aa-ee}{1}$	$\frac{ab}{d}$
	2	$\frac{cd}{e}$	$\frac{cd}{e}$	$\frac{aa+ee}{fg+mn}$	$\frac{a+r+s}{1}$
1 * 2	3		$\frac{acd+bcd}{e}$	$\frac{a^4-e^4}{fg+mn}$	$\frac{aab+abr+abs}{d}$

R. IV. When either or both Ingredients are *mixed Quantities,* then *reduce* the *Integer* to the Denominator of the Fraction.

	1	$a+\frac{b+c}{d}$	Stand reduced thus, $\frac{ad+b+c}{d}$
	2	$d+\frac{f+k}{2e}$	$\frac{2de+f+k}{2e}$
1 * 2	3	$ad+b+c\,\frac{+aak+bk+ck+cf+bf+adf}{2ed}$	

R. V. If

R. V. If you *cast away* the Denominator of a Fraction, it is *all one* as if you had *multiplied* the Fraction by the Denominator.

$\dfrac{a+b+c}{aa-bb}$	$\dfrac{aa-rr+xy+fz}{a+b-c-r}$	$\dfrac{aa-af-x-y}{tt+bb+ll+mm}$	1	
$aa-bb$	$a+b-c-r$	$tt+bb+ll+mm$	2	
$a+b+c$	$aa-rr+xy+fz$	$aa-af-x-y$	3	1 * 2

Division in Fractions.

§ 7.

R. I. WHen the Fractions are of the same Denomination, divide the numerators by each other, and cast away the Denominator. $\dfrac{aabb}{d} \div \dfrac{ab}{d}$ gives ab.

R. II. But if there be a quantity in *one Numerator*, that is not in the *other*, remove *that quantity* into the place of the Denominator. $\dfrac{aabb}{d} \div \dfrac{abc}{d}$ gives quot. $\dfrac{ab}{c}$

R. III. If the Fractions be of *diverse Denominations*, the work is done by multiplying *cross-wise*.

$$\frac{ff-bb}{d+c} \div \frac{d-c}{ff+bb} \text{ gives quot. } \frac{ffff-bbbb}{dd-cc}$$

$$\frac{b^3-d^3}{b+d} \times \frac{bb-bd+dd}{fd} \text{ gives } \frac{b^3df-d^3df}{bbb+ddd}$$

R. IV. If

R. IV. If the Numerator will be contracted with the Numerator, or the Denominator with the Denominator (*i.e.* if they be *commensurable* by any common measure, and so will be abbreviated by it) let that Contraction be made for the ease of the Operation.

1		$\frac{bdd+bee+add+aee}{dm-fm+dn-fn} = = = = = = = \frac{dd+ee}{d-f}$
2		$\frac{aa+2ab+bb}{am+an+bm+bn+cm+cn} = = = = = = = \frac{a+b}{a+b+c}$
3	$1 \div 2$	$\frac{aee+bee+cee+add+bdd+cdd}{ad+bd-af-bf}$.

R. V. If Integers come to be ingredients, express them in form of a Fraction (as afore) by subscribing an Unity.

1		$a-b = \frac{a-b}{1}$	$a+xy = \frac{a+xy}{1}$
2		$\frac{c-d}{a+b} = \frac{c-d}{a+b}$	$\frac{ff+bb}{a-xy} = \frac{ff+bb}{a-xy}$
3	$1 \div 2$	$\frac{aa-bb}{c-d}$	$\frac{aa-xxyy}{ff+bb}$

Involution

Involution in Fractions.

§ 8.

This is done altogether as Multiplication.

$\frac{3a}{b}$	$\frac{2bc}{d}$	$\frac{a+b}{d-f}$	1	
$\frac{9aa}{bb}$	$\frac{4bbcc}{dd}$	$\frac{aa+2ab+bb}{dd-2df+ff}$	2	1 ⊚ 2.
$\frac{27aaa}{bbb}$	$\frac{8b^3c^3}{ddd}$	$\frac{a^3+3aab+3abb+b^3}{d^3-3ddf+3dff-fff}$	3	1 ⊚ 3

$\frac{a-b}{x+y}$	1	
$\frac{aa-2ab+bb}{xx+2xy+yy}$	2	1 ⊚ 2
$\frac{aaa-3aab+3abb-bbb}{xxx+3xxy+3xyy+yyy}$	3	1 ⊚ 3

Evolution

Q. 9.

Evolution in Fractions.

R. I. If the Numerators and Denominators have *just Roots*, set those *Roots* down *respectively* for the Numerator and Denominator of the *Evolute* Fraction (or Root.) But if they have no exact Roots, then prefix to the whole Fraction the *Sign* of the Root.

	1	$\frac{aabb}{cc}$	$\frac{a^3b^3}{c^3}$	$\frac{aa+2ab+bb}{aa-2ab+bb}$
1 ω 2	2	$\frac{ab}{c}$	$\sqrt{\frac{a^3b^3}{ccc}}$	$\frac{a+b}{a-b}$
1 ω 3	3	$\sqrt{c}\,\frac{aabb}{cc}$	$\frac{ab}{c}$	$\sqrt{c}\,\frac{aa+2ab+bb}{aa-2ab+bb}$

	1	$\frac{aaa+3aab+3abb+bbb}{aaa-3aab+3abb-bbb}$
1 ω 2	2	$\sqrt{\frac{aaa+3aab+3abb+bbb}{aaa-3aab+3abb-bbb}}$
1 ω 3	3	$\frac{a+b}{a-b}$

R. II. When

℞ II. When the Num. or Denomin. hath not any just Root, prefix to it the Sign of the Root intended. And such Fraction is a *Surd Quantity*.

$\frac{a^4 - 2aacc + c^4}{a+d}$	$\frac{a-f}{aa - 2ad + dd}$	1	
$\frac{aa - cc}{\sqrt{a+d}}$	$\frac{\sqrt{a-f}}{a-d}$	2	1 ω 2
$\sqrt{c}\,\frac{a^4 - 2aacc + c^4}{a+d}$	$\sqrt{c}.\,\frac{a-f}{aa - 2ad + dd}$	3	1 ω 3

$\frac{aaa + 3aab + 3abb + bbb}{aa - 2ad + dd}$	1	
$\frac{\sqrt{aaa + 3aab + 3abb + bbb}}{a-d}$	2	1 ω 2
$\frac{a+b}{\sqrt{c\,aa - 2ad + dd}}$	3	1 ω 3

And thus much of *Rational Quantities*, *Surds* follow.

CHAP.

CHAP. IV.

Of Surd Quantities.

Reduction.

§ 1.

R. I. THE *Surd Quantities* that have *unlike Radical Signs* must be reduced to one, thus; Find the *smallest Number* that will be *just* divided by the Numbers of the Powers of the given Quantities. Let $\surd a$ and $\surd c.a$ be two such quantities. Now $\surd$ is the Sign of the second Power, and $\surd c.$ the Sign of the third Power. Seek then the smallest Number which will be just divided by 2 and 3, which is 6. This 6 divided by 2 gives 3, by 3 gives 2 in the Quotient. And so from $\surd a$ (the second Power) comes [aaa] the third. and from $\surd c.a$ (the third Power) comes [aa] the second Power, and both these refer to the Root of the sixth Power, *sc. Cubocubick.*

The Square $\surd$ of a is the Cubo-cubick $\surd$ of aaa
The Cubick $\surd$ of a is the Cubo-cubick $\surd$ of aa.

R. II. If you would reduce any *rational quantity* to any *assigned* radical Sign, involve it as often as is the Number of the assigned Power, and prefix its Sign. (And this is of frequent use in Equations.) Thus [$b-d$] shall have the *same Radical Sign* with $\surd\,\overline{bb-d}$, if it be twice involved, and then it is thus, $\surd bb-2bd+dd=b-d$.

Again, [$b-d$] shall have the *same radical Sign* with $\surd c.\,bb-d$ if it be involved three times; so $\surd c.\ bbb-3bbd+3bdd-ddd=b-d$.

R. III. *Surd quantities* sometimes may be reduced to lower terms, if the quantities (placed under one radical Sign) be

be divided by ſome *Square* or *Cube*, &c. from whence they proceed. And then ſhall the Root of ſuch Cube: Square, *&c.* loſe its radical Sign; but the Quotient of ſuch Diviſion muſt be ſubjoyned after the Form of Multiplication.

$\sqrt{48aa}$	$\sqrt{aabc+aab}$	1	
$\sqrt{16aa}$	$\sqrt{aa}$	2	
$\sqrt{3}$	$\sqrt{bc+b}$	3	1 ÷ 2

These two examples reduced ſtand thus:

$$\left.\begin{array}{c} 4a\sqrt{3}=\sqrt{48aa} \\ \text{or} \\ 4a*\sqrt{3}=\sqrt{48aa} \end{array}\right\} \text{Alſo} \left\{\begin{array}{l} a\sqrt{bc+b}=\sqrt{aabc+aab} \\ \\ a*\sqrt{bc+b}=\sqrt{aabc+aab} \end{array}\right.$$

Two other Examples.

$\dfrac{\sqrt{\sqrt{9aaabb}}}{4b-4a}$	$\sqrt{aabbc+2afbbc+ffbbc}$	1	
$\dfrac{\sqrt{\sqrt{9aabb}}}{4}$	$\sqrt{aa+2af+ff}$	2	
$\dfrac{\sqrt{\sqrt{a}}}{b-a}$	$\sqrt{bbc}$	3	1 ÷ 2

These Examples ſtand thus abbreviated.

$\sqrt{\dfrac{3ab}{2}} * \sqrt{\dfrac{a}{b-a}}$ | Alſo $a+f*\sqrt{bbc}$

F But

§. 2. But it is oftentimes very troublesome to find a Square, Cube, *&c.* whereby this *Abbreviation* may be performed. Find therefore all the *Partes aliquotæ* or *just Dividers*, and these will tell us whether, and how often any Cube, Square, *&c.* is contained in the Quantity assigned.

Forasmuch then as the Discovery of the *Partes aliquotæ* is many waies useful in *Vulgar Arithmetick*, I have adjoyned a Table in the End of this Book, which discovers them in all uneven Numbers as far as 100,000.

In which Table [p] stands for a *Prime Number* throughout.

The Use of that Table is

To discover *at view* whether any given Quantity be compound or simple, *i. e.* be divisible or indivisible, and how many *Partes aliquotæ* it hath. On the left side you see, run down all the odd Numbers to *99*, which must be set after the Numbers in the Head-Row, as Occasion is, thus. Let the Number given be 21449, seek 49 in the side, and the other 214 in the head, then run downward, and side-waies till their Rows meet in a Square, where we find 89, which is a *Pars aliquota*, which dividing 21449 gives Quotient 241. With this 241 do as before (*i.e.* seek 41 on the side, and 2 in the head) and in its Square you find (P) which shews that it is an indivisible or *Prime* Number. Wherefore the aliquot Parts of this 21449 stand thus.

1
89 · 241
21449.

If the even Number 21696 were given, *subdivide* it continually by 2 till the Quotient be an odd Number (as at the sixth Time you will here find 339) Seek this 339 in the Table as you are directed above. In its Square we find 3, which dividing

dividing 339 gives Quotient 113, which 113 we find to be a *Prime* Number. The *Partes aliquotæ* of the Number 21696 stand as follows. Out of 1, 2, 3, 113 we may find the rest.

1

2 . 2 . 2 . 2 . 2 . 2

4. 8. 16. 32. 64

3. 6. 12. 24. 48. 96. 192

113. 226. 452. 904. 1808. 3616. 7232. 339. 678. 1356. 2712. 5424

10848. 21696.

How those *Principal* Divisors (1, 2, 3, 113) are multiplied into each other, and into their Products, lies plain before the Eyes without any more words.

The same Work in *Algebra.*

IN the Quantity [*aadee—aafe*,] first note what like Letters be in both parts of the *Binomial* [as here *a a e.*] Herewith divide, the quotient is [*de—f*] which is a prime Quantity Hence the *Partes aliquotæ* first found are [1, *a*, *a* *a*, *e*. *de—f.*] by which the rest are also found.

1

a *a*

aa

e . *ae* . *aae*.

de—f. ade—af. aade—aaf. dee—fe. aadee—aafe.

Amongst these there is one *Quadratick Divisor* wherewith the quantity may be depressed and abbreviated according to the Rules aforegoing, thus.

$a\sqrt{.}dee-fe$, or, $a * \sqrt{dee-fe} = \sqrt{aadee-aafe}$.

With *Cubick* and *Biqu.* quantities we have the same way to

deal,

§ 3.

deal, only then amongst the *Partes aliquotæ* seek such as be *Cubick*, or *Biquadratick*, and divide by them as afore.

Note here, that by the help of these *just Divisors* the quantities of Fractions may be brought to lower terms more handsomly than by the way taught in the Doctrine of Fractions. (p.21,22.)

$$\left.\frac{bdd - cdd + bee - cee}{bb - cc}\right\}\text{their first Divisors be}\left\{\begin{array}{l}1\,.\,b - c\,.\,dd + ee\\ 1\,.\,b - c\,.\,b + c\,.\end{array}\right.$$

So the *aliquotæ partes* of the

$$\overbrace{\textit{Numerator.} \qquad\qquad\qquad \textit{Denominator.}}$$

$$\frac{\frac{1}{b - c \cdot dd + ee}}{bdd - cdd + bee - cee} \qquad\qquad \frac{\frac{1}{b - c \cdot b + c}}{bb - cc}$$

Now because in both places the same *Residue* [$b - c$] is found, the Fraction may be abbreviated by the same, and therefore

$$\frac{dd + ee}{b + c} = \frac{bdd - cdd + bee - cee}{bb - cc}$$

When divers *Partes aliquotæ* are found in both places, then we must take the greatest of them all, and therewith perform the Division; for else the quantities would not be *prime* without further *Subdivision*, and so make us new work. If among the *Partes aliquotæ* no *Square*, *Cube*, &c. be found, then can the quantity be brought *no lower*, but wil be handled Fraction-wise.

Let

Let the quantity be $\sqrt{35aa}$. Divide this by any quadratick Number [4 . 25 . 36, &c.] and set the Divisor also before the quantity as in Multiplication.

$$4\sqrt{\frac{35aa}{4}} = 25\sqrt{\frac{35aa}{25}} = 36\sqrt{\frac{35aa}{36}} = 36 \cdot \frac{\sqrt{35aa}}{6}$$

Hence comes $2a\sqrt{\frac{35}{4}} = 5a\sqrt{\frac{35}{25}} = 6a\sqrt{\frac{35}{36}} = 6a * \frac{\sqrt{35}}{6}$

Although these kind of Quantities before mentioned are *surd*, yet are they related to each other in a double property. For they are either *Commensurable* (and *Communicant*) or *Incommensurable* to each other. The *commensurable* Quantities are related to some *rational* quantities by an exact Cognation, *i. e.* They respect each other as *two rational quantities* do each other. But the *Incommensurable* are such as bear no proportion either to *one another*, or *any rational quantity* You may know *Commensurable* quantities thus, *viz.* When that which stands under the Radical Sign is in both the same.

As in these two. [$a\sqrt{bc}$. and $d\sqrt{bc}$] for the Proportion is [$a\sqrt{bc}$. $d\sqrt{bc}$: : a . d.]

And here comes in the Note of Proportion [: :] which notes four terms standing *related* as in the Golden *Rule of Proportion.*

And when hereafter, in the Margin you see [*anal.*] ser, know that it hintsan analogy or the relation of four proportional terms. But the above-mentioned Proportion may be thus expressed in words. As [$a\sqrt{bc}$ is to $d\sqrt{bc}$: so is [a] to [d.]

$$\therefore ad\sqrt{bc} = ad\sqrt{bc}.$$

According to the Nature of Proportion, the Product of the two middle terms is equal to the Product made under the two Extreams. Which Deduction is hereafter noted with [$\therefore$] that

that is as much as [*ergo.*] only [∵] is set in the work, [*ergo*] in the Margin.

1 As in these. $18\sqrt{\overline{dd-cc}}$ and $c+d\sqrt{\overline{dd-cc}}$ they stand thus,

1 Analog. 2 $18\sqrt{\overline{dd-cc}} \,.\, c+d\sqrt{\overline{dd-cc}} :: 18 \,.\, c+d$

2 Ergo. 3 $\because 18c+18d * \sqrt{\overline{dd-cc}} = 18c+18d * \sqrt{\overline{dd-cc}}$

Again.

1 $\frac{r}{d}\sqrt{\overline{gg+nq}}$ and $\frac{dn}{qr}\sqrt{\overline{gg+nq}}$

1 Anal. 2 $\frac{r}{d}\sqrt{\overline{gg+nq}} \quad \frac{dn}{qr}\sqrt{\overline{gg+nq}} :: \frac{r}{d} \,.\, \frac{dn}{rq}$

2 Ergo. 3 $\because \frac{n}{q}\sqrt{\overline{gg+nq}} = \frac{n}{q}\sqrt{\overline{gg+nq}}$

Addition

Addition and Subtraction in Surd Quantities. § 4.

R. I. If the Quantities be *commensurable*, then let the *rational part* be added or subtracted, and subjoyn the *irrational*.

Addition.

$3\sqrt{b}$	$2a\sqrt{b}$	$d\sqrt{bb-cc}$	$-\frac{r}{d}\sqrt{gg+nq}$	1	
$5\sqrt{b}$	$5a\sqrt{b}$	$-5f\sqrt{bb-cc}$	$+\frac{dn}{qr}\sqrt{gg+nq}$	2	
$8\sqrt{b}$	$7a\sqrt{b}$	$d-5f\sqrt{bb-cc}$	$-\frac{r}{d}+\frac{dn}{qr}*\sqrt{gg+nq}$	3	1+2

Subtraction.

$5\sqrt{b}$	$5a\sqrt{b}$	$d\sqrt{bb-cc}$	$-\frac{r}{d}\sqrt{gg+nq}$
$3\sqrt{b}$	$2a\sqrt{b}$	$-5f\sqrt{bb-cc}$	$\frac{dn}{qr}\sqrt{gg+nq}$
$2\sqrt{b}$	$3a\sqrt{b}$	$d+5f*\sqrt{bb-cc}$	$-\frac{r}{d}-\frac{dn}{qr}*\sqrt{gg+nq}$

R. II. If the quantities be *incommensurable*, add and subtract them by the Signs + and —; whence arise a *Surd Binomial and Residue*.

Examples

Examples of Addition.

	1	$\sqrt{ab}$	$\sqrt{dd-e}$	$ab+cc$
	2	$\sqrt{dc}$	$\sqrt{f+g}$	$\sqrt{-dd+ff}$
1+2	3	$\sqrt{ab}+\sqrt{dc}$	$\sqrt{dd-e}+\sqrt{f+g}$	$ab+cc+\sqrt{-\ast dd+ff}$

	1	$\frac{-a+f}{g}$
	2	$\sqrt{ff-e}$
1+2	3	$\frac{-a+f}{g}+\sqrt{ff-e}$

Examples of Subtraction.

	1	$\sqrt{ab}$	$\sqrt{dd-e}$	$ab+cc$
	2	$\sqrt{dc}$	$\sqrt{-f+g}$	$\sqrt{+dd+ff}$
1 — 2	3	$\sqrt{ab}-\sqrt{dc}$	$\sqrt{dd-e}-\sqrt{-f+g}$	$ab+cc-\sqrt{+dd+ff}$

	1	$-a+f$
	2	$\sqrt{ff-e}+b$
1 — 2	3	$f-a-\sqrt{ff-e}-b$

Multipli-

Multiplication in Surd quantities.

§ 5.

R. I. IF the Quantities be *commensurable*, multiply the *Rational* part by the Rational, and that also by the *Irrational* part, casting away its *Radical Sign*. Hence the Product will be *entirely* rational.

$3\sqrt{b}$	$2a\sqrt{b}$	$d\sqrt{\overline{bb-cc}}$	$\frac{r}{d}\sqrt{\overline{gg+nq}}$	$3d+\sqrt{\overline{gg}}$	1	
$5\sqrt{b}$	$5a\sqrt{b}$	$5\sqrt{\overline{bb-cc}}$	$-\frac{dn}{qr}\sqrt{\overline{gg+nq}}$	$+\sqrt{\overline{gg}}$	2	
$15b$	$10aab$	$5bbd-5dcc$	$\frac{-ngg-nnq}{q}$	$3d\sqrt{\overline{gg}}+gg$	3	1 * 2

R. II. If the Quantities be *only Surd*, and altogether alike, then only cast away the *Radical Sign*, so have you the Product: as

$\sqrt{\overline{aa-bb}} * \sqrt{\overline{aa-bb}}$ makes the Product $aa-bb$, which indeed is nothing but *Quadratick Involution.*

100

R. III. If the Quantity be *unlike* and *wholly Surd*, or *in part rational* but *incommensurable*, multiply the rational by the rational, and the Surd by the Surd, and in the Quotient *connect* the Surd to the Rational with the Sign of Multiplication.

	1	$\sqrt{ab}$	$\sqrt{\overline{d-e}}$	$a+b$	$2\sqrt{ab}$
	2	$\sqrt{bc}$	$\sqrt{\overline{d+e}}$	$\sqrt{bbb}$	$3c\sqrt{bc}$
1 * 2	3	$\sqrt{abbc}$	$\sqrt{\overline{dd-ee}}$	$\overline{a+b} * \sqrt{bbb}$	$6c\sqrt{abbc}$

	1	$3z * \sqrt{\overline{ff-gg}}$	$a+b$
	2	$\sqrt{\overline{ff+gg}}$	$2\sqrt{b}$
1 * 2	3	$3z * \sqrt{\overline{f^4-g^4}}$	$\overline{2a+2b} * \sqrt{b}$

Division in Surd Quantities.

§ 6.

R. I. IF the quantities be *commensurate*, then only divide the *rational* by the *rational* part, and what ariseth is the Quotient.

	1	$3\sqrt{b}$	$2a\sqrt{b}$	$d\sqrt{\overline{dd-cc}}$	$-\frac{r}{d}\sqrt{\overline{gg+nq}}$	$3d\sqrt{g}$
	2	$2\sqrt{b}$	$5a\sqrt{b}$	$-5f\sqrt{\overline{dd-cc}}$	$\frac{dn}{qr}\sqrt{\overline{gg+nq}}$	$\sqrt{g}$
1 ÷ 2	3	$\frac{3}{2}$	$\frac{2}{5}$	$-\frac{d}{5f}$	$-\frac{qrr}{ddn}$	$3d$

R. II.

R. II. If the Quantities be *incommensurable*, then divide the *rational* by the *rational* (if any such there be) and *surd* by *surd*, and in the quotient subjoyn the *surd part* with its first *radical Sign*.

$4a\sqrt{fb}$	$12ag\sqrt{gh}$	$\sqrt{aab^3-b^3dd}$	1	
$2a\sqrt{f}$	$12a\sqrt{hi}$	$\sqrt{aa-dd}$	2	
$2\sqrt{b}$	$g\sqrt{\frac{g}{i}}$	$\sqrt{bbb}$	3	$1 \div 2$

$\sqrt{dde+eef-ef-ff+eff+ddf}$	1	
$\sqrt{dd+ef-f}$	2	
$\sqrt{f+e}$	3	$1 \div 2$

R. III. Divide a *Square* by its *Root*, and the Quotient is the *Root* too.

$aa+2ab+bb$	a^4+aaff	1	
$\sqrt{aa+2ab+bb}$	$a\sqrt{aa+ff}$	2	
$\sqrt{aa+2ab+bb}$	$a\sqrt{aa+ff}$	3	$1 \div 2$

More Examples.

	1	$af+f\sqrt{fx}$	$bde\begin{matrix}+bf\\+de\end{matrix}\sqrt{dc+de}+fdc+fde$
	2	$a+\sqrt{fx}$	$de+f\sqrt{dc+de}$
$1 \div 2$	3	f	$b+\sqrt{dc+de}$

	1	$\sqrt{5kk+zzz}$
	2	$\sqrt{13xx-yy}$
$1 \div 2$	3	$\dfrac{\sqrt{5kk+zzz}}{\sqrt{13xx-yy}} = \sqrt{\dfrac{5kk+z^3}{13xx-yy}}$

Again.

	1	$dfgh+\begin{matrix}ghx\\ddf\end{matrix}\sqrt{gg-kk}+dggx-dkkx$	$aa+af+fx+\begin{matrix}2a\\f\end{matrix}\sqrt{fx}$
	2	$df+x\sqrt{gg-kk}$	$a+f+\sqrt{fx}$
$1 \div 2$	3	$gh+d\sqrt{gg-kk}$	$a+\sqrt{fx}$

Substitution.

§ 7. WHen in tedious Operations, *in stead* of a surd or long Quantity, you would take one that is rational or shorter, and deal with it as was to be done with the other; or when, for *any other respect*, you would put one quantity for another

ther (though thereby the quantity be made neither rational nor ſhorter) this is called *Subſtitution*.

Suppoſe this quantity $\sqrt{aa-bb}*d-e$. For $\sqrt{aa-bb}$ ſubſtitute c, ſo it will be $cd-ce=\sqrt{aa-bb}*d-e$. For $c=\sqrt{aa-bb}$

Again, $\frac{3bbs+bcc-ccc}{fg}$ and for $[b, c, f]$ take $[r, t, x]$ thence comes $\frac{3rrs+rtt-ttt.}{xg}$

So in this, $zz+3Bz+BB=0$, and for z ſet $h-b$.

Wherefore $z=h-B$	1	
$zz=hh-2Bh+BB$	2	1 ⊚ 2.
$3z=3h-3B$	3	$1 * \overline{3}$
$3Bz=3Bh-3BB$	4	$3 * B$
$BB=\quad +BB$	5	B ⊚ $\overline{\cancel{3}}$ 2
$zz+3Bz-BB=hh+Bh-BB$	6	$2+4+5$

So in this, $\frac{CCDE}{ff}$ in ſtead of $\left\{\begin{matrix} C \\ f \end{matrix}\right\}$ ſet $\left\{\begin{matrix} \sqrt{aa+bb} \\ \sqrt{DD+EE} \end{matrix}\right\}$ (or any two quantities greater or leſs at pleaſure.)

So here, $c=\sqrt{aa+bb}$	1	
$f=\sqrt{DD+EE}$	2	
$cc=aa+bb$	3	1 ⊚ 2
$ff=DD+EE$	4	2 ⊚ 2
$ccDE=aaDE+bbDE$	5	$3 * DE$
$\frac{ccDE}{ff}=\frac{aaDE+bbDE}{DD+EE}$	6	$5 \div 4$

Reſtitution.

§ 8.

Restitution.

IF you would reduce the *substitute* Quantities to the first Form, work back just contrary to *Substitution.* As in $[\overline{cd-ce}]$ for c set $\sqrt{\overline{aa-bb}}$, hence comes out $\overline{d-e} * \sqrt{\overline{aa-bb}}$, as above.

Again $hh+Bh-BB$; for $h-B$ set z.

	1	$h-B=z$
$1+B$	2	$h=z+B$
$2 \odot 2$	3	$hh=zz+2Bz+BB$
$2-z$	4	$h-z=B$
$2 * B$	5	$hB=Bz+BB$
$B \odot 2$	6	$BB= \quad +BB$
$3+5-6$	7	$hh+Bh-BB=zz+3Bz+BB$

And so in the other Examples of *Substitution.*

The

The Second Part.

Resolution of Problemes.

The Use of this ALGEBRA.

CHAP. I.

Of *Equations*, and their Properties: And first,

How many Roots any Equation may contain.

AS great as is the *Number* of the highest *Power* in any *Equation*, so many Roots doth it contain; either *Affirmative* or *Nega-tive*, or *Impossible*. These Negative Roots *Des Cartes* cals *False Roots*. But forasmuch as they are *therefore* negative, because they express Quantities of a Denomination opposite to those that are counted Affirmative, and in *Geometrical Projection* they require a *situation* contrary to the Affirmative Roots; therefore they are no less *true* than the Affirmative ones. For which cause I forbear to call them *False Roots*, and content my self with the Term of *Negative Roots*, according to the import

import of the Sign [—] prefixed to ſuch roots. Roots that are *Impoſſible*, are ſuch as by their own Nature or their Signs are ſo ordered and *entangled* as that they render the Probleme *unreaſonable* and *impoſſible*. Such are marked with this Sign [ↃΙϹ]

As many Dimenſions or Powers as any Equation hath, ſo many Diviſors it hath, which are ſome *Binomium* or *Reſiduum* conſiſting of the unknown quantity and the Root. As the Equation [$x^4 - 4x^3 - 19xx + 106x - 120 = 0$] may be divided by [$x-4$, $x-3$, $x-2$, and $x+5$] and therefore

$$+ \quad x = \left\{\begin{matrix} +4 \\ +3 \\ +2 \\ -5 \end{matrix}\right\}$$ as may be eaſily tried.

CHAP.

CHAP. II.

Of casting away the Second Term of the Equation.

BY the power of the first Term divide the known quantity of the Second Term. Then if the Signs of the First and Second Terms be unlike, *subtract* the Quotient out of the Affirmative Root; but if the Signs be like, then *add* the same Quotient to the Affirmative Root. Then rectifie the *Substitution*, and you have your Desire.

Let the Equation proposed be $zz-6z-bb=0$. Here the *Power* of the first Term is 2, and the *known quantity* of the second Term 6. This divided by 2, gives 3, which taken from the Root z, leaves $z-3$ equal to any qu. taken at pleasure, as suppose that be x, then

$z-3=x$ and	The new root x is 3 less than the root z.	1		
$z=x+3$		2		$1+\overline{3}$
$zz=xx+6x+9$		3		$2 \circledcirc \overline{2}$
$6z= \quad 6x+18$		4	2	$1 * \overline{6}$
$zz-6z=xx(*)-9$		5		$3-4$
$zz-6z-bb=xx-9-bb=0$		6		$5-bb$

Again, let the Equation be $zz+6z-bb=0$.

Because the first and second Terms of the Equation have the same Sign $+$, increase the root z with 3, hence comes $z+3=x$.

$z+3=x$	The new root x is 3 more than z.	1	
$z=x-3$		2	$1-\overline{3}$
$zz=xx-6x+9$		3	$2 \circledcirc \overline{2}$
$6z= \quad +6x-18$		4	$2 * \overline{6}$
$zz+6z=xx(*)-9$		5	$3+4$
$zz+6z-bb=xx-9-bb=0$		6	$5-bb$

An Example of a Cubick Equation.

As this $zzz + 12yzz + 27yyz - 27yyy = 0$

	1	$12y =$ the known qu. of the second place.
	2	$3 =$ the power of the first term.
$1 \div 2$	3	$4y =$ the quotient sought.
$z + 4y$	4	$z + 4y = x$ the new root.
$4 - 4y$	5	$z = x - 4y$
$5 \circledcirc 2$	6	$zz = xx - 8yx + 16yy$
$5 \circledcirc 3$	7	$zzz = xxx - 12yxx + 48yyx - 64yyy$
$6 * 12y$	8	$12yzz = + 12yxx - 96yyx + 192yyy$
$5 * 27yy$	9	$27yyz = \qquad + 27yyx - 108yyy$
$7 + 8 + 9$	10	$z^3 + 12yzz + 27yyz = xxx (*) - 21yyx + 20yyy$
$10 + 27y^3$	11	$z^3 + 12yzz + 27yyz - 27yyy = xxx (*) - 21yyx - 7yyy.$

The new root x is greater than the root z by $4y$.

As here the second place is voided, so may the third and fourth by other Rules. But the questions of this Treatise do give no occasion to use such Rules, and therefore I refer the Reader to *Des Cartes* and others, where all is sufficiently explained. There also he may learn *how to discover how many affirmative or negative Roots any Equation contains. How to increase or lessen an Equation, and variously to sift and mould it. How to supply the Defect of any places in any incompleat Equation. And what in each place hath relation to the Root*, &c.

CHAP.

CHAP. III.

Resolution of Equations.

SOme Præcognita *necessary for Resolution of Equations.*

All six *Species* of *Algebraick Arithmetick* are employed in this Resolution. According to these Rules.

The Rule for Addition.

When any Term is connected with [—] it is added by casting away [—] and putting the qu. into the other side with the Sign [+]

$z - a - 13 = 20.$	1	$z = ?$
$z - a = 33$	2	$1 + 13$
$z = a + 33$	3	$2 - a$

Rule for Subtraction.

Where any Term is connected with [+] it is subtracted by casting away [+] and putting the quantity into the other side with the Sign [—].

$z + a + 13 = b$	1	$z = ?$
$z + a = b - 13$	2	$1 - 13$
$z = b - a - 13.$	3	$2 - a$

Rule for Multiplication.

By Multiplication Fractions are reduced to integers.

$\dfrac{z + A}{3} = 29$	1	$z = ?$
$z + A = 87$	2	$1 * 3$
$z = 87 - A$	3	$2 - A$

Again.

$z = ?$	1	$\frac{z+B-C}{AB} = \frac{B+2c}{13}$
$1 * X$	2	$13z+13B-13c = ABB+2ABc$
$2 + 13C$	3	$13z+13B = ABB+2ABc+13c$
$3 - 13B$	4	$13z = ABB+2ABc+13c-13B$

Rule for Division.

The three first *Species* (*Addition*, *Subtraction* and *Multiplication*) may serve well for the *lessening* and contracting an Equation, but cannot *take down* any *degree of Power*: But now *Division* and *Evolution* perform *this Office*, and bring down the Equation to the lowest possible degree.

$z = ?$	1	$zdd - zee = 5A$
$1 \div dd - ee$	2	$Z = \frac{5A}{dd-ee}$

Again.

$z = ?$	1	$AA + Az = 30A$
$1 \div A$	2	$A + z = 30$
$2 - A$	3	$Z = 30 - A.$

Rule for Involution.

Surd Quantities are reduced to rational ones by Involution.

$Z = ?$	1	$A\sqrt{z} = \sqrt{BD}$
$1 \odot 2$	2	$AAz = BD$
$2 \div AA$	3	$Z == \frac{BD}{AA}$

Again.

Again.

$A\sqrt{c.}z=\frac{BD}{A}$	1	$Z=?$
$AAAz==\frac{BBBDDD}{AAA}$ ~~(or $2\div A^3$ gives $Z=\frac{B^3D^3}{A^6}$)~~ wrong'ly placed	2	$1 \odot 3$
$A^6z=BBBDDD$	3	$2 * A^3$
$Z=\frac{B^3D^3}{A^6}$ (or $2\div A^3$ gives $Z=\frac{B^3D^3}{A^6}$)	4	$3\div A^6$

Rule for Evolution.

By Evolution Degrees of power are taken down and brought to the lowest.

$zz+2Az=3AA$	1	$Z=?$
$zz+2AZ+AA=4AA$	2	$1+AA$
$Z+A=2A$ and $=-2A$ (for $-2A$ involved is also $4AA$)	3	$2\ \omega\ 2$
$Z=A$ and also $=-3A$	4	$3-A$

Again.

$\frac{1}{4}zz-\frac{3}{2}Bz+\frac{9}{4}BB=\frac{9}{4}BB$	1	$Z=?$
$\frac{1}{2}z-\frac{3}{2}B=\frac{3}{2}B$, (also $=-\frac{3}{2}B$)	2	$1\ \omega\ 2$
$\frac{1}{2}z=3B$, (also $=0$)	3	$2+\frac{3}{2}B$

Note here that the same Equation may have one of its sides *ambiguous* ($\frac{1}{2}Z=3B=0$) which is sometimes expressed in the Margin by a [,] set after its Number, which shews no *new* work, but hints the Equation immediately precedent in *another way* expressed. [2 ,]

That

That in the first Example $Z = A$ and $-3A$ is thus proved.

	1	Let $A = 6$
1 ⊙ 2	2	$AA = 36$
2 * 3	3	$3AA = 108$
1 * 2	4	$2A = 12$
4 * z	5	$2AZ = 12Z$
Since then	6	$Z = A$
6, 1.	7	$Z = 6$
7 ⊙ 2	8	$ZZ = 36$
4 * 7	9	$2AZ = 72$
8 + 9	10	$ZZ + 2AZ = 108 = 3AA$ (as in the Probleme)
Because	11	$Z = -3A = -18$ (1 * 3)
11 ⊙ 2	12	$ZZ = 9AA = 324$
11 * 4	13	$2AZ = -216$
12 + 13	14	$ZZ + 2AZ = 108 = 3AA$ (altogether as in the Question.) Here A in this Proof may be $=$ any quantity at pleasure.

CHAP. IV.

FRanciscus à Schooten in the Conclusion of his *Principia Math. Univers.* (p. 45.) brings this following Equation for Exercise sake, wherein will happen *all sorts* of Reductions. Which also I shall undertake to resolve after this aforesaid Method,

He expresses it thus. $\frac{\sqrt{\overline{zz+3aa}}}{4} - \frac{\sqrt{\overline{zz-3aa}}}{4} = \frac{\sqrt{\overline{azz}}}{b}$ But in this, (as elsewhere in his Book) is a Mistake of the *Printer*, and forgotten in the *Errata*. However, we shall express it

it rightly, as may be gathered from the whole Procefs, *viz.* thus:

	$\sqrt{\frac{zz+3aa}{4}} - \sqrt{\frac{zz-3aa}{4}} = \sqrt{\frac{azz}{b}}$	1	$z = ?$
For a Model for the work take	$c-d=e$	2	
	$cc-2cd+dd=ee$	3	$2 ⊙ 2$
All is brought out of the *Surdity* thus,	$\frac{zz+3aa}{4} - \sqrt{\frac{z^4-9a^4}{4}} + \frac{zz-3aa}{4} = \frac{azz}{b}$	4	$1, 3$
	$\frac{1}{2}zz - \sqrt{\frac{z^4-9a^4}{4}} = \frac{azz}{b}$ *	5	4
	$\frac{1}{2}zz - \frac{azz}{b} = \sqrt{\frac{z^4-9a^4}{4}}$	6	$5 \pm$
	$\frac{1}{4}z^4 - \frac{az^4}{b} + \frac{aaz^4}{bb} = \frac{z^4-9a^4}{4}$	7	$6 ⊙ 2$
Brought out of the *Fractions* thus,	$-\frac{az^4}{b} + \frac{aaz^4}{bb} = \frac{-9a^4}{4}$	8	$7 - \frac{z^4}{4}$
	$\frac{az^4}{b} - \frac{aaz^4}{bb} = \frac{9a^4}{4}$	9	$0 - 8$
	$4az^4b - 4aaz^4 = 9a^4bb$	10	$9 * 4bb$

* Derived from the 4th

$\frac{zz+3aa}{4} + \frac{zz-3aa}{4} = \frac{1}{2}zz$

Then

$10 \div 4ab \cdot 4aa$	11		$zzzz = \frac{9aaabb}{4b - 4a}$
$11,$	12	Then the quantity z^4 is thus depressed to z.	$z^4 = \frac{9aabb}{4} * \frac{a}{b - a}$
$11 \, \omega \, 2$	13		$zz = \frac{3ab}{2} * \sqrt{\frac{a}{b - a}}$
$13 \, \omega \, 2$	14		$z = \sqrt{\frac{3ab}{2}} * \sqrt{\frac{a}{b - a}}$

CHAP. V.

The Resolution of divers Arithmetical and Geometrical Problemes.

DES *Cartes's* way is to signifie ***known*** quantities by the *former* Letters of the Alphabet, and ***unknown*** by the *later*. [$z\,y\,x$, &c.] But I choose to signifie the ***unknown*** quantities by *small* Letters, and the ***known*** by *Capitals*. Because methinks it is more convenient; for thus will all the concerned quantities be expressed according to the Order both of the Question and the Alphabet too. That when we compare a *Geometrical* Figure with its Resolution, every quantity which is requisite for ordering the Work, may presently and commodiously be expressed by these *Substitutes*.

And when at last the question is brought to an Equation, it may easily be changed [in its *Form* or Figure] without altering the Letters. As, $aa + bb = DD$. $aa - dd = BB$. $bb - dd = AA$. Here the Equations keep their Names, and yet each same quantity in these three dispositions is known and unknown.

When

When any question is to be resolved, in the *first* place the meaning of it must be clearly comprehended; *Then* all the quantities concern'd, as well unknown as known, must be expressed by Letters; *Next*, *these Letters* must be ordered, disposed, and moulded according to the Demand of the *Probleme*, and whatever is required must be so, and no otherwise expressed, than if it were *already* discovered; *Lastly*, as many particular quantities as are contained in the Question (or as many Lines as be in the Figure) so many *several* Letters must you take; but when divers quantities are granted to be equal, they may be expressed by the same Letter.

The Questions follow.

Probl. I. THere be six quantities [$a+b$: $a-b$: ab: $\frac{a}{b}$ $aa+bb$: and $aa-bb$:] Now by any two of these given, find the other four.

$$\text{For}\left\{\begin{matrix} a+b \\ a-b \\ ab \\ \frac{a}{b} \\ aa+bb \\ aa-bb \end{matrix}\right\}\text{substitute}\left\{\begin{matrix} D \\ E \\ F \\ G \\ T \\ R \end{matrix}\right\}\quad \begin{matrix} a=? \\ b=? \end{matrix}$$

First then, by D and E (that is, the Sum and Difference of two quantities) given, find F, G, T, R, (that is, the Product, Quotient, and Sum and Difference of the Squares.)

$a = ?$	1	$a+b=D$ } as in the Probleme.
$b = ?$	2	$a-b=E$
$1+2$	3	$2a=D+E$
$3 \,\,\, \overline{2}$	4	$A=\frac{D+E}{2}=A$.
$1-2$	5	$2b=D-E$
$5 \div \overline{2}$	6	$B=\frac{D-E}{2}=B$
or $1-4$	7	$B=\frac{D-E}{2}=B$
$ab=?$ $4 * 6$	8	$AB=\frac{DD-EE}{4}=$ the Product $=F$
$\frac{a}{b}=?$ $4 \div 6$	9	$\frac{A}{B}=\frac{D+E}{D-E}=$ Quotient $=G$
$aa+bb?$ $4 \,\, 2$	10	$aa=\frac{DD+2DE+EE}{4}$
$6 \,\, 2$	11	$bb=\frac{DD-2DE+EE}{4}$
$10+11$	12	$AA+BB=\frac{DD+EE}{2}$ *Sum of the Squares* $=T$
$aa-bb=?$ $10-11$	13	$AA-BB=DE$ *Differ. of Squares.* $=R$

And so A and B are both explained by D & E, which was first to be found. The rest follows easily.

Illustration

Illustration in Numbers.

$2+b$	14	$a=b+E$
Ergo 14	15	$a>b$
Let	16	$a=4$ *ergo* $aa=16$
	17	$b=2$ *ergo* $bb=4$
$16+17$	18	$a+b=6=D$
$16-17$	19	$a-b=2=E$
$18+19$	20	$D+E=8$
$18-19$	21	$D-E=4$
$18 \odot 2$	22	$DD=36$
$19 \odot 2$	23	$EE=4$
$18 * 19$	24	$DE=12$
$20 \div 2$	25	$\frac{D+E}{2}=4=A$ (*Num* 4.) *
$21 \div 2$	26	$\frac{D-E}{2}=2=B\ (6)$
$20 \div 21$	27	$\frac{D+E}{D-E}=2=\frac{A}{B}\ (9)$
$22-23$	28	$DD-EE=32$
$28 \div 4$	29	$\frac{DD-EE}{4}=8=AB\ (8)$
$22+23 \div 2$	30	$\frac{DD+EE}{2}=20=AA+BB\ (12)$
$18 * 19$	31	$\cancel{B}E=12=AA-BB\ (13)$ D

This Sign $>$ signifies [*more than*:] as if a be equal to $b+E$ if you take away E then is a more or greater than b which is expressed $a>b$ And for a and b you may take away any Number at pleasure.

So also the Sign $<$ notes [*less than*] and this way of working is called *Collatio Quantitatum*, a Comparing of quantities, as truly as an Equation, which is a comparing of equal quantities.

* *N. B.* This refers to the Numbers in the small Column in the Margin. And so in other places of the Book.

In this *Illustration* by Numbers we might have taken any Numbers at pleasure, for D and E, so that $D > E$. Because $\frac{D-E}{2} = B$ (*per 7 hujus*) *ergo* $D - E = 2B$, *ergo* $D = E + 2B$. (*ergo* $D > E$.

By D and F given to find the rest.

$a = ?$	1	$a + b = D$
$b = ?$	2	$ab = F$
1 ⊙ 2	3	$aa + 2ab + bb = DD$
2 $*$ 4	4	$4ab = 4F$
3 — 4	5	$aa - 2ab + bb = DD - 4F$
5 ω 2	6	$a - b = \sqrt{DD - 4F}$

When $a + b$ and $a - b$ are found, it is superfluous to go further, for the manner of process lies plain in the preceding Operation.

By D and G. find the rest.

$a = ?$	1	$a + b = D$
$b = ?$	2	$\frac{a}{b} = G$
1 — b	3	$a = D - b$
2 $*$ b	4	$a = bG$
3, 4	5	$D - b = bG$
5 + B	6	$D = b + bG$
6 ÷ 1 + G	7	$\frac{D}{1+G} = B$
1 — 7	8	$\frac{DG}{1+G} = A$, &c.

By D and T, &c.

$a=?$	1	$a+b=D$
$b=?$	2	$aa+bb=T$
$1 \odot 2$	3	$aa+bb+2ab=DD$
$3-2$	4	$2ab=DD-T$
$4 * 2$	5	$4ab=2DD-2T$
$3-5$	6	$aa+bb-2ab=2T-DD$
$6 \omega 2$	7	$a-b=\sqrt{2T-DD}$, &c.

By D and R.

$a=?$	1	$a+b=D$
$b=?$	2	$aa-bb=R$
$2 \div 1$	3	$a-b=\frac{D}{R}$ &c. $\frac{R}{D}$

By E and F.

$a=?$	1	$a-b=E$
$b=?$	2	$ab=F$
$1 \odot 2$	3	$aa-2ab+bb=EE$
$2 * 4$	4	$4ab=4F$
$3+4$	5	$aa+2ab+bb=EE+4F$
$5 \omega 2$	6	$a+b=\sqrt{EE+4F}$, &c.

By

By *E* and *G*.

$a = ?$	1	$a - b = E$
$b = ?$	2	$\frac{a}{b} = G$
$1 + b$	3	$a = b + E$
$2 * b$	4	$a = bG$
$3, 4$	5	$b + E = bG$
$5 - b$	6	$E = bG - b$
$6 \div \overline{G - 1}$	7	$\frac{E}{G - 1} = B$
$1 + 7$	8	$\frac{EG}{G - 1} = A$, &c.

By *E* and *T*.

$a = ?$	1	$a - b = E$
$b = ?$	2	$aa + bb = T$
$1 \odot 2$	3	$aa - 2ab + bb = EE$
$2 - 3$	4	$+ 2ab = T - EE$
$4 * 2$	5	$4ab = 2T - 2EE$
$3 + 5$	6	$aa + 2ab + bb = 2T - EE$
$6 \omega 2$	7	$a + b = \sqrt{2T - EE}$, &c.

By *E* and *R*.

$a = ?$	1	$a - b = E$
$b = ?$	2	$aa - bb = R$
$2 \div 1$	3	$a + b = \frac{R}{E}$ &c.

By *F* and *G*.

$a = ?$	1	$ab = F$
$b = ?$	2	$\frac{a}{b} = G$
$1 * 2$	3	$aa = FG$
$1 \div 2$	4	$bb = \frac{F}{G}$ &c.

By *F* and *T*.

$a = ?$	1	$ab = F$
$b = ?$	2	$aa + bb = T$
$1 * 2$	3	$2ab = 2F$
$2 + 3$	4	$aa + 2ab + bb = T + 2F$
$2 - 3$	5	$aa - 2ab + bb = T - 2F$
$4 \omega 2$	6	$a + b = \sqrt{T + 2F}$
$5 \omega 2$	7	$a - b = \sqrt{T - 2F}$

By *F* and *R*.

$a = ?$	1	$ab = F$
$b = ?$	2	$aa - bb = R$
$2 \odot 2$	3	$a^4 - 2aabb + b^4 = RR$
$1 \odot 2$	4	$aabb = FF$
$4 * 4$	5	$4aabb = 4FF$
$3 + 5$	6	$a^4 + 2aabb + b^4 = 4FF + RR$
$6 \omega 2$	7	$aa + bb = \sqrt{4FF + RR}$ &c,

By

By *G* and *T*.

$a = ?$	1	$\frac{a}{b} = G$
$b = ?$	2	$aa + bb = T$
$1 \odot 2$	3	$\frac{aa}{bb} = GG$
$3 * bb$	4	$aa = GGbb$
$2 - bb$	5	$aa = T - bb$
$4, 5$	6	$bbGG = T - bb$
$6 + bb$	7	$bbGG + bb = T$
$7 \div \overline{GG + 1}$	8	$bb = \frac{T}{GG + 1}$ &c.

By *G* and *R*.

$a = ?$	1	$\frac{a}{b} = G$
$b = ?$	2	$aa - bb = R$
$1 * b$	3	$a = bG$
$3 \odot 2$	4	$aa = bbGG$
$2 + bb$	5	$aa = R + bb$
$4, 5$	6	$bbGG = R + bb$
$6 - bb$	7	$bbGG - bb = R$
$7 \div \overline{GG - 1}$	8	$BB = \frac{R}{GG - 1}$ &c.

By

By T and R.

$a=?$	1	$aa+bb=T$
$b=?$	2	$aa-bb=R$
$1+2$	3	$2aa=T+R$
$1-2$	4	$2bb=T-R$, &c.

N. For the resolving of several of the following Problemes and their Equations, these two Propositions must be well understood.

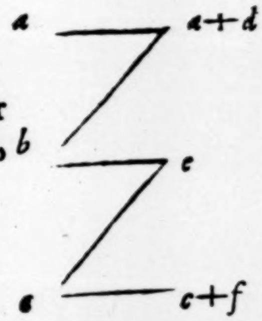
Prop. I. *In all rect-angle Triangles the Square of the greatest side is equal to the Sum of the Squares of the two other sides*, Eucl. 1. 47. (*Fig.* I.)

$$\left.\begin{array}{l} h = \text{Hypotenuse.} \\ b = \text{Base} \\ c = \text{Perpendicular} \end{array}\right\} \text{say then} \left\{\begin{array}{l} hh=bb+cc \\ bb=hh-cc \\ cc=hh-bb \end{array}\right.$$

Prop. II. *The sides of like Triangles are respectively proportional to each other*. Eucl. 6. 4. (*Fig.* II.)

$$\text{Let the like Triangles be} \left\{\begin{array}{l} a,\ b,\ c. \\ ad,\ e,\ cf. \end{array}\right. \quad a+d.\ e.\ c+f.$$

$$\text{Say then} \left\{\begin{array}{l} a\,.\,b :: a+d\,.\,e \\ a\,.\,a+d :: e\,.\,c+f \\ b\,.\,e :: e\,.\,c+f \end{array}\right\} \text{Or thus,}$$



Des Cartes, in a Letter not yet printed, writes thus, *In searching the Solution of Geometrical Questions I alwaies make use of Lines Parallel and Perpendicular, as much as is possible: and I consider no other Theoremes but these two.* [The sides of like

like Triangles have like Proportions.] *And* [In rect-angle Triangles the square of the greatest side is equal to the squares of the two other sides.] *And I am not afraid to suppose many unknown Quantities, that I may reduce the proposed Question to such terms, as that it depends on no other Theoremes but these two.*

Prob. II. In a right angl'd plain Triangle the side

$c = ?$	1	$B = 3\sqrt{2} + 3$	$b = 3\sqrt{2} + 3$	how much is each?
$h = ?$	2	$h + c = 9\sqrt{2} + 9$	$c + h = 6\sqrt{2} + 9$ 9	
	3	$hh - cc = BB$ (out of the Figure)	(*Fig.* I.)	
1,2 Analog.	4	$B . h + c :: 1 . 3$		
4 Ergo.	5	$3B = h + c$		
$5 - c$	6	$3B - c = h$		
6 ⊙ 2	7	$9BB - 6Bc + cc = hh$		
$3 + cc$	8	$BB + cc = hh$		
$7 - 8$	9	$8BB - 6Bc = 0$		
$9 \div B$	10	$8B - 6c = 0$		
$10 + 6c$	11	$8B = 6c$		
$1 * 8$	12	$8B = 24\sqrt{2} + 24$		
11, 12	13	$6c = 24\sqrt{2} + 24$		
$12 \div 6$	14	$C = 4\sqrt{2} + 4$		
$2 - 14$	15	$H = 5\sqrt{2} + 5$		

9 x

Prob.

Prob. III. There is a Circle whose Diameter is 120, and the Sine is $\sqrt{2925-\sqrt{405000}}$, I would know the versed sine and Cosine. (*Fig.* III.)

$b=?$	1	$A=\sqrt{2925-\sqrt{405000}}$
$c=?$	2	$b+c=60$
1 ⊙ 2	3	$AA=2925-\sqrt{405000}$
Anal.	4	$60+c \,.\, A :: A \,.\, b$ (out of the Figure.)
∵	5	$60b+bc=AA$
3, 5	6	$60b+bc=2925-\sqrt{405000}$
$6 \div \overline{60+c}$	7	$b=\dfrac{2925-\sqrt{405000}}{60+c}$
$2-c$	8	$b=60-c$
7, 8	9	$60-c=\dfrac{2925-\sqrt{405000}}{60+c}$
$9 * \overline{60+c}$	10	$3600-cc=2925-\sqrt{405000}$
$10-\overline{2925}+cc$	11	$675=cc-\sqrt{405000}$
$11+\sqrt{405000}$	12	$675+\sqrt{405000}=cc$
12 ω 2	13	$\sqrt{675+\sqrt{405000}}=C=$ Cosine
$2-13$	14	$60-\sqrt{675+\sqrt{405000}}=B$ the versed sine.

Prob. IV. A Sine cutteth the Diameter in extream and mean proportion, and the double sine of half the Angle $=150-30\sqrt{5}$. I would know the *versed Sine, Cosine* and *Radius*. (*Fig* IV.)

$$\text{Set for}\left\{\begin{array}{l}\text{Versed sine}\\ \text{Cosine with Rad.}\\ \text{Diameter}\\ \text{Right Sine}\\ \text{Double Sine}\frac{1}{2}\text{Angle}\end{array}\right\}\begin{array}{l}a\\ b\\ a+b\\ d\\ c\end{array}$$

$a=?$	1	$c=150-30\sqrt{5}$
$b=?$	2	$a+b\,.\,b :: b\,.\,a$
$c=?$	3	$c=b$
	4	$a\,.\,d :: d\,.\,b$ (out of the Figure)
4 Ergo	5	$ab=dd$
2 Ergo	6	$ab+aa=bb$
3 ⊙ 2	7	$cc = bb$
6, 7	8	$cc=aa+ab$
3, 8	9	$cc=aa+ac$
1 ⊙ 2	10	$cc=27000-9000\sqrt{5}$
10 — aa	11	$cc-aa=27000-9000\sqrt{5}-aa$
1 * a	12	$ac=150a-30a\sqrt{5}$
9 — aa	13	$ac=cc-aa$
11, 12, 13.	14	$27000-9000\sqrt{5}-aa=150a-30a\sqrt{5}$
+		$+150a$
14	15	$aa=27000-9000\sqrt{5}$
—		$-30a\sqrt{5}$
		$+150a+6750$
15 C □	16	$aa=33750-11250\sqrt{5}$
		$-30a\sqrt{5}-2250\sqrt{5}$
		$+75$
16 ω 2	17	$a=\sqrt{33750-11250\sqrt{5}}$
		$-15\sqrt{5}$

+ 17 —	18	$\overline{}$ +15√5 $A=\sqrt{33570-11250\sqrt{5}}$ −75
1, 3.	19	$B=150-30\sqrt{5}$

More briefly thus,

1, 3.	20	$b=150-30\sqrt{5}$
$a+20$	21	$a+b=150-30\sqrt{5}+a$
2, 20, 21	22	$150-30\sqrt{5}+a \,.\, 150-30\sqrt{5} :: 150-30\sqrt{5} \,.\, a$
22 Ergo	23	$150a-30a\sqrt{5}+aa=27000-9000\sqrt{5}$, as in (14.)

Prob. V. In an equicrural right angl. Triangle $h+b=6$, how much is each? (*Fig.* I.)

$h=?$	1	$h+b=6$
$b=?$	2	$b=c$
$c=?$	3	$bb+cc=hh$ (by the Figure.)
1 − b	4	$h=6-b$
4 ◎ 2	5	$hh=36-12b+bb$
3 — 5	6	$0=cc-36+12b$
2, 6	7	$0=cc-36+12c$
$7+\overline{72}$	8	$72=cc+12c+36$
8 ω 2	9	+ — $\sqrt{72}=c+6$ —
$9-\overline{6}$	10	+ — $\sqrt{72}-6=C=B$ —
1 — 10	11	+ — 12 $\sqrt{72}=H$ —

Prob.

Prob. VI. In an Equicrural right angl. Triangle $C * h - b = 15$? What are See Figure (I.)

h and b^2

b c

$h = ?$	1	$hc - c = 15$
$b = ?$	2	$b = c$
	3	$bb + cc = hh$ (out of the Figure.)
1, 2	4	$hb - bb = 15$
4 $\pm$	5	$bb = hb - 15$
5 $*$ 2	6	$2bb = 2hb - 30$
2, 3	7	$2bb = hh$
7 — 6	8	$0 = hh - 2hb + 30$
8 — $\overline{30}$	9	$-30 = hh - 2hb$
9 + bb	10	$bb - 30 = hh - 2hb + bb$
10 ω 2	11	$\sqrt{bb - 30} = h - b$
11 $*$ b	12	$b\sqrt{bb - 30} = hb - bb$
4, 12	13	$b\sqrt{bb - 30} = 15$
13 ⊙ 2	14	$b^4 - 30bb = 225$
14 + $\overline{225}$	15	$bbbb - 30bb + 225 = 450$
15 ω 2	16	$bb - 15 = \pm\sqrt{450}$
16 + $\overline{15}$	17	$bb = 15 \pm \sqrt{450}$
17 ω 2	18	$B = \sqrt{15 \pm \sqrt{450}}$
17 $*$ 2	19	$2bb = 30 \pm 2\sqrt{450}$
19, 7	20	$hh = 30 \pm 2\sqrt{450}$
20 ω $\overline{2}$	21	$H = \sqrt{30 \pm 2\sqrt{450}}$

Prob.

Prob. VII. Three men have Money ;

$a=?$	1	$a-100=\frac{b+c}{4}$ The first $-100=\frac{1}{4}$
$b=?$	2	$b-100=\frac{a+c}{3}$ The second $-100=\frac{1}{3}$ of The other two :
$c=?$	3	$c-100=\frac{a+b}{2}$ The third $-100=\frac{1}{2}$ How much each ?
$1 * \overline{4}$	4	$4a-400=b+c$
$2 * \overline{3}$	5	$3b-300=a+c$
$3 * \overline{2}$	6	$2c-200=a+b$
$4 \overline{+}$	7	$4a-b-c=400$
$5 \overline{+}$	8	$-a+3b-c=300$
$6 \overline{+}$	9	$-a-b+2c=200$
$7+8+9$	10	$2a+b=900$
$7-8$	11	$5a-4b=100$
$10 * \overline{4}$	12	$8a+4b=3600$
$11+12$	13	$13a=3700$
$13 \div \overline{13}$	14	$A=\frac{3700}{13}=$ the sum of the first.
$14 * \overline{2}$	15	$2A=\frac{7400}{13}$
$10-15$	16	$B=\frac{4300}{13}$ the sum of the second.
$14+16$	17	$A+B=\frac{8000}{13}$

2c

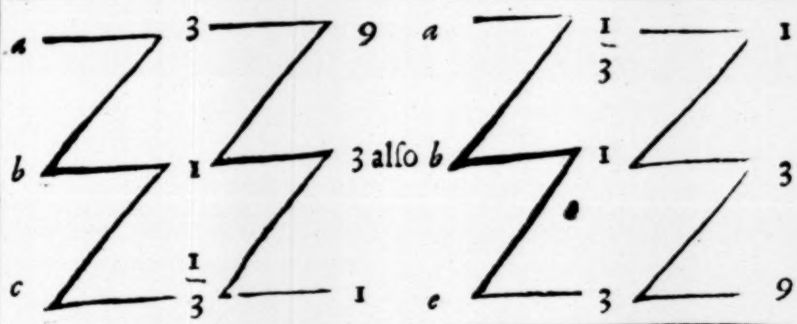
$9+17$	18	$2c=\frac{10600}{13}$
$18 \div 2$	19	$C=\frac{5300}{13}$ the sum of the third.

third
second

Prob. VIII. Three Numbers be continually proportional; and the sum of the first and [redacted] multiplied by the difference of the said sum, and the [redacted] is equal to 90720; but the aforesaid difference multiplied into the sum of all three is 117936. Which be the Numbers?

$\left.\begin{matrix} a \\ b \\ c \end{matrix}\right\} = ?$	1	$\overline{a+c} * \overline{a+c-b}=90720$
	2	$\overline{a+b+c} * \overline{a+c-b}=117936$
	3	$a.b::b.c$
1, 2	4	$\overline{a+c}.\overline{a+b+c}::90720.117936$ (because of the com. Factor $a+c-b$)
4 Ergo	5	$117936a+117936c=90720a+90720b+90720c$
$5-90720a$ $-90720c$	6	$\left.\begin{matrix} 27216a \\ +27216c \end{matrix}\right\}=90720b$
$6 \div 27216$	7	$a+c=\frac{10}{3}b$
7 ⊙ 2	8	$aa+2ac+cc=\frac{100}{9}bb$
3 Ergo	9	$ac=bb$
$9 * 4$	10	$4ac=\frac{36}{9}bb$

aa

8 — 10	11	$aa-2ac+cc=\frac{64}{9}bb$ X
11 ω 2	12	$a-c=\pm\frac{8}{3}b$
7 + 12	13	$2a=6b$. also $=\frac{2}{3}b$
13 ÷ $\overline{2}$	14	$a = 3b$ ∵ also $=\frac{1}{3}b$
7 — 12	15	$2C=\frac{2}{3}b$ also $=6b$
15 ÷ $\overline{2}$	16	$C=\frac{1}{3}b$ ∵ also $=3b$
14, 16	17	
2, 17	18	$9a+3a+a * 9a+a-3a=117936$. or $\boxed{\frac{a+3a+9a}{a+9a-3a}}$
18,	19	$91aa=117936$
19 ÷ 91	20	$aa = 1296$
20 ω 2	21	$A=\pm 36=C$
21 * $\overline{3}$	22	$B=\pm 108=B$
21 * $\overline{9}$	22	$C=\pm 324=A$

a — c

Prob. IX. There be three Numbers in ***continual*** Proportion; their ſum is 74, and the ſum of their Squares 1924.

$a = ?$	1	$a+b+c=74$
$b = ?$	2	$aa+bb+cc=1924$
$c = ?$	3	$a.b::b.c$
1 ☉ 2	4	$aa+bb+cc+2ab+2ac+2bc=5476$
4 − 2	5	$2ab+2ac+2bc=3552$
3 ∴	6	$ac=bb$
5, 6	7	$2ab+2bb+2bc=3552$
1 ✱ 2	8	$2a+2b+2c=148$
7 ÷ 8	9	$B=24$
1 − 9	10	$a+c=50$
9 ☉ 2	11	$bb=576=ac$ (3)
10 ☉ 2	12	$aa+2ac+cc=2500$
11 ✱ 4	13	$4ac=2304$
12 − 13	14	$aa-2ac+cc=196$
14 ω 2	15	$a-c = \pm 14$
10 + 15	16	$2a=64$ Alſo $=36$
16 ÷ 2	17	$A=32$ Alſo $=18$
10 − 17	18	$C=18$ Alſo $=32$

Prob. X. The Difference of two Numbers is 12, and their Product multiplied into their ſum is $=$ 14560.

$a = ?$	1	$a-b=12$
$b = ?$	2	$ab * \overline{a+b}=14560$
$c = ?$	3	$a+b=2c$ [$c=\frac{1}{2}$ Sum]
1 + b	4	$a=12+b$
1 + 3	5	$2a=2c+12$
5 ÷ 2	6	$a=c+6$
6 − 1	7	$b=c-6$
6 ✱ 7	8	$ab=cc-36$

ab

2, 8, 3,	9	$ab * a+b = cc-36 * 2c = 14560$
9,	10	$2ccc-72c = 14560$
$10 \div \overline{2}$	11	$ccc-36c = 7280$
$11 - \overline{7280}$	12	$ccc-36c-7280 = 0$
$13 \div c - \overline{20}$	13	$cc+20c+364 = 0$ ∴ *ergo* $c = 20$ (For by this Division I find that had I divided the Divident $ccc-36c-7280$, by $cc+2cc+364$, the Quotient must have been $c-20 = 0$.
$13 * \overline{2}$	14	$2c = 40$
3, 14	15	$a+b = 40$
$1 + 15$	16	$2a = 52$
$16 \div \overline{2}$	17	$A = 26$
$15 - 17$	18	$B = 14$

Since the value of the quantity *c* is expressed in a Cubick Equation, therefore *c* hath two roots more.

$13 - \overline{364}$	19	$cc+20c = -364$
19 C □	20	$cc+20c+100 = -264$
20 ω 2	21	$c+10 = \pm\sqrt{-264}$
$21 - \overline{10}$	22	$C = +\sqrt{-264}-10 =$ ƆI
$21 - \overline{10}$	23	$C = -\sqrt{-264}-10 =$ ƆI

Prob. XI. Three men [A. B. C.] divide a sum, A. gives both the other as much as they had before; B. then gives both the other as much as now they have, and C. doth the like, and at last each man had 8. How much had each man at first?

	First.	Second.	Third
1	a	b	c
2	$a-b-c$	$2b$	$2c$
3	$2a-2b-2c$	$-a+3b-c$	$4c$
4	$4a-4b-4c$	$-2a+6b-2c$	$-a-b+7c$.

And thus much had each after last Division.

Prob. IX. There be three Numbers in ***continual*** Proportion; their sum is 74, and the sum of their Squares 1924.

$a = ?$	1	$a+b+c=74$
$b = ?$	2	$aa+bb+cc=1924$
$c = ?$	3	$a . b :: b . c$
1 ⊙ 2	4	$aa+bb+cc+2ab+2ac+2bc=5476$
4 — 2	5	$2ab+2ac+2bc=3552$
3 ∵	6	$ac=bb$
5, 6	7	$2ab+2bb+2bc=3552$
1 * $\bar{2}$	8	$2a+2b+2c=148$
7 ÷ 8	9	$B=24$
1 — 9	10	$a+c=50$
9 ⊙ 2	11	$bb=576=ac$ (3)
10 ⊙ 2	12	$aa+2ac+cc=2500$
11 * $\bar{4}$	13	$4ac=2304$
12 — 13	14	$aa-2ac+cc=196$
14 ω 2	15	$a-c = \pm 14$
10 + 15	16	$2a=64$ Also $=36$
16 ÷ $\bar{2}$	17	$A=32$ Also $=18$
10 — 17	18	$C=18$ Also $=32$

Prob. X. The Difference of two Numbers is 12, and their Product multiplied into their sum is = 14560.

$a = ?$	1	$a-b=12$
$b = ?$	2	$ab * a+b=14560$
$c = ?$	3	$a+b=2c$
1 + b	4	$a=12+b$ [$c = \frac{1}{2}$ Sum]
1 + $\bar{3}$	5	$2a=2c+12$
5 ÷ $\bar{2}$	6	$a=c+6$
6 — 1	7	$b=c-6$
6 * 7	8	$ab=cc-36$

ab

2, 8, 3,	9	$ab * \overline{a+b} = cc - 36 * 2c = 14560$
9,	10	$2ccc - 72c = 14560$
$10 \div \overline{2}$	11	$ccc - 36c = 7280$
$11 - \overline{7280}$	12	$ccc - 36c - 7280 = 0$
$12 \div \overline{c - 20}$	13	$cc + 20c + 364 = 0$ ∴ *ergo* $c = 20$ (For by this Division I
$13 * \overline{2}$	14	$2c = 40$ find that had I divided the Divident $ccc - 36c$
3, 14	15	$a + b = 40$ -7280, by $cc + 20c + 364$, the Quotient
$1 + 15$	16	$2a = 52$ must have been $c - 20 = 0$.
$16 \div \overline{2}$	17	$A = 26$
$15 - 17$	18	$B = 14$

Since the value of the quantity *c* is expressed in a Cubick Equation, therefore *c* hath two roots more.

$13 - \overline{364}$	19	$cc + 20c = -364$
19 C □	20	$cc + 20c + 100 = -264$
20 ω 2	21	$c + 10 = \pm\sqrt{-264}$
$21 - \overline{10}$	22	$C = +\sqrt{-264} - 10 =$ ƆI
$21 - \overline{10}$	23	$C = -\sqrt{-264} - 10 =$ ƆI

Prob. XI. Three men [A. B. C.] divide a sum, A. gives both the other as much as they had before; B. then gives both the other as much as now they have, and C. doth the like, and at last each man had 8. How much had each man at first?

	First.	Second.	Third
1	a	b	c
2	$a - b - c$	$2b$	$2c$
3	$2a - 2b - 2c$	$-a + 3b - c$	$4c$
4	$4a - 4b - 4c$	$-2a + 6b - 2c$	$-a - b + 7c$.

And thus much had each after last Division.

4,	5	$4a-4b-4c=8$
4,	6	$-2a+6b-2c=8$
4,	7	$-a-b+7c=8$
$5 \div 4$	8	$a-b\cancel{-c}=2$ $\quad -c$
$6 \div 2$	9	$-a+3b-c=4$
$7+8$	10	$-2b+6c=10$
$8+9$	11	$+2b-2c=6$
$10+11$	12	$4c=16$
$12 \div 4$	13	$C=4$
$11 \ast 3$	14	$+6b-6c=18$
$10+14$	15	$4b == 28$
$15 \div 4$	16	$B = 7$
$13+16$	17	$B+c=11$
$8+17$	18	$A = 13$

So that A. B. C. had 13, 7, 4.

Prob. XII. In a right-ang. Triangle the Difference between h and b or c being given, find the reason of the rest.

$h=?$	1	$h=b+D$ (Let the Difference be D.)
$b=?$	2	$hh=bb+cc$ (*Fig.* I.)
$c=?$	3	(✱) See *Probleme* XV.
1 ⊙ 2	4	$hh=bb+2bD+DD$
$2-4$	5	$0\ast=cc-2bD-DD$
$5+2bD+DD$	6	$cc=2bD+DD$
6,	7	$cc>DD$ (For c and D you may take any Numbers at plea-
7 ω 2	8	$c>D$ sure, provided c be $>D$.

Examples

Examples.

Let	9	$c=3$	$=7$	4	
and	10	$D=1$	$=5$	2	
9 ⓖ 2	11	$cc=9$	$=49$	16	
10 ⓖ 2	12	$DD=1$	$=25$	4	
6, 11, 12	13	$9=2b+1$	$49=10b+25$	$16=4b+4$	
13 — 1	14	$8=2b$	$24=10b$	$12=4b$	&c.
14 ÷ 2	15	$4=B$	$\frac{12}{5}=B$	$3=B$	
10 + 15	16	$5=B+D$	$7\frac{2}{5}=B+D$	$5=B+D$	
1, 16	17	$5=H$	$7\frac{2}{5}=H$	$5=H$	

Prob. XIII. In a right-ang. Triangle the difference of b and c being known, find the rest.

$h=?$	1	$hh-bb=cc$
$b=?$	2	$b-D=c$ (The Difference is D) (*Fig.* V.
$c=?$	3	(∗) See *Probleme* XV.
2 ⓖ 2	4	$bb-2bD+DD=cc$
4 — 1	5	$2bb-2bD+DD-hh=0$
5 + hh	6	$2bb-2bD+DD=hh$
6 ÷ 2	7	$bb-bD+\frac{1}{2}DD=\frac{1}{2}hh$
$7-\frac{1}{4}DD$	8	$bb-bD+\frac{1}{4}DD=\frac{1}{2}hh-\frac{1}{4}DD$

$b-$

8 ω 2	9	$b - \frac{1}{2}D = \pm\sqrt{\frac{1}{2}hh - \frac{1}{4}DD}$
$9 + \frac{1}{2}D$	10	$= \frac{1}{2}D \pm \sqrt{\frac{1}{2}hh - \frac{1}{4}DD}$
2 + D	11	$b = D + c$ } For B and D may be taken any Numbers at pleasure, provided $B > D$.
11 − c	12	$B > D$
Let	13	$B = 8$
and	14	$D = 2$
13 − 14	15	$B - D = 6$
2, 13, 15	16	$c = 6$
13 ⊙ 2	17	$BB = 64$
16 ⊙ 2	18	$cc = 36$
17 + 18	19	$BB + cc = 100$
1 + BB	20	$BB + cc = hh$
19, 20	21	$100 = hh$
21 ω 2	22	$\pm 10 = H$

Or else thus.

	13	$B = 8$
	14	$D = 2$
14 ÷ 2	15	$\frac{1}{2}D = 1$
13 − 15	16	$B - \frac{1}{2}D = 7$

16, 9	17	$+\text{or}-\sqrt{-\frac{1}{2}bb-\frac{1}{4}DD}=7$	$19\div 4$	20	$\frac{1}{4}DD=1$
17 ⊙ 2	18	$\frac{1}{2}bb-\frac{1}{4}DD=49$	$18+20$	21	$\frac{1}{2}bb=50$
14 ⊙ 2	19	$DD=4$	$21 * 2$	22	$bb=100$. As in pag. 78

Probl. XIV. *Having all the sides of any right-lined Triangle to find its* Area *[as $\frac{1}{2}$ Ad in Fig. VI.]*

$d=?$	1	$dd+ee=BB$	$3+5$	6	$2e=\frac{AA+BB-CC}{A}$
$e=?$	2	$dd+ff=CC$	$3-5$	7	$2f=\frac{AA-BB+CC}{A}$
$f=?$	3	$e+f=A$	$6\div 2$	8	$E=\frac{AA+BB-CC}{2A}$
$1-2$	4	$ee-ff=BB-CC$	$7\div 2$	9	$F=\frac{AA-BB+CC}{2A}$
$4\div 3$	5	$e-f=\frac{BB-CC}{A}$			

By *B* and *E*, or by *C* and *F*, you may find *d*. Then *DA* is the double of the *Area*. But without the help of [*d*] the perpendicular, or [*e*, *f*] the segments of the Base, you may find the *Area*, working thus,

$6 * A$	10	$2Ae=AA+BB-CC$ (Or $7 * A$	10	$2Af=AA-BB+CC$
$1-ee$	11	$dd=BB-ee$ $2-ff$	11	$dd=CC-ff$ &c.
10 ⊙ 2	12	$4AAee=aaaa+2aabb+bbbb-2aacc-2bbcc+cccc$.
$11 * 4aa$	13	$4AAdd=4AABB-4AAee$
$4aabb-12$	14	$4AABB-4AAee=2aabb+2aacc+2bbcc-aaaa-bbbb-cccc$.
13, 14	15	$4AAdd=2aabb+2aacc+2bbcc-aaaa-bbbb-cccc$.
15 ω 2	16	$2Ad=\sqrt{2aabb+2aacc+2bbcc-aaaa-bbbb-cccc}$

That is to say, this root is equal to the *quadruple of the Area* of the Triangle, whose sides are *A.B.C.*

	17	$4AABB-4AAee=2AB+2Ae$ upon $2AB-2Ae$	Pag. 12. Examp. 1.
$2ab+10$	18	$2AB+2Ae=+aa+2ab+bb-cc=a+b+c$ upon $a+b-c$	
$2ab-10$	19	$2AB-2Ae=-aa+2ab-bb+cc=a-b+c$ upon $-a+b+c$	
16, 14	20	Therefore the right lined Triangle, whose sides are *A. B. C.* hath an *Area*, whose Quadruple is the root of the Product $a+b+c * a+b-c * a-b+c * -a+b+c$	
16, 20	21	Therefore, from [$a+b+c$] the Perimeter, substract severally the double of each of the three sides. Multiply the Perimeter upon that which is made by continual multiplication of those three remainders, you have the Product whose root is the quadruple of the Area of that given Triangle.	

Probl

Probl. XV. *To make a Rectangled Triangle, whereof one leg is equal to the square of the other leg.*

Understand it Arithmetically, as if it had been thus; A number is equal to the square of *another* number: The square of a *third* number is equal to the summ of the squares of the other two. What are those 3 numbers?

$h=?$	1	$hh=bb+cc$
$b=?$	2	$c=bb$
$c=?$	3	(*)

Of this (*) see Pag. 12 Note 1. and Pag. 49. l. 18. and pag. 50 lin. 13. But in Pag. 76 and 77, as here it shews *the defect of an Equation.* For in Probl. 12 and 13, and this fifteenth, the Margin shews that three Equations are required; but the Question affords not *more than two. One* Equation is wanting, to make up the number of *given* Equations, *equal* to the number of *sought* Equations. For such *Equality* is necessary for the *limiting* of a question, that the answers may not be *innumerable.*

Whensoever the number of *required* Equations is *greater* than the number of *given* ones; The Question is capable of *innumerable* Answers. Such was Probl. 12, and 13. And such is this XV Problem.

If a *given* Equation can be drawn out of one or all the rest of those that are *given*; it is to be accounted as *not given*, in this Rule for limitations. If in this XV Problem, $hh=cc+c$ had been given for a *third* Equat. the Problem would have remained as unlimited as before, because this, $hh=cc+c$, ariseth from Equat. 1 and 2.

A Problem hath a certain determinable number of Answers, when the *given* Equations, *not mutually depending upon one another*, are as many as the *sought* Equations, or Terms unknown in the Question. For then each term hath one single value. (How plurality of dimensions makes a term capable of 2, or 3, or more values, is to be shewn in a fitter place.)

In this XV Problem, for a *third* Equation, you may *assume* $B=$ any number. For then $c=BB$ $hh=BB+CC \therefore H=\sqrt{BB+CC}=\sqrt{BB+BBBB}$. As, $B=1 \therefore C=1 \therefore H=\sqrt{2}$. Or $B=2 \therefore C=4 \therefore H=\sqrt{20}$. Or $B=3 \therefore C=9 \therefore H=\sqrt{90}$

We may *avoid* these *surd* values of H, by working thus,

2 ⊛ 2	4	$cc=bbbb$	$a=?$	9	Let $\frac{h}{b}+b=a$	$9-10$	12	$2b=\frac{aa-1}{a}$
$bb+4$	5	$bb+cc=b^4+bb$						
1, 5	6	$hh=bbbb+bb$	$8\div 9$	10	$\frac{h}{b}-b=\frac{1}{a}$	$11\div 2$	13	$\frac{h}{b}=\frac{aa+1}{2a}$
$6\div bb$	7	$hh\div bb=bb+1$						
$7-bb$	8	$\frac{hh}{bb}-bb=1$	$9+10$	11	$\frac{2h}{b}=\frac{aa+1}{a}$	$12\div 2$	14	$b=\frac{aa-1}{2a}$ $h=$

13 ✱ 14	15	$b=\frac{aaaa-1}{4aa}$		25	$b>0$						
				26	$a>0$						
14 ⊕ 2	16	$bb=\frac{aaaa-2aa+1}{4aa}$	24, 25, 26	27	$aa-1>0$						
			$27+\bar{1}$	28	$aa>1$						
2, 16	17	$c=\frac{aaaa-2aa+1}{4aa}$	26, 28	29	$A>1$						
			29	30	A	2	3	4	5	6	7
14	18	$b=\frac{2aaa-2a}{4aa}$	$30 ✱ \bar{2}$	31	$2A$	4	6	8	10	12	14
			30 ⊕ 2	32	AA	4	9	16	25	36	49
15, 18, 17	9	b — $aaaa-1$	$32-\bar{1}$	33	$AA-1$	3	8	15	24	35	48
		b — $2aaa-2a$	$33\div31$	34	$\frac{AA-1}{2A}$	$\frac{3}{4}$	$\frac{4}{3}$	$\frac{15}{8}$	$\frac{12}{5}$	$\frac{35}{12}$	$\frac{24}{7}$
		c — $aaaa-2aa+1$									
		Proof.	14, 34	35	B	$\frac{3}{4}$	$\frac{4}{3}$	$\frac{15}{8}$	$\frac{12}{5}$	$\frac{35}{12}$	$\frac{24}{7}$
$\frac{19}{aa-1}$	20	h — $aa+1$; b — $2a$; c — $aa-1$	35 ⊕ 2	36	BB	$\frac{9}{16}$	$\frac{16}{9}$	$\frac{225}{64}$	$\frac{144}{25}$	$\frac{1225}{144}$	$\frac{576}{49}$
20 ⊕ 2	21	hh — $\frac{aaaa+2aa+1}{4aa}$; bb; cc — $aaaa-2aa+1$	36, 2	37	C	$\frac{9}{16}$	$\frac{16}{9}$	$\frac{225}{64}$	$\frac{144}{25}$	$\frac{1225}{144}$	$\frac{576}{49}$
			$32 ✱ \bar{4}$	38	$4AA$	16	36	64	100	144	196
21	22	$hh=bb+cc$	$32+\bar{1}$	39	$AA+1$	5	10	17	26	37	50
16, 17	23	$c=bb$ As was required	38, 39 }	40	H	$\frac{15}{16}$	$\frac{20}{9}$	$\frac{255}{64}$	$\frac{156}{25}$	$\frac{129[illegible]}{144}$	$\frac{6[illegible]}{49}$
15	24	$4aa.aa+1::a-1.h$	33, 24 }								
						5.4.3.	5.3.4	17.8.15	13.5.12	37.12.35	25 7.24

Another way; without A.

b,c,b [sides of any rectangled Triangle] multiplied by $\frac{c}{bb}$, or by $\frac{b}{cc}$ will give H, C, B fit answers to this XV Problem. As in these 14 Examples:

b	5	13	17	25	29	37
c	4	12	8	24	20	12
b	3	5	15	7	21	35
$hc\div bb=H$	$20\div9$	$156\div25$	$136\div225$	$600\div49$	$580\div441$	$444\div1225$
$cc\div bb=C$	$16\div9$	$144\div25$	$64\div225$	$576\div49$	$400\div441$	$144\div1225$
$bc\div bb=B$	$12\div9$	$60\div25$	$120\div225$	$168\div49$	$420\div441$	$420\div1225$
$c\div b=B$	$4\div3$	$12\div5$	$8\div15$	$24\div7$	$20\div21$	$12\div35$
$hb\div cc=H$	$15\div16$	$65\div144$	$255\div64$	$175\div576$	$609\div400$	$1295\div144$
$bb\div cc=C$	$9\div16$	$25\div144$	$225\div64$	$49\div576$	$441\div400$	$1225\div144$
$cb\div cc=B$	$12\div16$	$60\div144$	$120\div64$	$168\div576$	$420\div400$	$420\div144$
$b\div c=B$	$3\div4$	$5\div12$	$15\div8$	$7\div24$	$21\div20$	$35\div12$

Probl. XVI. *To make a right-angled Triangle whose longest side may be* a cube *with its own root; one of the legs may be a* cube; *the other leg may be* a cube *wanting its own root.*

$h=?$	1	$hh=bb+cc$
$b=?$	2	$h=ddd+d$
$c=?$	3	$b=eee$
$d=?$	4	$c=fff-f$
$e=?$	5	(*)
$f=?$	6	(*)

This must be understood of Numbers, as Probl. XV. was If *the same Cube* had been meant in all three places, there had been but four terms in the question [h,b,c,d]. But here being expressed as largely as the words will bear, *it requires six* Equations, though it *affords but four.* So that two are wanting (see pag. 80. lin. 12.) and therefore the *fifth* and *sixth* Equations are noted with a sign of defect.

6 (*)	7	Let $d=f$
7, 8, 4	8	$ddd-d=fff-f$
	9	$c=ddd-d$
$2 \odot 2$	10	$hh=d^6+2d^4+dd$
$9 \odot 2$	11	$cc=d^6-2d^4+dd$
$10-11$	12	$hh-cc=4dddd$
$1-cc$	13	$hh-cc=bb$
12, 13	14	$4dddd=bb$
$14\ \omega\ 2$	15	$2dd=b$
15, 3	16	$2dd=eee$
$g=?$	17	Let $e=gd$
$17 \odot 3$	18	$eee=gggddd$
16, 18	19	$2dd=gggddd$
$19 \div dd$	20	$gggd=2$
$20 \div d$	21	$ggg=\frac{2}{d}$
$20 \div ggg$	22	$d=2 \div ggg$
$22 \odot 2$	23	$dd=4 \div g^6$
$23 * 2$	24	$2dd=8 \div g^6$
16, 24	25	$eee=8 \div g^6$
15, 3	26	$b=8 \div g^6$
$22 * 23$	27	$ddd=8 \div g^9$
22	28	$d=2g^6 \div g^9$
$27+28$	29	$ddd+d=\frac{2g^6+8}{g^9}$

6 (*) In the margin of the 7 Equation signifies a liberty of *assuming* an Equation, because *the sixth is wanting.* As here I assume $f=d$, that is, of the *Cubes* mentioned in the Question, the *third* is equal to the *first*. So f is rejected: But the *number of answers* is thereby much *diminished.* Yet, after this supply of the *sixth*, the defect of the *fifth* continuing, is sufficient to cause the remaining answers to be innumerable. See pag. 80. lin. 15.

The *habitude* of *h. b. c. e. f.* to *d*
is in Equation 2. 15. 9. 16. 7

To find what *ratio* of *d* to *e*, may stand with all these habitudes, I may safely say *d* is to *e* as 1 is to some unknown quantity, which I call *g*; and therefore I make $d.e::1.g$, or $e=gd$ the 17th. Equation, setting $g=?$ in the margine to signifie that *g* is *yet unknown*, but is to be *sought.*

An Equation so *assumed* to express the *habitude* of a *new* term to some of the *old* ones, alters not the number of answers. So Equ. 9. was assumed in pag 80. without any inconvenience.

d

27 — 28	30	$d^3 - d = \frac{8-2g^6}{g^9}$	
2, 29	31	$H = \frac{2g^6+8}{g^9}$	
9, 30	32	$C = \frac{8-2g^6}{g^9}$	
	33	$C > 0$	
32, 33	34	$\frac{8-2g^6}{g^9} > 0$	
$34 * g^9$	35	$8-2g^6 > 0$	
$35 + 2g^6$	36	$8 > 2g^6$	
$36 \div 2$	37	$4 > g^6$	
37 ω 2	38	$2 > g^3$	
38 ω 3	39	$\sqrt{c.2} > g$	[Therefore $\sqrt{c.2}$ (or 1 | 259) is $> g$.]

Illustration in Numbers.

g may be taken at pleasure, provided it be less than 1 | 259

Let	40	$g = \frac{1}{3}$	$\frac{1}{2}$
40 ⊙ 3	41	$ggg = \frac{1}{27}$	$\frac{1}{8}$
21, 41	42	$\frac{2}{d} = \frac{1}{27}$	$\frac{1}{8}$

$42 * X$	43	$54 = D$	16
$43 \odot 2$	44	$2916 = DD$	256
$44 * 2$	45	$5832 = DD$	512
15, 45	46	$5832 = B$	512
$43 \odot 3$	47	$157464 = DDD$	4096
$43 + 47$	48	$157518 = DDD + D$	4112
2, 48	49	$157518 = H$	4112
$47 - 43$	50	$157410 = DDD - D$	4080
9, 50	51	$157410 = C$	4080

Prob. XVII. How to find a Theoreme according to which all three sides of a Right-ang. Triangle shall be *Rational*?

Case I. When one side is given.

	1	$bb + cc = hh$
Let	2	$b + d = h$ (i. e. $b + d = \sqrt{bb + cc}$)
$2 \odot 2$	3	$bb + 2bd + dd = hh$
$3 - 1$	4	$2bd + dd - cc = 0$ (*Fig.* I.)
$4 + cc$	5	$2bd + dd = cc$
$5 - dd$	6	$2bd = cc - dd$
$6 \div 2d$	7	$B = \frac{cc - dd}{2d}$ or, $C = \frac{bb - dd}{2d}$ by the same reason.

The use of this Theoreme.

Let now	8	$B > 0$
7, 8	9	$\frac{cc - dd}{2d} > 0$
$9 * 2d$	10	$cc - dd > 0$
$10 + dd$	11	$cc > dd$ } So that therefore c must be greater than d, but otherwise it may be taken at pleasure.
$11 \omega 2$	12	$c > d$

Illustration

Illustration in Numbers.

Let	13	$c=4$	7	80
and	14	$D=2$	6	20
13 ⓖ 2	15	$cc=16$	49	6400
14 ⓖ 2	16	$DD=4$	36	400
15 − 16	17	$cc-DD=12$	13	6000
14 * 2	18	$2D=4$	12	40
17 ÷ 18	19	$\frac{cc-DD+3}{2D}$	$\frac{13}{12}$	150
7, 19	20	$B=3$	$\frac{13}{12}$	150
20 ⓖ 2	21	$BB=9$	$\frac{169}{144}$	225000
15 + 21	22	$BB+CC=25$	$\frac{7225}{144}$	28900
22, 1	23	$hh=25$	$\frac{7235}{144}$	28900
23 ω 2	24	$H=5$	$\frac{85}{12}$	170

Case II.

Case II. When no side is given.

	1	$B = \frac{cc - dd}{2d}$ = the Side found just above.
	2	$C = \frac{2cd}{2d}$ = the other side, so that c is brought under the same Letters.
$1 \odot 2$	3	$bb = \frac{cccc - 2ccdd + d^4}{4dd}$
$2 \odot 2$	4	$cc = \frac{4ccdd}{4dd}$
$3 + 4$	5	$bb + cc = \frac{c^4 + 2ccdd + d^4}{4dd} = hh$
$5 \omega 2$	6	$\frac{cc + dd}{2d} = H =$ the third side.
1, 2, 6	7	$\left\{\begin{array}{lcc} B & \frac{cc - dd}{2d} & cc - dd \\ C & \frac{2cd}{2d} & 2cd \\ H & \frac{cc + dd}{2d} & cc + dd \end{array}\right\}$ By help of this Theorem innumerable right-angled Triangles may be discovered, whose sides be all rational and Integers.

Use

Use of this Theoreme.

1	$B > 0$	*c* and *d* may be taken at pleasure, yet so as *c* be greater than *d*.
2	$cc - dd > 0$	
3	$cc > dd$	
4	$c > c$	

Illustration in Numbers.

Let	5	$c=2$	3	5	10	8	12	13
	6	$d=1$	1	1	3	7	8	12
5 ⊙ 2	7	$cc=4$	9	25	100	64	144	169
6 ⊙ 2	8	$dd=1$	1	1	9	49	64	144
7 + 8	9	$cc+dd=5=h$	10	26	109	113	208	313 $=h$
7 − 8	10	$cc-dd=3=b$	8	24	91	15	80	25 $=b$
5 * 6	11	$cd=2$	3	5	30	56	96	156
11 * 2	12	$2cd-4=c$	6	10	60	112	192	312 $=C$

Prob. XVIII. To find a Theoreme by help of which the Perpendicular and Segments of the Base of an oblique-ang. Triangle may be Rational.

I. *Case.*

When Perpend. is without the Triangle. (*Fig.* VII.)

$a, b, c, d, e = ?$	1	$dd = bb + cc$
	2	$aa + 2ab + bb + cc = ee$
	3	(*)
	4	(*)
	5	(*)
Let	6	$d = b + f$
6 ⊙ 2	7	$dd = bb + 2bf + ff$
7 − 1	8	$0 = \quad 2bf + ff - cc$
8 + cc	9	$cc = \quad 2bf + ff$

cc

$9 - ff$	10	$cc - ff = 2bf$
$10 \div 2f$	11	$\dfrac{cc - ff}{2f} = B$
Let	12	$a + b + g = e$
$12 ⊙ 2$	13	$aa + 2ab + bb + gg + 2ag + 2bg = ee$
$13 - 2$	14	$gg + 2ag + 2bg - cc = 0$
$14 + cc$	15	$gg + 2ag + 2bg = cc$
$15 -- gg - 2bg$	16	$2ag = cc - gg - 2bg$
$11 * 2g$	17	$\dfrac{2ccg - 2ffg}{2f} = 2bg$
16, 17	18	$2ag = cc - gg - \dfrac{2ccg + 2ffg}{2f}$
18,	19	$2ag = \dfrac{2ccf - 2fgg - 2ccg + 2ffg}{2f}$
$19 \div 2g$	20	$A = \dfrac{ccf - fgg - ccg + ffg}{2fg}$

The Use of these Theoremes.

	21	$b > 0$
11	22	$\dfrac{cc - ff}{2f} > 0$
$22 * 2f$	23	$cc - ff > 0$
$23 + ff$	24	$cc > ff$
$24\, ω\, 2$	25	$C > f$ So that c must be greater than f
$16 \div 2g$	26	$a = \dfrac{cc - gg - 2bg}{2g}$
26,	27	$a > 0$
26, 27	28	$\dfrac{cc - gg - 2bg}{2g} > 0$

cc

$28 * 2g$	29	$cc - gg - 2bg > 0$
$29 + g^2\ 2bg$	30	$cc > gg + 2bg$
30,	1	$cc > gg$
$1\,3\ \omega\ 2$	2	$C > g$. So that C must be greater than g.
	33	$A > 0$
20	34	$\frac{ccf - fgg - ccg + ffg}{2fg} > 0$
$34 * 2fg$	35	$ccf - fgg - ccg + ffg > 0$
$35 +$	36	$ffg - fgg > ccg - ccf$
$36 \div g - f$	37	$\frac{ffg - ffgg}{g - f} > cc$
37,	38	$\frac{ffg - fgg}{g - f} > 0$
$38 * g - f$	39	$ffg - fgg > 0$
$39 \div -gf$	40	$f - g > 0$
$40 + g$	41	$F > G$

Here c, f, and g may be taken at pleasure, provided that c be greater than f, and than g; also that f be greater than g

Illustration in Numbers.

Let	42	$f = 5$ (Fig.IX.)	4 (Fig. X.)	2 (Fig.VIII.) 4
	43	$g = 4$	3	1
	44	$c = 6$	5	3
$42 \circledcirc 2$	45	$ff = 25$	16	4
$43 \circledcirc 2$	46	$gg = 16$	9	1
$44 \circledcirc 2$	47	$cc = 36$	25	9
$45 * 43$	48	$ffg = 100$	48	4
$46 * 42$	49	$fgg = 80$	36	2
$47 * 43$	50	$ccg = 144$	75	9
$47 * 42$	51	$fcc = 180$	100	18
$51 - 49 - 50 + 48$	52	$ccf - fgg - ccg + ffg = 56$	37	11
$42 * 43$	53	$fg = 20$	12	2
$53 * 2$	54	$2fg = 40$	24	4
$52 \div 54$	55	$\frac{ccf - ffg - ccg + ffg}{2fg} = \frac{7}{5}$	$\frac{37}{24}$	$\frac{11}{4}$

20, 55	56	$A=\frac{7}{5}=\frac{14}{10}$	or 14	$\frac{37}{24}$, 37	$\frac{11}{4}$, 11 = 1 side
47 − 45	57	$cc-ff=11$		$\frac{9}{8}$	$\frac{5}{4}$
42 * 2	58	$2f=10$			
57 ÷ 58	59	$\frac{cc-ff}{2f}=\frac{11}{10}$		$\frac{9}{8}$	$\frac{5}{4}$
11, 59	60	$B=\frac{11}{10}$	11	$\frac{9}{8}$, 27	$\frac{5}{4}$, 5 = segment
56 + 60	61	$A+B=\frac{25}{10}$		$\frac{8}{3}$	4
44,	62	$C=\frac{60}{10}$	60	$\frac{15}{3}$ 120	3, 12 = perp.
43 + 61	63	$A+B+g=\frac{65}{10}$		$\frac{17}{3}$	5
12, 63	64	$E=\frac{65}{10}$		$\frac{17}{3}$ 136	5, 20 = 2 side
42 + 60	65	$B+F=\frac{61}{10}$		$\frac{41}{8}$	$\frac{13}{4}$
6, 65	66	$D=\frac{61}{10}$	61	$\frac{41}{8}$, 143	$\frac{13}{4}$, 13 = 3 side

2. *Case, when the perpend. falls within the Triangle.*

(Fig. XI.)

$\left.\begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix}\right\} = ?$	1	$cc=dd-bb$
	2	$cc=-aa+ee$
	3	(*)
	4	(*)
	5	(*)

b+-

$f=?$	6	$b+f=d$
$6 \circledcirc \overline{2}$	7	$bb+2bf+ff=dd$
$1+bb$	8	$cc+bb=dd$
$7-8$	9	$2bf+ff-cc=0$
$9+cc$	10	$2bf+ff=cc$
$10-ff$	11	$2bf=cc-ff$
$11 \div 2f$	12	$b=\dfrac{cc-ff}{2f}$

$2+aa$	13	$aa+cc=ee$
$g=?$	14	$a+g=e$
$14 \circledcirc \overline{2}$	15	$aa+2ag+gg=ee$
13, 15	16	$aa+2ag+gg=aa+cc$
$16-aa$	17	$2ag+gg=cc$
$17-gg$	18	$2ag=cc-gg$
$18 \div 2g$	19	$a=\dfrac{cc-gg}{2g}$

Thd Vse of these Theoremes.

In the Probleme next aforegoing c is greater than f and than g, which is also here true upon the same ground.

But in this case f may be greater than g, and g greater than f.

Illustration in Numbers.

Let	20	$f=1$	5
	21	$g=2$	2
	22	$c=3$	6 or = 60 perpendicular
$22 \circledcirc \overline{2}$	23	$cc=9$	36
$20 \circledcirc \overline{2}$	24	$ff=1$	25
$20 * \overline{2}$	25	$2f=2$	10
$23-24$	26	$cc-ff=8$	11
$26 \div 25$	72	$\dfrac{cc-ff}{2f}=4$	$\dfrac{11}{10}$

12, 27	28	$B = 4 = 1$ segment	$\frac{11}{10} = 1$ segm. or 11
21 ⊖ $\overline{2}$	29	$gg = 4$	4
23 — 29	30	$cc - gg = 5$	32
21 * $\overline{2}$	31	$2g = 4$	4
30 ÷ 31	32	$\frac{cc - gg}{2g} = \frac{5}{4}$	8
32, 19	33	$A = \frac{5}{4} = 2$ segment	$8 = 2$ segm. $= 80$
28 + 33	34	$A + B = \frac{21}{4} = 1$ side	$\frac{91}{10} = 1$ side $= 91$
33 + 21	35	$A + G = \frac{13}{4}$	10
14, 35	36	$E = \frac{13}{4} = 2$ side	$10 = 2$ side $= 100$
20 + 28	37	$B + F = 5$	$\frac{61}{10}$
6, 37,	38	$D = 5 = 3$ side	$\frac{61}{10} = 3$ side $= 61$

See Fig. (XII)

A Pro-

A Proposition.

If in an oblique triangle, a perpendicular be let fall from any angle (B) upon the opposite side within or without the triangle; then is the difference of the squares of the sides (AB, and BC) conteining the said angle, equal to the difference of the lines (AD and CD) conteined between the perpendicular and both ends of the Base (or opposite side aforesaid.)

AB

(Fig. XIII.)

	1	$A = a + b$
	2	$AC = d$
	3	$CD = e$
	4	$BC = c$
	5	$BD = f$
$1 \circledcirc \overline{2}$	6	$\square AB = aa + 2ab + bb$
$2 + 3$	7	$AC + CD = d + e = AD$
$7 \circledcirc \overline{2}$	8	$\square AD = dd + 2de + ee$
$4 \circledcirc \overline{2}$	9	$\square BC = cc$
$3 \circledcirc \overline{2}$	10	$\square CD = ee$
$6 - 9$	11	$\square AB - \square BC = aa + 2ab + bb - cc$
$8 - 10$	12	$\square AD - \square CD = dd + 2de$
11, 12	13	$aa + 2ab + bb - cc = dd + 2de$ as sayes the Proposition

Demonstration.

$6 - 8$	14	$aa + 2ab + bb - dd - ee - 2de = \square BD$
$9 - 10$	15	$cc - ee = \square BD$
$14 - 15$	16	$aa + 2ab + bb - dd - 2de - cc = 0$
$16 + dd + 2de$	17	$aa + 2ab + bb - cc = dd + 2de$ as above in (13) *q. e. d.*

And after this way may all sorts of Geometrical Propositions be demonstrated.

The

The Use of this Proposition.

Prob. XIX. Three sides of an oblique Triangle being given, thence to find the perpendicular and segments of the base?

Let	1	$AB=20$
	2	$BC=15$
	3	$AC=8$
	4	$CD=e$
	5	$BD=f$
$1 \circledcirc \overline{2}$	6	$\square AB=400$
$2 \circledcirc \overline{2}$	7	$\square BC=225$
$6-7$	8	$\square AB-\square BC=175$
$3+4$	9	$AC+CD=8+e=AD$
$9 \circledcirc \overline{2}$	10	$\square AD=64+16e+ee$
$4 \circledcirc \overline{2}$	11	$\square CD= = = ee$
$10-11$	12	$\square AD-\square CD=64+16e$
$8, 12$	13	$175= \quad 64+16e$
$13-64$	14	$111=16e$
$14 \div \overline{16}$	15	$\frac{111}{16}=e=CD$
$15+\overline{8}$	16	$\frac{239}{16}=8+e=AD$
$6,$	17	$\frac{6400}{16}=\square AB$
$17-16$	18	$\frac{6161}{16}=\square BD$
$18\ \omega\ \overline{2}$	19	$\sqrt{\frac{6161}{16}}=BD$

And thus in the right angled Triangle *ABD*, are all 3 sides discovered.

Probl.

Probl. XX. If the Base of an oblique Triangle be 39, the Perpendicular 24; also as 8 to 5 :: so *AB* to *BC*, to find *BC*, *AD*, and *DC*?

(Fig. XIV)

AB, *BC*, *AD*, *DC*, *BD* } = ?	1	$AD = e$
	2	$DC = f$
	3	$BD = d$
	4	$AB = c$
	5	$BC = b$
b, *c*, *e*, *f* } = ?	6	$e + f = 39$ } by the Probleme
	7	$d = 24$ } by the Probleme
	8	$b . c :: 5 . 8$ } by the Probleme
	9	$dd + ee = cc$ } by the Figure
	10	$dd + ff = bb$ } by the Figure
8,	11	$bb\ cc :: 25 . 64$
7 ⊕ 2	12	$dd = 576$
6 — *e*	13	$f = 39 - e$
13 ⊕ 2	14	$ff = 1521 - 78e + ee$
12 + 14	15	$dd + ff = 2097 - 78e + ee$
10, 15	16	$bb = 2097 - 78e + ee$
12 + *ee*	17	$dd + ee = 576 + ee$
9, 17	18	$cc = 576 + ee$
16, 11, 18	19	$2097 - 78e + ee . 576 + ee :: 25 . 64$
19 ∴	20	$134208 - 4992e + 64ee = 14400 + 25ee$
20 ±	21	$119808 - 4992e + 39ee = 0$
21 ÷ 39	22	$3072 - 128e + ee = 0$
23 C □	23	$4096 - 128e + ee = 1024$
23 ω 2	24	$64 - e = 32$
24 — 32	25	$32 - e = 0$
25 + *e*	26	$32 = e = AD$
6 — 26	27	$7 = f = DC$
26 ⊕ 2	28	$1024 = ee$
27 ⊕ 2	29	$49 = ff$
12 + 28	30	$1600 = dd + ee$
9, 30	31	$1600 = cc$

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The Uſe of this Propoſition.

Prob. XIX. Three ſides of an oblique Triangle being given, thence to find the perpendicular and ſegments of the baſe?

Let	1	$AB = 20$
	2	$BC = 15$
	3	$AC = 8$
	4	$CD = e$
	5	$BD = f$
1 ⊙ 2	6	$\square AB = 400$
2 ⊙ 2	7	$\square BC = 225$
6 — 7	8	$\square AB - \square BC = 175$
3 + 4	9	$AC + CD = 8 + e = AD$
9 ⊙ 2	10	$\square AD = 64 + 16e + ee$
4 ⊙ 2	11	$\square CD = = = ee$
10 — 11	12	$\square AD - \square CD = 64 + 16e$
8, 12	13	$175 = 64 + 16e$
13 — 64	14	$111 = 16e$
14 ÷ 16	15	$\frac{111}{16} = e = CD$
15 + 8	16	$\frac{239}{16} = 8 + e = AD$
6,	17	$\frac{6400}{16} = \square AB$
17 — 16	18	$\frac{6161}{16} = \square BD$
18 ω 2	19	$\sqrt{\frac{6161}{16}} = BD$

And thus in the right angled Triangle *ABD*, are all 3 ſides diſcovered.

Probl.

Probl. XX. If the Base of an oblique Triangle be 39, the Perpendicular 24; also as 8 to 5 :: so *AB* to *BC*, to find *BC*, *AD*, and *DC*?

(Fig. XIV)

AB, BC, AD, DC, BD = ?	1	$AD=e$
	2	$DC=f$
	3	$BD=d$
	4	$AB=c$
	5	$BC=b$
b, c, e, f = ?	6	$e+f=39$ by the Probleme
	7	$d=24$ by the Probleme
	8	$b.\ c :: 5.8$ by the Probleme
	9	$dd+ee=cc$ by the Figure
	10	$dd+ff=bb$ by the Figure
8,	11	$bb\ cc :: 25.64$
7 ⊛ 2	12	$dd=576$
6 — e	13	$f=39-e$
13 ⊛ 2	14	$ff=1521-78e+ee$
12 + 14	15	$dd+ff=2097-78e+ee$
10, 15	16	$bb=2097-78e+ee$
12 + ee	17	$dd+ee=576+ee$
9, 17	18	$cc=576+ee$
16, 11, 18	19	$2097-78e+ee.576+ee :: 25.64$
19 ∴	20	$134208-4992e+64ee=14400+25ee$
20 ∓	21	$119808-4992e+39ee=0$
21 ÷ 39	22	$3072-128e+ee=0$
23 C □	23	$4096-128e+ee=1024$
23 ɯ 2	24	$64-e=32$
24 — 32	25	$32-e=0$
25 + e	26	$32=E=AD$
6 — 26	27	$7=F=DC$
26 ⊛ 2	28	$1024=ee$
27 ⊛ 2	29	$49=ff$
12 + 28	30	$1600=dd+ee$
9, 30	31	$1600=cc$

31 ω 2	32	$40 = C = AB$
12 + 29	33	$625 = dd + ff$
10, 33	34	$625 = bb$
~~32~~ ω 2 34	35	$25 = B = BC$

Probl. XXI. When in an oblique Triangle BD is taken at pleasure, and BD (for example) $= 26$, $AD = 22$, $DC = 17$, also, $AB . BC :: 8 . 5$. find the rest, viz. AB, BC, GC, DG, IG ? (Fig. XV.)

Given		
AB = ?	1	$BC = b$
BC = ?	2	$AB = c$
GC = ?	3	$BD = d$
DG = ?	4	$AD = e$
BG = ?	5	$DG = f$
	6	$GC = g$
	7	$BG = h$

Required		
b = ?	8	$d = 26$ by the Probleme
c = ?	9	$e = 22$
f = ?	10	$f + g = 17$
g = ?	11	$c . b :: 8 . 5$
h = ?	12	$ee + 2ef + ff + hh = cc$ by the figure
	13	$ff + hh = dd$
	14	$gg + hh = bb$
10 − g	15	$f = 17 - g$
15 ⊙ 2	16	$ff = 289 - 34g + gg$
8 ⊙ 2	17	$dd = 676$
13 − ff	18	$dd - ff = hh$
17 − 16	19	$dd - ff = 387 + 34g - gg$
18, 19	20	$hh = 387 + 34g - gg$
20 + gg	21	$gg + hh = 387 + 34g$
14, 21	22	$bb = 387 + 34g$
9 + 10	23	$e + f + g = 39$
23 − g	24	$e + f = 39 - g$
24 ⊙ 2	25	$ee + 2ef + ff = 1521 - 78g + gg$
25 + hh	26	$ee + 2ef + ff + hh = 1521 - 78g + gg + hh$
12, 26	27	$cc = 1521 - 78g + gg + hh$
27 ±	28	$cc - 1521 + 78g - gg = hh$

cc

20, 28	29	$cc-1521+78g-gg=387+34g-gg$
29 $\pm$	30	$1908=cc+44g$
30 $-$ 44g	31	$1908-44g=cc$
11 ⊚ 2	32	$cc \,.\, bb :: 64 \,.\, 25$
32, 22, 31	33	$1908-44g \,.\, 387+34g :: 64 \,.\, 25$
33 *ergo*	34	$47700-1100g=24768+2176g$
34 $\pm$	35	$22932=3276g$
35 $\div$ 3276	36	$7=G=GC$ } In the Figure
10 $-$ 36	37	$10=F=DG$ }
36 $*$ 34	38	$238=34g$
38 + 387	39	$625=34g+387$
22, 39	40	$625=bb$
40 ω 2	41	$25=B=BC$ in the figure
11, 41	42	$c \,.\, 25 :: 8 \,.\, 5$
42 *ergo*	43	$5c=200$
43 $\div$ 5	44	$40=C=AB=$ in the figure
37 ⊚ 2	45	$100=ff$
17 $-$ 45	46	$576=dd-ff$
18, 46	47	$576=bb$
47 ω 2	48	$24=H=BG$ in the figure

Probl. XXII. When of three quantities in continual proportion *a. c. b.* there be given either $a+b$ and c, $a-b$ and c, $b+c$ and a, $c-b$ and a, find the rest.

$\left.\begin{matrix}a\\b\end{matrix}\right\}=?$	1	$a+b=15$ } (*Fig.* XVI)
	2	$c=6$ }
by the figure	3	$a \,.\, c :: c \,.\, b$ By $a+b$ and c given.
3 *ergo*	4	$ab=cc$
2 ⊚ 2	5	$36=cc$
4, 5	6	$ab=36$
1 ⊚ 2	7	$aa+2ab+bb=225$
6 $*$ 4	8	$4ab == 144$
7 $-$ 8	9	$aa-2ab+bb=81$
9 ω 2	10	$a-b=\pm 9$

1 + 10	11	$2a=24$ ⎫ and ⎧ $+6$
11 ÷ 2	12	$A=12$ ⎬ and ⎨ $+3$
1 — 12	13	$b=3$ ⎭ and ⎩ $+12$

by $a-b$ and c given

	1	$a-b=9$
	2	$c=6$
	3	$a.c::c.b$
3 *ergo*	4	$ab=cc$
1 ⊕ 2	5	$aa-2ab+bb=81$
4 * 4	6	$4ab=4cc$
2 ⊕ 2	7	$cc=36$
7 * 4	8	$4cc=144$
8, 6	9	$4ab=144$
5 + 9	10	$aa+2ab+bb=225$
10 ω 2	11	$a+b=\pm 15$ &c.

by a and $b+c$ given.

	1	$a=12$
	2	$b+c=9$
1 * 2	3	$ab+ac=108$
	4	$a.c::c.b$
4 *ergo*	5	$ab=cc$
3, 5	6	$cc+ac=108$
6, 1	7	$cc+12c=108$
7 + 36	8	$cc+12c+36=144$
8 ω 2	9	$c+6=\pm 12$
9 — 6	10	$C=\begin{cases}+6\\-18\end{cases}$
2 — 10	11	$B=\begin{cases}+3\\+27\end{cases}$

by a and $c-b$ given

	1	$a=12$
	2	$c-b=3$
	3	$a.c::c.b$
1 * 2	4	$ac-ab=36$
3 *ergo*	5	$cc=ab$

ac

4, 5	6	$ac - cc = 36$
1, 6	7	$12c - cc = 36$
0 — 7	8	$-12c + cc = -36$
8 + 36	9	$36 - 12c + cc = 0$
9 ω 2	10	$-6 + c = 0$
10 + 6	11	$C = 6$
11 — 2	12	$B = 3$

Probl. XXIII. When in a right angle Triangle one side, and the sum of the Hypot. and *area* are given, find the rest.

$b = ?$	1	$C = 3$ (Fig. I.)
$h = ?$	2	$\frac{3b}{2} = area \quad \frac{3b}{2} + h = 11$
	3	$\frac{3b}{2} + h = 11 = hyp + area.$
$3 - \frac{3b}{2}$	4	$h = 11 - \frac{3b}{2}$
4 ⊙ 2	5	$hh = 121 + \frac{9}{4}bb - 33b$
	6	$hh = bb + 9$ in the figure
5 — 6	7	$0 = 112 + \frac{5}{4}bb - 33b$
$7 * \frac{4}{5}$	8	$0 = \frac{448}{5} + bb - \frac{132b}{5} = 0$
8, C □	9	$\frac{2116}{25} = \frac{4356}{25} - 26\frac{2}{5}b + bb$
9 ω 2	10	$\frac{66}{5} - b = \frac{46}{5}$

$10 - \frac{46}{5}$	11	$4 - b = 0$
$11 + b$	12	$4 = B$
$12 * \frac{3}{2}$	13	$6 = \frac{3b}{2}$
$3 - 13$	14	$5 = H$
$12 \div 2$	15	$2 = \frac{B}{2}$
$1 * 15$	16	$6 = \frac{1}{2} Bc = area$

Probl. XXIV. To Find two Numbers, either of which being subtracted from the square of their summe, will leav a remainder, that is a *square Number*.

$a = ?$	1	$a + b = c$
$b = ?$	2	$cc - a = dd$
$c = ?$	3	$cc - b = ee$
$d = ?$	4	(*)
$e = ?$	5	(*)
$2 + -$	6	$cc - dd = a$
$3 + -$	7	$cc - ee = b$
4 (*)	8	Let $e = 2d$
8 ⊙ 2	9	$ee = 4dd$
3, 9	10	$cc - b = 4dd$
10,	11	$cc > 4dd$
11 ω 2	12	$c > 2d$
5 (*) 12	13	Let $c = 3d$
13 ⊙ 2	14	$cc = 9dd$
$14 - 9$	15	$cc - ee = 5dd$
7 15,	16	$b = 5dd$
$14 - dd$	17	$cc - dd = 8dd$
6 17,	18	$a = 8dd$
$16 + 18$	19	$a + b = 13dd$

$c =$

1, 19	20	$c=13dd$
13, 20	21	$3d=13dd$
$21 \div d$	22	$3=13d$
$22 \div 13$	23	$\frac{3}{13}=D$
$23 * \bar{2}$	24	$2D=\frac{6}{13}$
$23+24$	25	$3D=\frac{9}{13}$
23 ⊚ 2	26	$DD=\frac{9}{169}$
$26 * \bar{5}$	27	$5DD=\frac{45}{169}$
$26 * \bar{8}$	28	$8DD=\frac{72}{169}$

The Answers.

18, 28	29	$A=\frac{72}{169}$
16, 27	30	$B=\frac{45}{169}$
13, 25	31	$C=\frac{9}{13}$
23,	.	$D=\frac{3}{13}$
8, 24	32	$E=\frac{6}{13}$

Th

The Proofs.

$29+30$	33	$\frac{72+45}{169}=\frac{117}{169}=\frac{9}{13}$
31 ⊕ 2	34	$CC=\frac{81}{169}$
$34-29$	35	$\frac{81-72}{169}=\frac{9}{169}$
$34-30$	36	$\frac{81-45}{169}=\frac{36}{169}$ as was required.

Probl. XXV. To Find three Numbers, which being subtracted from the square of their summe, will leav three remainders, that are *square numbers.*

$a=?$	1	$a+b+c=d$
$b=?$	2	$dd-a=ee$
$c=?$	3	$dd-b=ff$
$d=?$	4	$dd-c=gg$
$e=?$	5	(∗)
$f=?$	6	(∗)
$g=?$	7	(∗)
$2+-$	8	$dd-ee=a$
$3+-$	9	$dd-ff=b$
$4+-$	10	$dd-gg=c$
5 (∗)	11	Let $g=2e$
11 ⊕ 2	12	$gg=4ee$
4, 12	13	$dd-c=4ee$
13	14	$dd>4ee$
14 ω 2	15	$d>2e$
6 (∗), 15	16	Let $d=3e$
16 ⊕ 2	17	$dd=9ee$
$17-ee$	18	$dd-ee=8ee$
8, 18	19	$a=8ee$
$17-12$	20	$dd-gg=5ee$
10, 20	21	$c=5ee$

17, 3	22	$9ee-b=ff$
22,	23	$9ee>ff$
23 ɯ 2	24	$3e>f$
7 (*), 24	25	Let $f=2\frac{1}{2}e$
25 ⊚ 2	26	$ff=\frac{25}{4}ee$
17 − 26	27	$dd-ff=\frac{11}{4}ee$
9, 27	28	$b=\frac{11}{4}ee$
19 + 28 + 21	29	$a+b+c=\frac{63}{4}ee$
1, 29	30	$d=\frac{63}{4}ee$
30, 16	31	$\frac{63}{4}ee=3e$
31 * $\overline{4}$	32	$63ee=12e$
32 ÷ 63e	33	$E=\frac{4}{21}$
33 ÷ $\overline{2}$	34	$\frac{1}{2}E=\frac{2}{21}$
33 * $\overline{2}$	35	$2E=\frac{8}{21}$
33 + 35	36	$3E=\frac{12}{21}=\frac{4}{7}$

$34 + 35$	37	$2\frac{1}{2}E = \frac{10}{21}$
$33 \odot 2$	38	$EE = \frac{16}{441}$.
$38 * 5$	39	$5EE = \frac{80}{441}$
$38 * 8$	40	$8EE = \frac{128}{441}$
$38 * 11$	41	$11EE = \frac{176}{441}$
$41 \div 4$	42	$\frac{11}{4}EE = \frac{44}{441}$
		The Answers.
19, 40	43	$A = \frac{128}{441}$
28, 42	44	$B = \frac{44}{441}$
21, 39	45	$C = \frac{80}{441}$
16, 36	46	$D = \frac{4}{7}$
33,	.	$E = \frac{4}{21}$
25, 37	47	$F = \frac{10}{21}$
11, 35	48	$G = \frac{8}{21}$

The

The Proofs.

$43+44+45$	49	$\frac{128+44+80}{441} = \frac{4}{7}$
$46 \circledast 2$	50	$DD = \frac{16}{49} = \frac{144}{441}$
$50-43$	51	$\frac{144-128}{441} = \frac{16}{441} = EE$
$50-44$	52	$\frac{144-44}{441} = \frac{100}{441} = FF$
$50-45$	53	$\frac{144-80}{441} = \frac{64}{441} = GG$ as was required

Probl. XXVI. To find *three Numbers*, which will *make as many Cubes*, by *adding* each of them to the Cube of their Sum. (*Diophantus* V. 18.)

$a=?$	1	$a+b+c=d$
$b=?$	2	$ddd+a=eee$
$c=?$	3	$ddd+b=fff$
$d=?$	4	$ddd+c=ggg$
$e=?$	5	$(*)$
$f=?$	6	$(*)$
$g=?$	7	$(*)$
$h=?$	8	Let $e=hd$
$k=?$	9	Let $f=kd$
$l=?$	10	Let $g=ld$
$8 \circledast 3$	11	$eee=hhhddd$
$9 \circledast 3$	12	$fff=kkkddd$
$10 \circledast 3$	13	$ggg=lllddd$

2, 11	14	$ddd + a = hhhddd$
3, 12	15	$ddd + b = kkkddd$
4, 13	16	$ddd + c = llldd d$
$14 - ddd$	17	$a = hhhddd - ddd$
$15 - ddd$	18	$b = kkkddd - ddd$
$16 - ddd$	19	$c = lllddd - ddd$
$17 + 18 + 19$	20	$a + b + c = \dfrac{hhh + kkk + lll - 3}{ldd}$
1, 20	21	$d = ddd * \overline{hhh + kkk + lll - 3}$
$21 \div d$	22	$1 = dd * \overline{hhh + kkk + lll - 3}$
$m = ?$	23	Let $hhh + kkk + lll - 3 = mm$
$n = ?$	24	Let $hhh - 1 = n$
$p = ?$	25	Let $kkk - 1 = p$
$q = ?$	26	Let $lll - 1 = q$
$24 + 25 + 26$	27	$h^3 + k^3 + l^3 - 3 = n + p + q$
23, 27	28	$mm = n + p + q$
$r = ?$	29	Let $h = r + 1$
5 (*)	30	Let $k = 2 - r$
6 (*)	31	Let $L = 2$
$29 \circledcirc 3$	32	$hhh = rrr + 3rr + 3r + 1$
$30 \circledcirc 3$	33	$kkk = 8 - 12r + 6rr - rrr$
$31 \circledcirc 3$	34	$LLL = 8$
$32 - \overline{1}$	35	$hhh - 1 = rrr + 3rr + 3r$
$33 - \overline{1}$	36	$kkk - 1 = -rrr + 6rr - 12r + 7$
$34 - \overline{1}$	37	$LLL - 1 = 7$
24, 35	38	$n = rrr + 3rr + 3r$
25, 36	39	$p = -rrr + 6rr - 12r + 7$
26, 37	40	$Q = = = 7$
$38 + 39 + 40$	41	$n + p + Q = 9rr - 9r + 14$
28, 41	42	$mm = 9rr - 9r + 14$
7 (*)	43	Let $m = 3r - 4$ [or $4 - 3r$
$43 \circledcirc 2$	44	$mm = 9rr - 24r + 16$
$44 - 42$	45	$4 = -15r + 2$
$45 + 15.r$	46	$15r = 2$
$46 \div \overline{15}$	47	$R = \dfrac{2}{15}$

$R + 1$

$47 + \bar{1}$	48	$R + 1 = \frac{17}{15}$
$\bar{2} - 47$	49	$2 - R = \frac{28}{15}$
29, 48	50	$H = \frac{17}{15}$
30, 49	51	$K = \frac{28}{15}$
50 ⊚ 3	52	$HHH = \frac{4913}{3375}$
51 ⊚ 3	53	$KKK = \frac{21952}{3375}$
$52 - \bar{1}$	54	$HHH - 1 = \frac{1538}{3375}$
$53 - \bar{1}$	55	$KKK - 1 = \frac{18577}{3375}$.
24, 54	56	$N = \frac{1538}{3375}$
25, 55	57	$P = \frac{18577}{3375}$.
40,	58	$Q = \frac{23625}{3375}$.
$56 + 57 + 58$	59	$N + P + Q = \frac{43740}{3375}$.

28, 59	60	$mm = \frac{43740}{3375} = \frac{324}{25}$
22, 23	61	$mmdd = 1$
61 ω 2	62	$md = 1$
60 ω 2	63	$M = \frac{18}{5}$
62 ÷ 63	64	$D = \frac{5}{18}$ Here *Diophantus* breaks off.
50 * 64	65	$HD = \frac{17}{54}$
51 * 64	66	$KD = \frac{28}{54}$
31 * 64	67	$LD = \frac{5}{9} = \frac{30}{54}$
17, 24	68	$a = Nddd$
18, 25	69	$b = Pddd$
19, 26	70	$c = Qddd$
64 ⊙ 3	71	$DDD = \frac{125}{5832}$
56 * 71	72	$NDDD = \frac{1538}{157464}$ (3365 = 27 * 125)
57 * 71	73	$PDDD = \frac{18577}{157464}$
58 * 71	74	$QDDD = \frac{23625}{157464}$

The

The Answers

68,72	75	$A=\frac{1538}{157464}$
69,73	76	$B=\frac{18577}{157464}$
70 74	77	$C=\frac{23625}{157464}$
64,		$D=\frac{5}{18}$
8,65	78	$E=\frac{17}{54}$
9,66	79	$F=\frac{28}{54}=\frac{14}{27}$
10,67	80	$G=\frac{5}{9}=\frac{30}{54}$

$\left(\begin{array}{ccccccc} H \cdot & K \cdot & L \cdot & M \cdot & N \cdot & P \cdot & Q \\ 50 \cdot & 51 \cdot & 31 \cdot & 63 \cdot & 56 \cdot & 57 \cdot & 58 \end{array}\right)$

The Proofs.

75+76+77	81	$A+B+C=\frac{43740}{157464}$
71,	82	$DDD=\frac{125}{5832}=\frac{3375}{157464}$
82+75	83	$DDD+A=\frac{4913}{157464}$
82+76	84	$DDD+B=\frac{21952}{157464}$

DDD+C

82 + 77	85	$DDD+C=\frac{27000}{157464}$	
78 ⊙ 3	86	$EEE=\frac{4913}{157464}$	
79 ⊙ 3	87	$FFF=\frac{21952}{157464}$	
80 ⊙ 3	88	$GGG=\frac{27000}{157464}$	
81, 64	89	$A+B+C=D$	as was required in (1)
83, 86	90	$DDD+A=EEE$	(2)
84, 87	91	$DDD+B=FFF$	(3)
85, 88	92	$DDD-C=GGG$	(4)

Probl. XXVII. To find *three Numbers* which will *leave* as many Cubes after the *Subtraction* of each from the Cube of their Sum. (*Diophantus* V. 19.)

$a=?$	1	$a+b+c=d$
$b=?$	2	$ddd-a=eee$
$c=?$	3	$ddd-b=fff$
$d=?$	4	$ddd-c=ggg$
$e=?$	5	(*)
$f=?$	6	(*)
$g=?$	7	(*)
$h=?$	8	Let $e=hd$
$k=?$	9	Let $f=kd$
$l=?$	10	Let $g=ld$
8 ⊙ 3	11	$eee=hhhddd$
9 ⊙ 3	12	$fff=kkkddd$
10 ⊙ 3	13	$ggg=lllddd$
2, 11	14	$ddd-a=hhhddd$

$ddd-b$

3, 12	15	$ddd - b = kkkddd$
4, 13	16	$ddd - c = lllddd$
$ddd - 14$	17	$a = ddd - hhhddd$
$ddd - 15$	18	$b = ddd - kkkddd$
$ddd - 16$	19	$c = ddd - lllddd$
$17 + 18 + 19$	20	$a + b + c = \dfrac{3 - hhh - kkk - lll}{ddd}$
1, 20	21	$d = ddd * \sqrt{3 - hhh - kkk - lll}$
$21 \div d$	22	$1 = dd * \sqrt{3 - hhh - kkk - lll}$
m?	23	Let $3 - hhh - kkk - lll = mm$
	24	$1 - hhh > 0$
	25	$1 - kkk > 0$
	26	$1 - lll > 0$
$24 + hhh$	27	$1 > hhh$
$25 + kkk$	28	$1 > kkk$
$26 + lll$	29	$1 > lll$
27, 28, 29	30	$1 > hhh + kkk + lll$
$\overline{3} - 30$	31	$3 - hhh - kkk - lll > 2$
23, 31	32	$mm > 2$
32, 5 (*)	33	Let $mm = 2\frac{1}{4} = \frac{9}{4}$
23, 33	34	$3 - hhh - kkk - lll = \frac{9}{4}$
$\overline{3} - 34$	35	$hhh + kkk + lll = \frac{3}{4} = \frac{48}{64} = \frac{162}{216}$.
$35 * \overline{216}$	36	$216h^3 + 216k^3 216lll = 162$
	37	$162 = 125 + 37$
	38	$37 = 64 - 27$.
		Here the Manuscripts of *Diophantus* want a considerable part of the Inquiry. It is hard to supply the defect by conjecture. I proceed thus.
37 (*)	39	Let $216lll = 125$
$36 - 39$	40	$216hhh + 216kkk = 37$

216*hhh*

40, 38	41	$216hhh + 216kkk = 64 - 27$
	42	$\left.\begin{array}{c}6h\\6h\\6h\end{array}\right] + \left[\begin{array}{c}6k\\6k\\6k\end{array}\right. = \left.\begin{array}{c}4\\4\\4\end{array}\right] - \left[\begin{array}{c}3\\3\\3\end{array}\right.$
7 (*)	43	Let $6h = \dfrac{40}{91}$
43 ⊙ 3	44	$216hhh = \dfrac{64000}{753571}$
	45	$37 = \dfrac{27882127}{753571}$
45 − 44	46	$37 - 216hhh = \dfrac{27818127}{753571}$
41, 46	47	$216kkk = \dfrac{27818127}{753571}$
47 ω 3	48	$6k = \dfrac{303}{91}$
39 ω 2	49	$6l = 5$
43 ÷ $\overline{6}$	50	$H = \dfrac{40}{546}$
48 ÷ $\overline{6}$	51	$K = \dfrac{303}{546}$
49 ÷ $\overline{6}$	52	$L = \dfrac{5}{6} = \dfrac{455}{546}$
33 ω 2	53	$M = \dfrac{3}{2}$
22, 23	54	$MMdd = 1$
54 ω 2	55	$Md = 1$

$D =$

55 ÷ 53	56	$D=\frac{2}{3}$ Here I meet again with *Diophantus* and he breaks off.
50 * 56	57	$HD=\frac{80}{1638}=\frac{40}{819}$
51 * 56	58	$KD=\frac{606}{1638}=\frac{303}{819}$
52 * 56	59	$LD=\frac{10}{18}=\frac{455}{819}$
8, 57	60	$E=\frac{40}{819}$
9, 58	61	$F=\frac{303}{819}$
10, 59	62	$G=\frac{455}{819}$
ddd — 2	63	$a=DDD-EEE$
ddd — 3	64	$b=DDD-FFF$
ddd — 3	65	$c=DDD-GGG$
60 ⊙ 3	66	$EEE=\frac{64000}{594353259}$
61 ⊙ 3	67	$FFF=\frac{27818127}{549353259}$
62 ⊙ 3	68	$GGG=\frac{94196375}{549353259}$
56 ⊙ 3	69	$DDD=\frac{8}{27}=\frac{162771336}{549353259}$

69 — 66	70	$DDD - EEE = \frac{162707336}{549353259}$
69 — 67	71	$DDD - FFF = \frac{134953209}{549353259}$
69 — 68	72	$DDD - GGG = \frac{68574961}{549353259}$
63, 70	73	$A = \frac{162707336}{549353259}$
64, 71	74	$B = \frac{134953209}{549353259}$
65, 72	75	$C = \frac{68574961}{549353259}$
73+74+75	76	$A + B + C = \frac{366235506}{549353259} = \frac{2}{3}$
Proofs.		(76) = (56) (69) — (73) = (66). (69) — (74) = (67). (69 — (75) = (68). as was required in 1, 2, 3, and 4th. Equations.

Bachetus in his Comment upon this 19th. Probleme of *Diophantus*, seems to give *another answer* to it, but these values of *A. B. C.* are the same with his; save that he hath unnecessarily expressed those Fractions with Numbers twenty seven times greater than they need to be. He seems content that his Readers should believe that he can find *no other Answer* to it. He confesses he knows *no other Cubes* into which 37 is divisible than those whose sides are $\frac{40}{91}$ and $\frac{303}{91}$ which he found by *this* Rule.

To find two Cubes whose Sum is equal to the difference of two given

given Cubes; (the double of the lesser given Cube must not exceed the greater given Cube.) Multiply each of the given Cubes by the triple of the side of the other: by the Sum of the Cubes divide those Products. From the greater Quotient take the lesser side and subtract the lesser Quotient from the greater side. The Remainder shall be the sides of the Cubes desired. Bachet. ad Dioph. IIII. 2. *It may be expressed in letters and illustrated by an example thus.*

When $uuu+ttt.uuu-2ttt :: u.r$
And $uuuttt.2uuu-ttt :: t.s$
Then $rrr+sss=uuu-ttt$

If $uuu=64.ttt=27.u=4.$
$t=3.uuu-ttt=37.uuu+ttt=91$

$2ttt=54 \therefore uuu-2ttt=10$
$2uuu=128 \therefore 2uuu-ttt=101$
As 91 to 10 : so 4 to $\frac{40}{\cancel{90}}=R$ 91
As 91 to 101 : so 3 to $\frac{303}{91}=S$
$\therefore RRR+SSS=37$

But I by another way found that 37 is more equally divisible into two Cubes whose sides are $\frac{18}{7}$ & $\frac{19}{7}$ (For $5832+6859=12691=343*37$) And therefore in the foregoing search, in stead of following *Baehetus* I might have wroughr thus

$6h=\frac{18}{7}.6k=\frac{19}{7} \therefore H=\frac{3}{7}.K=\frac{19}{42}(L=\frac{5}{6}.M=\frac{3}{2}.D=\frac{2}{3}$ as before:) therefore $HD=\frac{2}{7}.KD=\frac{19}{63}.LD=\frac{5}{9}. \therefore D=\frac{42}{63}$

$E=\frac{18}{63}.G=\frac{35}{63}$ Then $DDD-EEE=\frac{68256}{250047}=A$

$$DDD - FFF = \frac{67229}{250047} = B.\quad DDD - GGG = \frac{31213}{250047} = C$$

Therefore $A + B + C = \frac{166698}{250047} = \frac{2}{3} = D$ as was required.

So that here we have *a second Answer* fitted to *Diophantus* his beginning. But the Probleme should have three Equations more given to limit it sufficiently. And therefore, as it is proposed by *Diophantus* it is capable of *innumerable Answers* which may be found by Searches very different from that of *Diophantus*. See the following Probleme.

Probl. XXVIII. To solve the foregoing Probleme after another manner.

$a = ?$	1	$a + b + c = d$
$b = ?$	2	$ddd - a = eee$
$c = ?$	3	$ddd - b = fff$
$d = ?$	4	$ddd - c = ggg$
$e = ?$	5	(∗)
$f = ?$	6	(∗)
$g = ?$	7	(∗)
$ddd - 2$	8	$a = ddd - eee$
$ddd - 3$	9	$b = ddd - fff$
$ddd - 4$	10	$c = ddd - ggg$
$n = ?$	11	Let $e = p - n$
$p = ?$	12	Let $f = 4n - p$
5 (∗)	13	Let $g = 2n$
11 ⊛ 3	14	$eee = ppp - 3ppn + 3pnn - nnn$
12 ⊛ 3	15	$fff = -ppp + 12ppn - 48pnn + 64nnn$
13 ⊛ 3	16	$ggg = 8nnn$

Let

6 (*)	17	Let $d = 4n$
17 ⊚ 3	18	$ddd = 64nnn$
18 − 14	19	$ddd - eee = 65nnn - 3pnn + 3ppn - ppp$
18 − 15	20	$ddd - fff = +48pnn - 12ppn + ppp$
18 − 16	21	$ddd - ggg = 56nnn$
8, 19	22	$a = 65nnn - 3pnn + 3ppn - ppp$
9, 20	23	$b = +48pnn - 12ppn + ppp$
10, 21	24	$c = 56nnn$
22 + 23 + 24	25	$a + b + c = 121nnn + 45pnn - 9ppn$
1, 25	26	$d = 121nnn + 45pnn - 9ppn$
26, 17	27	$121nnn + 45pnn - 9ppn = 4n$
27 ÷ n	28	$121nn + 45pn - 9pp = 4$
7 (*)	29	Let $11n + p = 2$
29 ⊚ 2	30	$121nn + 22pn + pp = 4$
28 − 30	31	$23np - 10pp = 0$
31 ÷ p	32	$23n - 10p = 0$
32 + 10 p	33	$23n = 10p$
33 ÷ 10	34	$p = \frac{23n}{10} = \frac{23}{10}n$
34 − n	35	$p - n = \frac{13n}{10} = \frac{13}{10}n$
11, 35	36	$e = \frac{13}{10}n$
•	37	$4n = \frac{40n}{10}$
37 − 34	38	$4n - p = \frac{17n}{10}$
12, 38	39	$f = \frac{17}{10}n$

$g =$

13,	40	$g=\frac{20}{10}n$
36 ⊙ 3	41	$eee=\frac{2197nnn}{1000}$
39 ⊙ 3	42	$fff=\frac{4913nnn}{1000}$
40 ⊙ 3	43	$ggg=\frac{8000nnn}{1000}$
18,	44	$ddd=\frac{64000nnn}{1000}$
44 — 41	45	$ddd-eee=\frac{61803nnn}{1000}$
44 — 42	46	$ddd-fff=\frac{59087nnn}{1000}$
44 — 43	47	$ddd-ggg=\frac{56000nnn}{1000}$
8, 45	48	$a=\frac{61803nnn}{1000}$
9, 46	49	$b=\frac{59087nnn}{1000}$
10, 47	50	$c=\frac{56000nnn}{8000}$
48 + 49 + 50	51	$a+b+c=\frac{176890nnn}{1000}$

d=

1, 51,	52	$d=\frac{17689nnn}{100}$
52, 17	53	$\frac{17689nnn}{100}=4n$
53 ÷ n	54	$\frac{17689nn}{100}=4$
54 ω 2	55	$\frac{133n}{10}=2$
55 * $\overline{10}$	56	$133n=20$
56 ÷ $\overline{133}$	57	$N=\frac{20}{133}$
57 * $\overline{2}$	58	$2N=\frac{40}{133}$
58 * $\overline{2}$	59	$4N=\frac{80}{133}$
57 ÷ $\overline{10}$	60	$\frac{N}{10}=\frac{2}{133}$
60 ⊕ 3	61	$\frac{NNN}{1000}=\frac{8}{2352637}$

The Answers.

48, 61	62	$A=\frac{61803*8}{2352637}=\frac{494424}{2352637}$
49, 61	63	$B=\frac{59087*8}{2352637}=\frac{472696}{2352637}$

$C=$

50, 61	64	$C=\frac{56000*8}{2352637}=\frac{448000}{2352637}$.
17, 59	65	$D=\frac{80}{133}$
36, 60	66	$E=\frac{26}{133}$
39, 60	67	$F=\frac{34}{133}$
13, 58	68	$G=\frac{40}{133}$
57,		$N=\frac{20}{133}$
34, 60	69	$P=\frac{46}{133}$

The Proofs.

62+63 +64	70	$A+B+C=\frac{1415120}{2352637}$
.	71	$\frac{1415120}{2352637}=\frac{17689*80}{17689*133}.=\frac{80}{133}$
65 ⊖ 3	72	$DDD=\frac{512000}{2352637}$
66 ⊖ 3	72	$EEE=\frac{17576}{2352637}$

FFF=

$67 \odot 3$	74	$FFF = \frac{39304}{2352637}$
$68 \odot 3$	75	$GGG = \frac{54000}{2352637}$
$72 - 62$	76	$512000 - 494424 = 17576$
$72 - 63$	77	$512000 - 472696 = 39304$
$72 - 64$	78	$512000 - 448000 = 64000$

In *Fr. Van Schooten*'s Book alled *Sectiones Miscellaneæ*, printed at *Leyden* 1657. the thirteenth Section hath this foregoing Question with *one Solution*; which he says, he took out of a Letter written by ***Ludolph van Keulen*** to one *Nicolas van Persiin*. ***Ludolph***'s Process is very near the way here expressed; and his values of *A. B. C. D. E. F. G.* are the same with these: save that he makes $G = \frac{240}{133}$ (which is only a false Print, 240 for 40.) It is not unlikely that he sought other Answers, but gave over before he had found any, being discouraged by meeting with ***numbers under nothing***. If in the 29th. Equation [instead of $11n + 1p$] you write $11n + 2p = 2$, and proceed, as there, upon this new ground; you will find ddd less than a, and so $e = \frac{-24}{145}$. If you suppose $11 + 0p = 2$, you will find ddd less than b, and so $f = \frac{-2}{11}$. So that you wil think that it was only ***Ludolph***'s good hap to steer between those two Rocks of Negation, and so to light upon such a Position, as afforded him an Answer wherein ***none*** of his Numbers fell under 0. Which might give him occasion to conclude as he did—*Constat ergo numeros ritè esse inventos. Cujus rei soli Deo debetur gloria.* Schoot. pag. 436. lin. 1.

This made *Van Persiin* think it a great matter that He had found ***one Answer more***: which *Van Schooten* hath there published: namely

$$A = \frac{15817815000}{865268334967}. \quad B = \frac{9568152000}{865268334967}. \quad C = \frac{8925120000}{865268334967}.$$

But

But he adds not a word concerning the way by which he sought them. Nor doth *Van Schooten* seem to have examined whether they be true Answers or no. At first sight its manifest, that his way of searching was none of the best; since it led him to Fractions expressed in such large Numbers, whereas *other* ways would have shewn him many Answers in shorter Numbers. The way that *Ludolph* had sent to him, by an easie improvement, would have been made fit to lead him to an *innumerable* multitude of Answers in Fractions, excluding not only Surds but also Negatives, or Numbers under o.

For the exclusion of Negatives I proceed thus. In the improvement of the Inquiry next preceding; in stead of its 29th. Equation. I take $11n+\underline{Q}p = +2$ or -2, and then I inquire what Numbers may be the values of $\underline{Q}$ that both e and f may be above o. To this purpose, having borrowed the 11th. 12th. and 28th. Equations I say

	11	$e=p-n$
	12	$f=4n-p$
	28	$121nn+45np-9pp=4$
29,	79	$11n+qp=+2$, or -2
79 ⊙ 2	80	$121nn+22qnp+qqpp=4$
28 — 80	81	$45np-9pp-22qnp-qqpp=0$
81 ÷ p	82	$45n-9p-22qn-qqp=0$
82 + +	83	$45n-22qn=9p+qqp$
83,	84	$n \,.\, p :: 9+qq \,.\, 45-22q$
First scope	85	$e=0$
11, 85	86	$p-n=0$
86 + n	87	$p=n$
84, 87	88	$9+qq=45-22q$
88 + —	89	$qq+22q=36$
89 + $\overline{121}$	90	$qq+22q+121=157$
90 ω 2	91	$q+11=+$ or $-\sqrt{157}$
91 — 11	92	$\underline{Q}=-11+\sqrt{157}$
91 — $\overline{11}$	93	$\underline{Q}=-11-\sqrt{157}$
	94	$\sqrt{157}=12\lfloor 529964$ &c.
92, 94	95	$\underline{Q}=+1\lfloor 529964$ &c.

$\underline{Q}=$

	96	$Q=-23\lfloor\underline{529964}$ &c.
95,96	97	*Ergo* Q between $+1\frac{529}{1000}$ &c. and $-23\frac{529}{1000}$ &c. makes $E>0$
95,96	68	Q not between those limits makes $E<0$
2d. *scope.*	99	$f=0$
12, 99	100	$4n-p=0$
$100+p$	101	$4n=p$
84	102	$4n \,.\, p :: 36+4qq \,.\, 45-22q$
101, 102	103	$36+4qq=45-22q$
$103 + -$	104	$4qq+22q=9=\frac{36}{4}$
$104+\frac{121}{4}$	105	$4qq+22q+\frac{121}{4}=\frac{157}{4}$
105 ω 2	106	$2q+\frac{11}{2}=\frac{+\text{ or }-\sqrt{157}}{2}$
$106 \div 2$	107	$q+\frac{11}{4}=\frac{+\text{ or }-\sqrt{157}}{4}$
$107-\frac{11}{4}$	108	$Q=\frac{-11+\sqrt{157}}{4}$
$107-\frac{11}{4}$	109	$Q=\frac{-11-\sqrt{157}}{4}$
$95 \div 4$	110	$Q=+0\lfloor\underline{382491}$ &c.
$96 \div 4$	111	$Q=-5\lfloor\underline{882491}$ &c.
110,111	112	*Ergo* Q between $+\frac{382}{1000}$ &c. and $-5\frac{882}{1000}$ makes $F<0$
110, 111	113	Q not between those limits, maketh $F>0$
97, 112	114	Both E and F will be greater than 0 when you take Q between $+1\lfloor\underline{529964}$ &c. and $+0\lfloor\underline{382491}$ &c. or between $-5\lfloor\underline{882491}$ &c. and $-23\lfloor\underline{529964}$ &c.
114,	115	Q may be $+1$, or any fraction between $\frac{1529964}{1000000}$ &c. and $\frac{382491}{1000000}$ &c.
114,	116	Q may be any of the 18 Integers between -5 and -24, or any fraction between $\frac{-5882491}{1000000}$ &c. and $\frac{-23529964}{1000000}$ &c.
	117	For Examples, I will suppose R 2 $q=\frac{1}{2}$

115,		$q=\frac{3}{2}$ as being, of small fractions, the next less than 1|529 &c.	
116,		$q=-6$, as being of Integers, the next after $-5\|882$ &c.	
		Of the first 28 Equations of the foregoing Process I shall need only these *nine*, in both the following Inquiries.	
	1	$a+b+c=d$	10 $c=d^3-g^3$; 13 $g=2n$
	8	$a=ddd-eee$	11 $e=p-n$; 17 $d=4n$ ∴ $ddd=64nnn$
	9	$b=ddd-fff$	12 $f=4n-p$; 28 $121nn+45np-9pp=4$
7(*)	29	Let $11n+\frac{3}{2}p=2$	Let $11n-6p=-2$
29 ⊛ 2	30	$121nn+33np+\frac{9}{4}pp=4$	$121nn-132np+36pp=4$
28 — 30	31	$12np-\frac{45}{4}pp=0$	$177np-45pp=0$
31 ÷ p	32	$12n-\frac{45}{4}p=0$	$177n-45p=0$
32 +	33	$12n=\frac{45}{4}p$	$177n=45p$
33 ÷	34	$p=\frac{16}{15}n$	$p=\frac{177n}{45}=\frac{59}{15}n$
	35	$p-n=\frac{1}{15}n$	$p-n=\left(p-\frac{15}{15}n\right)=\frac{44}{15}n$
	36	$e=\frac{1}{15}n$	$e=\frac{44}{15}n$
	37	$4n=\frac{60}{15}n$	$4n=\frac{60}{15}n$
37 — 34	38	$4n-p=\frac{44}{15}n$	$4n-p=\frac{1}{15}n$
12, 38	39	$f=\frac{44}{15}n$	$f=\frac{1}{15}n$
13,	40	$g=\frac{30}{15}n$	$g=\frac{30}{15}n$
36 ⊛ 3	41	$eee=\frac{1}{3375}nnn$	$eee=\frac{85184}{3375}nnn$

fff=

		$q=+\frac{3}{2}$	$q=-6$
39 ⊙ 3	42	$fff=\frac{85184}{3375}nnn$	$fff=\frac{1}{3375}nnn$
40 ⊙ 3	43	$ggg=\frac{27000}{3375}nnn$	$ggg=\frac{27000}{3375}nnn$
14,	44	$ddd=\frac{216000}{3375}nnn$	$ddd=\frac{216000}{3375}nnn$
44—41	45	$d^3-e^3=215999n^3\div3375$	$d^3-e^3=130816n^3\div3375$
41—42	46	$d^3-f^3=130816n^3\div3375$	$d^3-f^3=215999n^3\div3375$
44—43	47	$d^3-g^3=189000n^3\div3375$	$d^3-g^3=189000n^3\div3375$
8, 45	48	$a=215999n^3\div3365$	$a=130816n^3\div3375$
9, 46	49	$b=130816n^3\div3375$	$b=215999n^3\div3375$
10, 47	50	$c=189000n^3\div3375$	$c=189000n^3\div3375$
48, +49+50	51	$a+b+c=535815n^3\div3375$	$a+b+c=535815n^3\div3375$

The ten next Equations are common to both values of q

1, 51	52	$d=535815\,nnn\div3375=3969\,nnn\div25$			
52, 17	53	$\frac{3969nnn}{25}=4n$	57 * 2	58	$2N=\frac{20}{63}=\frac{60}{189}$
53 * 25	54	$3969nnn=100n$	58 * 2	59	$4N=\frac{40}{63}=\frac{120}{189}$
54 ÷ n	55	$3969nn=100$	57 ÷ 15	60	$\frac{N}{15}=\frac{2}{189}$
55 ω 2	56	$63n=10$			
56 ÷ 63	57	$N=\frac{10}{63}=\frac{30}{189}$	60 ⊙ 3	61	$\frac{NNN}{3375}=\frac{8}{6751269}$

130816 * 8 = 1046528 215999 * 8 = 1727992. 189 * 8 = 1512

48, 61	62	$A=1727992\div6751269$	$A=1046528\div6751269$
49 61	63	$B=1046528\div6751269$	$B=1727992\div6751269$
36, 60	64	$E=2\div189$	$E=88\div189$
39, 60	65	$F=88\div189$	$F=2\div189$
34, 60	66	$P=32\div189$	$P=118\div189$

50, 61	67	$C=1512000\div6751269$	13, 58	69	$G=20\div63=60\div189$
17, 59	68	$D=40\div63=120\div189$	57	.	$N=10\div63=30\div189$

These 4 last Equations are common to both values of q. That is to say; whether in Equation 29th., you suppose $q=+\frac{3}{2}$, or $q=-6$ you shall find the same values for C,D,G, and N. But the value of A in the one supposition is the value of B in the other, and therefore the value of E in the one, will be the value of F in the other. Which permutation causeth only P to have a new value For seeing E in the one is equal to F in the other that is (by Equation 11 and 12) $p-n=4n-P$, (n is the same in both;) therefore $P=5n-p$. And so here, $n=\frac{30}{189}$. $p=\frac{32}{189}$ ∵ $5n-p=\frac{150-32}{189}=\frac{118}{189}=P$; as it was found before by another way.

Nor is this a Concinnity belonging only to these two $\left(\frac{3}{2}\text{ and}-6.\right)$ What number soever you take for q, you shall still find $qQ=-9$ as it was in the utmost limits of q or Q (found before page 123) For there

$$q=-11+\sqrt{157}.\ Q=\frac{-11-\sqrt{157}}{4}\ ∵\ qQ=\frac{+121-157}{4}=\frac{-36}{4}=-9$$

$$q=\frac{-11+\sqrt{157}}{4}.\ Q=-11-\sqrt{157}\ ∵\ qQ=\frac{+121-157}{4}=\frac{-36}{4}=-9$$

Wherefore, to any supposed value of q, you may find its correspondent Supposition, thus. By any of value of q divide -9 the Quotient is the other value of Q. Thus $+\frac{3}{2})-9(-6$. And $+1)-9(-9$. which tells you that supposing $11n-9p=-2$ you shall find

$A=472696\div2352637$	$D=80\div133$	$G=40\div133$
$B=494424\div2352637$	$E=34\div133$	$N=20\div133$
$C=448000\div2352637$	$F=26\div133$	$P=54\div133$

Which numbers were all found before (page 120) by supposing $11n+1p=2$: saving that this $P=\frac{54}{133}$ is made of

N=

$N=\frac{20}{133}$ and $p=\frac{46}{133}$. For then $5n-p=\frac{100-46}{133}=\frac{54}{133}$.
As for the other eight, you see the repetition of C, D, G, N, and the permutations A for B, and $\mathcal{E}$ for F.

Of these permutations you have 36 Examples in the Table following. Whose *former* part consisteth of 6 Columns; whereof the first and sixt, do do shew the values of q in this Equation $11n+qp=2$. Or of Q in this $11n-Qp=-2$. By which those values of $e.\ f.\ g.\ x$ might be found in the other four Columns. (For $ex\div x=E.\ fx\div x=F.\ gx\div x=G.$) For example, the third line says, When $q=\frac{3}{2}$ then $x=189$. and $G=\frac{60}{189}$ $E=\frac{2}{189}$. $F=\frac{88}{189}$ But when $Q=-6$; then $x=189$. and $G=\frac{60}{189}$ as before; but $E=\frac{88}{189}$ and $F=\frac{2}{189}$.

$2G=d.$

$2G = d.\ D^3 - E^3 = a.\ D^3 - F^3 = b.\ D^3 - G^3 = c.\ A + B + C = D\ (n = \tfrac{1}{2}G\ p = E + N)$

q	ex	fx	gx	x		ex	fx	gx	x	÷
3÷2	2	88	60	189	−6	24	1056	720	2268	12
18÷13	136	1094	820	2633	−13÷2	136	1094	820	2633	1
9÷7	66	282	232	755	−7	264	1128	928	3020	4
6÷5	136	386	348	1143	−15÷2	408	1158	1044	3429	3
9÷8	142	296	292	965	−8	568	1184	1168	3860	4
18÷17	744	1206	1300	4313	−17÷2	744	1205	1300	4313	1
1	26	34	40	133	−9	935	1224	1440	4788	36
18÷19	1144	1238	1588	5285	−19÷2	1144	1238	1588	5285	1
9÷10	342	312	438	1451	−10	1368	1248	1744	5804	4
6÷7	536	418	636	2115	−21÷2	1608	1254	1908	6345	3
9÷11	466	314	520	1727	−11	1864	1256	2080	6908	4
18÷23	2136	1354	2260	7493	−23÷2	2135	1254	2260	7493	1
3÷4	202	104	204	675	−12	2424	1248	2448	8100	12
18÷25	2728	1238	2644	8729	−25÷2	2728	1238	2644	8729	1
9÷13	762	306	712	2345	−13	3048	1224	2848	9380	4
2÷3	376	134	340	1117	−27÷2	3384	1206	3060	10053	9
9÷14	934	296	820	2687	−14	3736	1184	3280	10748	4
18÷29	4104	1158	3508	11465	−29÷2	4104	1158	3508	11465	1
3÷5	374	94	312	1017	−15	4488	1128	3744	12204	12
18÷31	4888	1094	3988	12965	−31÷2	4888	1094	3988	12965	1
9÷16	1326	264	1060	3437	−16	5304	1056	4240	13748	4
6÷11	1912	338	1500	4851	−33÷2	5736	1014	4500	14553	3
9÷17	1546	242	1192	3845	−17	6184	958	4768	15380	4
18÷35	6648	918	5044	16229	−35÷2	6648	918	5044	16229	1
1÷2	198	24	148	475	−18	7128	864	5328	17100	36
18÷37	7624	806	5620	17993	−37÷2	7624	806	5620	17993	1
9÷19	2304	186	1480	4727	−19	8136	744	5920	18908	4
6÷13	2888	226	2076	6615	−39÷2	8564	678	6228	19845	3
9÷20	2302	152	1636	5201	−20	9208	608	6544	20804	4
18÷41	9768	534	6868	21785	−41÷2	9768	534	6868	21785	1
3÷7	862	38	600	1899	−21	10344	456	7200	22788	12
18÷43	10936	374	7540	23813	−43÷2	10936	374	7540	23813	1
9÷22	2886	72	1972	6215	−22	11544	288	7888	24860	4
2÷5	1352	22	916	2881	−45÷2	12168	198	8244	25929	9
9÷23	3202	26	2152	6755	−23	12808	104	8508	27020	4
18÷47	13464	6	8980	28133	47÷2	13464	6	8980	28133	1
	fx	ex	gx	x	Q	fx	ex	gx	x	÷

The *latter* part of this Table consisting of five Columns, gives some new light to the construction of the *former* part. For, whereas in *that* part the Numbers seem not to have been otherwise found, than by the ways hitherto described; this *later* part lets you see that almost all of them might be found more easily. For if by subtraction you seek their Differences, and differences of Differences thus,

ex	*Differ.*		*fx*	*Differ.*		*gx*	*Differ.*		*x*	*Differ.*	
24	112		1056	38		720	100		2268	365	
136	118	16	1094	34	−4	820	108	8	2633	387	22
264	144	16	1128	30	−4	928	116	8	3020	409	22
408			1158			1044			3429		
&c.			&c			&c.			&c.		

You shall see that the second Differences are equal, adding always 8, 16, or 22: But for *fx* always subtracting 4; so that having 3 or 4 of the upmost in each Column, the rest are found by aditions and subtractions.

The last Column, marked with the sign of Division conteins the *common Divisor*; by which if you divide the 4 Numbers of the same line, you shall find 4 Quotients equal to those of the same line in the former part of the Table. Thus the first of them (12) dividing 24, 1056, 720, and 2268 gives for Quotients 2, 88, 60, 189. Both parts of the Table do express the same *rationes* of *ex*, *fx*, *gx*, and *x* to one another. But the *former* part hath those *rationes* exprest in the smallest terms, to which they can be reduced; that so you may not be troubled with longer Numbers than needs, when out of them you would draw the Values of *A. B. C. D* (and *N*, *P* if you will.) which by the help of the uppermost line of the Table may be done as in the following Examples.

S *ex*

	1	ex	26	2	198
	2	fx	34	88	24
	3	gx	40	60	148
	4	x	133	189	475
$1 \div 4$	5	E	$26 \div 133$	$2 \div 189$	$198 \div 475$
$2 \div 4$	6	F	$34 \div 133$	$88 \div 189$	$24 \div 475$
$3 \div 4$	7	G	$40 \div 133$	$60 \div 189$	$148 \div 475$
$7 * 2$	8	$2G = D$	$80 \div 133$	$120 \div 189$	$296 \div 475$
8 ⊙ 3	9	DDD	$512000 \div x^3$	$1728000 \div x^3$	$25934336 \div x^3$
5 ⊙ 3	10	EEE	$17576 \div x^3$	$0 \div x$	$7762392 \div x^3$
6 ⊙ 3	11	FFF	$39304 \div x^3$	$681472 \div x^3$	$13824 \div x^3$
7 ⊙ 3	12	GGG	$64000 \div x^3$	$216000 \div x^3$	$3241792 \div x^3$
$9 - 10$	13	$D^3 - E^3 = A$	$494424 \div x^3$	$1727992 \div x^3$	$18171944 \div x^3$
$9 - 11$	14	$D^3 - F^3 = B$	$472696 \div x^3$	$1046528 \div x^3$	$25920512 \div x^3$
$9 - 12$	15	$D^3 - G^3 = C$	$448000 \div x^3$	$1512000 \div x^3$	$22692544 \div x^3$
$13 + 14 + 15$	16	$A + B + C$	$1415120 \div x^3$	$4286520 \div x^3$	$66785000 \div x$
	17		$17689 = xx$	$35721 = xx$	$225625 = xx$
$16 \div 17$	18	$A + B + C$	$80 \div x$	$120 \div x$	$296 \div x$

Therefore in all these, $A + B + C = D$, as was required.

In this manner you may out of that Table draw 36 several Answers to that Probleme. So that here *Franc. Van Schooten* might have found 19 Answers between *Ludolph* and *Van Persijn*. For you see the Column x in the first part hath those twenty Values 133. 189. 475. 675. 755. 965. 1017. 1117. 1143. 1451. 1727. 1899. 2115. 2345. 2633. 2687. 2881. 3437. 3845. 4313, which are all less than 4423. the Cubical Root of *Van Persijn's* common Denominator 86526834967. If, for your ease, you trust to printed Tables of Squares and Cubes, you may sometimes suspect these Numbers or your own work, when the fault is in those Tables and the imperfect enumeration of those faults with their amendments. Thus if you should have need of the Cube of 3214, *Guldin* tells you it is 33199996344, and amends it not in his Table, in which he saith *Errores summâ cum diligentiâ* omnes *notati sunt*. But you shall find that his 96 ought to be 64.

I should

I ſhould now ſhew how the compendious ways of finding $e. f. g.$ and x do ariſe out of the firſt Proceſs of Probleme 28; and how that Proceſs might be ſo improved as that it might direct you to make Innumerable changes of all the aſſured Equations, as

	for	to write
11	$e = p - n$	$e = p - 4n$
12	$f = 4n - p$	$f = 9n - p$
13	$g = 2n$	$g = n$
17	$d = 4n$	$d = 9n$
19	$11n + p = 2$	$39n + qp = 3$

And ſo, (for example, taking $q = 1$) to find $x = 247 \therefore x^3 = 15069223$.

$A = 2837107 \div xxx$	$D = 144 \div 247$
$B = 2966301 \div xxx$	$E = 53 \div 247$
$C = 2981888 \div xxx$	$F = 27 \div 247$
$D = 8785296 \div xxx$	$G = 16 \div 247$

So that by changing the Value of g you may have as great Varieties with theſe *Aſſumptæ* as with the former.

There might alſo *other* very different ways of ſolving this *Problema* 19. *V. Diophanti*, be added. But if all ſhould be ſet down that might pertinently be written concerning the ſupplys of this Defect in the Manuſcripts of *Diophantus*, it would make a large Treatiſe.

Probl. XXIX. To find two equicrural Triangles equal to one another in *Perimeter* and in *Area*; every one of their ſides and perpendiculars being to each of the reſt as ſome number is to ſome number.

(*Fran. Van Schooten*, in the XII. Section of his *Sectiones Miſcellaneæ*, tells us, that this Probleme having been openly propoſed at *Paris Anno* 1633, Monſieur *Des Cartes* gave that Solution which he there ſets down. I ſhall here expreſs it in the ſtyle of this book, not changing His letters, that his work may be eaſily compared with this.)

$e=?$	1	$ee=ff+gg$ See *Fig.* XVII.
$f=?$	2	$ll=mm+nn$
$g=?$	3	$e+f=l+m$
$l=?$	4	$fg=mn$
$m=?$	5	(∗) Let every one of these 6 [*e. f. g. l. m n.*] be to each other
$n=?$	6	(∗) five as some entire number is to some entire number.
$a=?$	7	Let $e=aa+bb$
$b=?$	8	Let $f=2ab$
1, 7, 8	9	$\therefore g=aa-bb$ See pag. 86
$k=?$	10	Let $l=kk+dd$ of this book.
$d=?$	11	Let $m=2kd$ (*a. b. k. d.* are numbers to be sought.)
2, 10, 11	12	$\therefore n=kk-dd$
7 + 8	13	$e+f=aa+2ab+bb$
10 + 11	14	$l+m=kk+2kd+dd$
3, 13, 14	15	$aa+2ab+bb=kk+2kd+dd$
15 ω 2	16	$a+b=k+d$
	17	suppose a less than k
17, $x=?$	18	Let $x=k-a$
18 + a	19	$a+x=k$
16 — 19	20	$b-x=d$
19 ∗ 2	21	$2k=\cancel{2a+2x}$ $2a+2x$ $-2xx$
21 ∗ 20	22	$2kd=2ab-2ax+2bx\cancel{+2xx}$
19 — 20	23	$k-d=a-b+2x$
16 ∗ 23	24	$kk-dd=aa-bb+2ax+2bx$
11, 22	25	$m=2ab-2ax+2xb-2xx$
12, 24	26	$n=aa-bb+2ax+2bx$
8 ∗ 9	27	$fg=2aaab-2abbb$
25 ∗ 26	28	$mn=\begin{cases}2a^3b-2a^3x+6aabx-6aaxx-2ab^3+6abbx\\-2b^3x+6bbxx-4ax^3-4bx^3\end{cases}$
4 — mn	29	$fg-mn=0$
27 — 28	30	$fg-mn=\begin{cases}2a^3x-6aabx+6aaxx-6abbx+2b^3x\\-6bbxx+4ax^3+4bx^3\end{cases}$

$4ax^3$

29, 30,	31	$4ax^3+4bx^3+6aaxx-6bbxx+2a^3x+2b^3x-6aabx-6abbx=0$
$31 \div x$	32	$4axx+4bxx+6aax-6bbx+2a^3+2b^3-6aab-6abb=0$
$32 \div a+b$	33	$4xx+6ax-6bx+2aa-8ab+2bb=0$
33 + —	34	$\left.\begin{matrix}4xx+6ax\\-6bx\end{matrix}\right\}=-2aa+8ab-2bb=\frac{-8aa+32ab-8bb}{4}$
34 +	35	$\begin{matrix}4xx+6ax\\-6bx\end{matrix}+\frac{9aa-18ab+9bb}{4}=\frac{aa+14ab+bb}{4}$
35 ω 2	36	$2x+\frac{3a-3b}{2}=+\text{or}-\frac{\sqrt{aa+14ab+bb}}{2}$
36 —	37	$2x=\frac{3b-3a+\sqrt{aa+14ab+bb}}{2}$
$37 \div 2$	38	$x=\frac{3b-3a+\sqrt{aa+14ab+bb}}{4}$
18	39	x is a Number, for it is the difference of the Numbers k, a
38, 39	40	$\sqrt{aa-14ab+bb}$ is a Number; part of the Number $4x$
	41	$12ab>0$
41 +	42	$aa+14ab+bb>aa+2ab+bb$
42 ω 2	43	$\sqrt{aa+14ab+bb}>a+b$
$43\ c=?$	44	Let $\sqrt{aa+14ab+bb}=a+b+c$
44 ⊙ 2	45	$aa+14ab+bb=aa+2ab+bb+2ac+2bc+cc$
45 — —	46	$12ab=2ac+2bc+cc$
$46-2ac$	47	$12ab-2ac=2bc+cc$
$\frac{47}{12b-2c}$	48	$=\frac{2bc+cc}{12b-2c}$. Here he tells us that *any Number may* be taken

for B and for C, *provided* that they permit A to be greater than B, because we had $aa-bb$ [in Equation 9] But he shews us not how to make that *Provision*. Know then, that there is a *necessity* that two Equations be assumed, because the first and sixt Equations are wanting all this while: and that you have a great *liberty* to make up these two *assumpta* with numbers taken at large. But this liberty is *limited* by your desire that all these sought Numbers *a. b. c. d. e. f. g. k. l. m. n. x.* may all fall above o. The Limits are found thus.

First Scope.	49	$a>0$	$57 + bb$	58	$aa>bb$
48, 49	50	$\frac{2bc+cc}{12b-2c}>0$	$58 \omega 2$	59	$a>b$
	51	$2bc+cc>0$	59, 48	60	$\frac{2bc+cc}{12b-2c}>b$
50, 51	52	$12b-2c>0$	$60 * 12b-2c$	61	$2bc+cc>12bb-2bc$
$52 \div 2$	53	$6b-c>0$	$61 + 2bc$	62	$4bc+cc>12bb$
$53 + c$	54	$6b>c$	$62 + 4bb$	63	$4bb+4bc+cc>16bb$
$54 \div 6$	55	$B>\frac{1}{6}c$	$63 \omega 2$	64	$2b+c>4b$
2d. Scope.	56	$g>0$	$64 - 2b$	65	$C>2B$
9, 56	57	$aa-bb>0$	$65 \div 2$	66	$\frac{1}{2}C>B$

54, 65	67	Make B equal to any Number; but then C must be less than $6B$ and greater than $2B$.
66, 55	68	Or, make C equal to any Number: but then B must be less than $\frac{1}{2}C$, and greater than $\frac{1}{6}C$.
59, 12, 23	69	Thus a being $>b$ will make $k>d$ and therefore $kk>dd$; So that we need not make a new search for a third scope, $n>0$.

5 (*) 67	70	Let $B=1$	9, 81	84	$G=\frac{21}{4}$
67, 70	71	$C<6.C>2$			
6 (*) 71	72	Let $C=3$	38, 44	85	$4x=3b-3a+a+b+c$
70, 72	73	$2BC+CC=15$	85,	86	$4x=4B-2A+C$
70, 72	74	$12B-2C=6$	$70 * 4$	87	$4B=4$
48, 73, 74	75	$A=\frac{15}{6}=\frac{5}{2}$	$87 + 72$	88	$4B+C=7$
			$88 - 76$	89	$4B-2A+C=2$
$75 * 2$	76	$2A=5$	86, 89	90	$4x=2$
$75 \odot 2$	77	$AA=25\div 4$			
$70 \odot 2$	78	$BB=1=4\div 4$			
$77 + 78$	79	$AA+BB=29\div 4$			
$76 * 70$	80	$2AB=5$			
$77 - 78$	81	$AA-BB=21\div 4$			
7, 79	82	$E=\frac{29}{4}$			
8, 80	83	$F=5$			

$x=$

90 ÷ 4	91	$x = \frac{1}{2}$
75 + 91	92	$A + x = 3$
70 — 91	93	$B - x = \frac{1}{2}$
19, 92	94	$K = 3$
20, 93	95	$D = \frac{1}{2}$
95 * 2	96	$2D = 1$
95 ⊙ 2	97	$DD = \frac{1}{4}$
94 ⊙ 2	98	$KK = 9 = \frac{36}{4}$
98 + 97	99	$KK + DD = \frac{37}{4}$
94 * 96	100	$2KD = 3$
98 — 97	101	$KK - DD = \frac{35}{4}$
10, 99	102	$L = \frac{37}{4}$
11, 100	103	$M = 3$
12, 101	104	$N = \frac{35}{4}$

		The Answers in Integers,			*The Proofes.*
82 * 4	105	$E = 29$	105 ⊙ 2	117	EE === 841
83 * 4	106	$F = 20$	106 ⊙ 2	118	FF === 400
84 * 4	107	$G = 21$	107 ⊙ 2	119	GG === 441
102 * 4	108	$L = 37$	108 ⊙ 2	120	LL === 1369
103 * 4	109	$M = 12$	109 ⊙ 2	121	MM === 144
104 * 4	110	$N = 35$	110 ⊙ 2	122	NN === 1225
75 * 2	111	$A = 5$	105 + 106	123	$E + F$ === 49
60 * 2	112	$B = 2$	108 + 109	124	$L + M$ == 49
72 * 2	113	$C = 6$	106 * 107	125	FG === 420
95 * 2	114	$D = 1$	109 * 110	126	MN === 420
94 * 2	115	$K = 6$	118 + 119	127	$FF + GG$ = 841
91 * 2	116	$X = 1$	121 + 122	128	$MM + NN$ = 1369

Comparing

Comparing Equation 117 and 127 with the *first*, and Equation 120 and 128 with the *second*, and Equation 123 and 124 with the *third*; and Equation 125 and 126 with the *fourth* Equation; you will see that *Monsieur des Cartes* gave a true answer to this Probleme, when he made 29, 29, 40 the sides of one Triangle, and 37, 37, 24 the sides of the other.

But *Van Schooten* says not that any notice was taken of *the other value of* x (in that Quadratick Equation $xx+\frac{3a}{2}x$ &c. in his 433[d] page) which might have been found by working thus.

$36 * 2$	129	$4x+3a-3b=-\sqrt{aa+14ab-bb}$
$129, 44$	130	$4x+3a-3b=a-b-c$
$130 + -$	131	$4x=2b-4a-c$
$112 * 2$	132	$2B=4$
$111 * 4$	133	$4A=20$
$132 - 133$	134	$2B-4A=-16$
$134 - 113$	135	$2B-4A-C=-22$
$131, 135$	136	$4x=-22$
$136 \div 4$	137	$X=\frac{-11}{2}$
$111 + 137$	138	$A+X=\frac{-1}{2}$
$112 - 137$	139	$B-X=\frac{+15}{2}$
$19, 138$	140	$K=\frac{-1}{2}$
$20, 139$	141	$D=\frac{15}{2}$
$140 * 2$	142	$2K=-1$
$140 \odot 2$	143	$KK=\frac{1}{4}$
$141 \odot 2$	144	$DD=\frac{225}{4}$
$134 + 144$	145	$KK+DD=\frac{226}{4}$
$141 * 142$	146	$2KD=\frac{-15}{2}$

KK—

143—144	147	$KK - DD = \frac{224}{4}$
10, 145	148	$L = \frac{113}{2} = 56\frac{1}{2}$
11, 146	149	$M = \frac{-15}{2} = -7\frac{1}{2}$
12, 147	150	$N = \frac{-112}{2} = -56$

The Proofs.

$148 \odot 2$	151	$LL = 12769 \div 4$
$149 \odot 2$	152	$MM = 225 \div 4$
$150 \odot 2$	153	$NN = 12544 \div 4$
$152 + 153$	154	$MM + NN = 12769 \div 4$
$148 + 149$	155	$L + M = 49$
$149 + 150$	156	$MN = 420$
151, 154	157	$LL = MM + NN$
155, 123	158	$L + M = E + F$
156, 125	159	$MN = FG$ as was prescribed in Equation 2. 3. 4.

Thus by this neglected value of x we have found *second* values of K, D, L, M, N: which second values of L, M, N I will call $P.\ Q.\ R.$ to avoid mistakes. I say this R is the hight of a *third* Equicrural Triangle (for each leg is equal to P and the Base equal to $2Q$) equal to either of the former Equicrurals in *Area*; and *Perimeter* also according to the rules of Algebraical Addition $\left[56\frac{1}{2}\text{and}-7\frac{1}{2}\text{ make a sum }=49\right]$ $PP = QQ + RR.\ P + Q = E + F.\ QR = FG.$

But to them that do not thorowly understand Negative Quantities, make no mention of the sign —, but tell them that *In this third Isosceles* [$P.\ P\ 2Q$] the Base ($2Q$) *subtracted* from ($2P$) the sum of the Legs, leaves a remainder equal to the common Perimeter of the other two: which in them is the summ of the Legs *adding* the Base. But the *Area's* of all three are equal to one another. $FG = MN = QR$.

Des Cartes worketh by a Quadratick Equation that hath *two values of x*. *One* of them he must find before he can have his *K* and *D*. ∵ then I may find *the other value of x*, and by it [*P. Q. R.*] the new values of *L. M. N.* Wherefore here is no place for suspicion, that this *third* Equicrural was but a casual concinnity happening to *this one pair* of Triangles: But rather here is sufficient ground to pronounce that this way of *Des Cartes* will lead you to *innumerable pairs* of such Equicrural Triangles as the *Parisian* Probleme requires. But always a *third* Isoscels *of the same area* will cleave to them as an inseperable Companion. Which was *the first* thing to be here admonished.

Secondly, If you have occasion to seek many such pairs, you need not run over all those 128 Steps. Rather imitate this following Pattern.

Sought.						
b . c	1	$b > 0$	5	$e = AA + BB$	10	$k = A + X$
A. B. D. E	2	$c > 2b$	6	$f = 2AB$	11	$d = K - X$
F. G. K. L	3	$c < 6b$	7	$g = AA - BB$	12	$l = KK + DD$
M. N. S. X	4	$\frac{2b + cc}{12bb - 2bc} = \frac{A}{B}$	8	$ss = E + 7F$	13	$m = 2KD$
			9	$4x = 3B - 3A + S$	14	$n = KK - DD$

1,	15	Let $= b$	1	1	1	2	2	2	2	3
15 ∗ 2	16	$2b =$	2	2	2	4	4	4	4	6
15 ∗ 6	17	$6b =$	6	6	6	12	12	12	12	18
2, 3, 16, 17	18	Let $c =$	3	4	5	5	7	9	11	7
16 + 18	19	$2b + c =$	5	6	7	9	11	13	15	13
17 — 18	20	$6b - c =$	3	2	1	7	5	3	1	11
19 ∗ 18	21	$2bc + cc =$	15	24	35	45	77	117	165	91
20 ∗ 16	22	$12bb - 2bc$	6	4	2	28	20	12	4	66
21, 22, 4	23	$\frac{A}{B} =$	$\frac{5}{2}$	$\frac{6}{1}$	$\frac{35}{2}$	$\frac{45}{28}$	$\frac{77}{20}$	$\frac{39}{4}$	$\frac{165}{4}$	$\frac{91}{66}$
23,	24	Let $A =$	5	6	35	45	77	39	165	91
23, 24	25	$B =$	2	1	2	28	20	4	4	66
24 ∗ 25	26	$AB =$	10	6	70	1260	1540	156	660	6006
24 ⊚ 2	27	$AA =$	25	36	1225	2025	5929	1521	27225	8281
25 ⊚ 2	28	$BB =$	4	1	4	784	400	16	16	4356

27 + 28	29	*AA+BB*	29	37	1229	2809	6329	1537	27241	12637
$26 * \overline{2}$	30	2*AB*=	20	12	140	2520	3080	312	1320	12012
27 — 28	31	*AA—BB*=	21	35	1221	1241	5529	1505	27209	3925
5, 29	32	*E*=	29	37	1229	2809	6329	1537	27241	12637
6, 30	33	*F*=	20	12	140	2520	3080	312	1320	12012
7, 31	34	*G* =	21	35	1221	1241	5529	1505	27209	3925
$33 * \overline{7}$	35	7*F*=	140	84	980	17640	21560	2184	9240	84084
32 + 35	36	*E*+7*F*	169	121	2209	20449	27889	3721	36481	96721
8, 36	37	*ſſ*=	169	121	2209	20449	27886	3721	36481	96721
37 ω 2	38	*S*=	13	11	47	143	167	61	191	311
$24 * \overline{3}$	39	3*A*=	15	18	105	135	231	117	495	273
$25 * \overline{3}$	40	3*B*=	6	3	6	84	60	12	12	198
38 + 40	41	2*B*+*S*=	19	14	53	227	227	73	203	509
41 — 39	42	3*B*+*S*−3*A*	4	—4	—52	92	—4	—44	—292	236
9, 42	43	4*x*=	4	—4	—52	92	—4	—44	—292	236
$43 \div \overline{4}$	44	*X*=	1	—1	—13	23	—1	—11	—73	59
24 + 44	45	*A+X*=	6	5	22	68	76	28	92	150
25 — 44	46	*B—X*=	1	2	15	5	21	15	77	7
10, 45	47	*K*=	6	5	22	68	76	28	92	150
11, 46	48	*D*=	1	2	15	5	21	15	77	7
47 * 48	49	*KD*=	6	10	330	340	1596	420	7084	1050
47 ◎ 2	50	*KK*=	36	25	484	4624	5776	784	8464	22500
48 ◎ 2	51	*DD*=	1	4	225	25	441	225	5929	49
50 + 51	52	*KK+DD*=	37	29	709	4549	6217	1009	14393	22549
$49 * \overline{2}$	53	2*KD*=	12	20	660	680	3192	840	14168	2100
50 — 51	54	*KK-DD*=	35	21	259	4599	5335	559	2535	22451
52, 12	55	*L*=	37	29	709	4649	6217	1009	14393	22549
53, 13	56	*M*=	12	20	660	680	3192	840	14168	2100
54, 14	57	*N*=	35	21	259	4599	5335	559	2535	22451

T 2 *The*

The Proofs.

32,	.	*E*	29	1229	1537	2809	6329
33,	.	*F*	20	140	312	2520	3080
34,	.	*G*	21	1221	1505	1241	5529
55,	.	*L*	37	709	1009	4649	6217
56,	.	*M*	12	660	840	680	3192
57,	.	*N*	35	259	559	4599	5335
32 + 33	58	*E+F*	49	1369	1849	5329	9409
45 + 56	59	*L+M*	49	1369	1849	5329	9409
33 * 34	60	*FG*	420	170940	469560	3127320	17029320
56 * 57	61	*MN*	420	170940	469560	3127320	17029320
32 ⊚ 2	62	*EE*	841	1510441	2362369	7890481	40056241
33 ⊚ 2	63	*FF*	400	19600	97344	6350400	9486400
34 ⊚ 2	64	*GG*	441	1490841	2265025	1540081	30569841
55 ⊚ 2	65	*LL*	1369	502681	1018081	21613201	38651089
56 ⊚ 2	66	*MM*	144	435600	705600	462400	10188864
57 ⊚ 2	67	*NN*	1225	67081	312481	21150801	28462225
63 + 64	68	*FF+GG*	841	1510441	2362369	7890481	40056241
66 + 67	69	*MM+NN*	1369	502681	1018081	21613201	38651089

1. In this Pattern being tyed to *Des Cartes* his Letters, but not willing to imitate him in making *B* an Integer, but *A* equal to some fraction, I avoid it, by beginning with b in a *less usual* shape; but changing it for a *more usual* as soon as I have found what Integer may express *B*, when I make *A* Integer.

2. In Equation 23, I reduce the *ratio* $\frac{A}{B}$ to the smallest terms [as $\frac{15}{6}$ to $\frac{5}{2}$, $\frac{24}{4}$ to $\frac{6}{1}$] that so the following work may not be cumbred with Numbers greater than needs, or require Reduction at the last. Yet making b $=3$. $c=14$, $\frac{A}{B}$ will be $\frac{280}{24}$ that is, $\frac{35}{3}$. Both these Numbers being odd (od *to* odd, and odd *from* odd, make even) will cause *E* and

and G to be even Numbers (F is always so.) Also K and D will be both even, which will cause L and N to be so. M is always an even Number. Wherefore all six ($E. F. G. L. M. N$) being even Numbers, they may be all divided by 2, aud so may be reduced to Numbers but half as great as they were. The same happens, when $b=5.\ c=18$. And when $b=7, c=22$. and when $b=9.\ c=26$.

3. When $b=2$ and therefore $c=$ any thing between 4 and 12, I make it 5, 7, 9, 11: but not 6. 8. or 10. because these are even Numbers as b is. By making $\frac{b}{c}=\frac{2}{6},\frac{2}{8},\frac{2}{10}$ I should produce no other Numbers than I had found before by making $\frac{b}{c}=\frac{1}{2},\frac{1}{3},\frac{1}{5}$. For the same reason, when b is 3, c will fall between 6 and 18. Yet I make not $c=9$, 12 or 15. For $\frac{b}{c}=\frac{3}{9},\frac{3}{12},\frac{3}{15}$ will produce no other values for $E\ F.\ G.\ L.\ M.\ N$, than those that I had found by $\frac{b}{c}=\frac{1}{3},\frac{1}{4},\frac{1}{5}$. If you take $b=15$, c may be any thing between 30 and 90; yet, for this reason, you must not let it be any Number dividable by 3 as 33, 36 .... 84, 87, or by 5 as 35, 40 .... 80, 85. Much less may it be dividable by both 3 and 5, as 45, 60, 75. For $\frac{b}{c}=\frac{15}{45},\frac{15}{60},\frac{15}{75}$ is but $\frac{1}{3},\frac{1}{4},\frac{1}{5}$ and will produce no other pairs of Triangles, than those did.

4. I wrought ont the whole columell of $b=1$, $c=4$ (though I foresaw that it would yield the same pair of Triangles that had found by $b=1, c=3$) that therein you might see, how some values of b and c would give you old answers, when you expected new, and could help you to no other pairs than those you had before. $b=1$, $c=3$, gave me 29, 20, 21 for the values of E, F, G; and 37, 12, 35 for L, M, N. But $b=1, c=4$, give me 37, 12, 35 for the values of E, F, G; and 29, 20, 21 for the values of L, M, N. The Triangles found by $\frac{b}{c}=\frac{1}{3},\frac{1}{5},\frac{2}{5},\frac{2}{9}$. You may find again by $\frac{b}{c}=\frac{1}{4},\frac{5}{12},\frac{5}{24},\frac{3}{8}$.

Every possible *ratio* of **b** to **c**, hath a correspondent *ratio* of **b** to **c**, which will make your labour fruitless; deluding you with this *inverted repetition*, if you have not *some Rule* to enable you to exclude all those Permutators at once, or to know them as they come in your way and so to avoid them.

5. this Pattern shews you a disorderly mixture of Answers in *Great* Numbers amongst *Smaller* Numbers. Though you take the values of **b** and **c** *in order* according to their greatness; yet the values of any side of your sought Triangles will not be sought in such order. $E=1537$ will fall between $E=6329$ and $E=27241$, next after which is $E=12637$. This adjoined Tablet shews how much greater Numbers do arise by supposing $C=13$ than by supposing $c=18$. So that here is need of *another Rule* for the orderly selecting of values of **b** and **c**, apt to lead us, in order, to Answers falling under any prescribed limit, [as for example 100, 000] that so we may not be cumbred with huge Numbers, when there are many smaller ones fit to answer the Question.

	b = 5	
	$C=13$	$c=18$
E	118301	233
F	101660	105
G	60501	208
L	186241	218
M	33540	120
N	183379	182

These *desired Rules* may be found by looking back upon what hath been already said concerning this Question. In which *Review* fearing that **b** and B may some where cause a mistake. I will leave out **b**, hence forward using y and z where I have hitherto used **b** and **c**. So that now I say *these* grounds instead of those of the 14 of the Pattern pag. 138

The Review.

Sought.	Given					
$D.E.F.G$	1	$y > 0$	6	$e=AA+BB$	11	$K=A+x$
$K.L.M.N$	2	$z > 2y$	7	$f=2AB$	12	$d=B-x$
$S.X.y.z$	3	$z < 6y$	8	$g=AA-BB$	13	$l=KK+DD$
	4	$A=2yz+zz$	9	$\int=E+7F$	14	$m=2KD$
	5	$B=12yy-2yz$	10	$4x=3B-3A+S$	15	$n=KK-DD$

$4 \circledcirc 2$	16	$AA = 4yyzz + 4yzzz + z^4$
$5 \circledcirc 2$	17	$BB = 144yyyy - 48yyyz + 4yyzz$
$4 * 5$	18	$AB = 24yyyz + 8yyzz - 2yz^3$
$16 + 17$	19	$AA + BB = +144yyyy - 48yyyz + 8yyzz + 4yz^3 + z^4$
$18 * \overline{2}$	20	$2AB = 48yyyz + 16yyzz - 4yz^3$
$16 - 17$	21	$AA - BB = -144y^4 + 48yyyz \ldots + 4yz^3 + z^4$
$19, 6$	22	$E = +144y^4 - 48y^3z + 8yyzz + 4yz^3 - z^4$
$20, 7$	23	$F = +48y^3z + 16yyzz - 4yz^3$
$21, 8$	24	$G = -144y^4 + 48y^3z \ldots + 4yz^3 + z^4$
$23 * \overline{7}$	25	$7F = 336y^3z + 112yyzz - 28yz^3$
$22 + 25$	26	$E + 7F = +144y^4 + 288y^3z + 120yyzz - 24yz^3 + z^4$
$9, 26$	27	$SS = +144y^4 + 288y^3z + 120yyzz - 24yz^3 + z^4$
$27\ \omega\ 2$	28	$S = 12yy + 12yz - zz$ (See the 48th of this.)
$5 * \overline{3}$	29	$3B = 36yy - 6yz$
$29 + 28$	30	$3B + S = 48yy + 6yz - zz$
$4 * \overline{3}$	31	$3A = 6yz + 3zz$
$30 - 31$	32	$3B - 3A + S = 48yy - 4zz$
$10, 32$	33	$4x = 48yy - 4zz$
$33 \div 4$	34	$x = 12yy - zz$
$4 + 34$	35	$A + x = 12yy + 2yz$
$5 - 34$	36	$B - x = zz - 2yz$
$11, 35$	37	$K = 12yy + 2yz$
$12, 36$	38	$D = zz - 2yz$
$37 \circledcirc 2$	39	$KK = 144yyyy + 48yyyz + 4yyzz$
$38 \circledcirc 2$	40	$DD = 4yyzz - 4yz^3 + z^4$
$37 * 38$	41	$KD = -24yyyz + 8yyzz + 2yz^3$
$39 + 40$	42	$KK + DD = 144y^4 + 48yyyz + 8yyzz - 4yz^3 + z^4$
$41 * \overline{2}$	43	$2KD = -48yyyz + 16yyzz + 4yz^3$
$39 - 40$	44	$KK - DD = 144y^4 + 48yyyz \ldots + 4yz^3 - z^4$
$13, 42$	45	$L = 144y^4 + 48yyyz + 8yyzz - 4yz^3 + z^4$
$14, 43$	46	$M = -48yyyz + 16yyzz + 4yz^3$
$15, 44$	47	$N = 144y^4 + 48yyyz \ldots + 4yz^3 - z^4$

Now

Now we have the habitudes of *A. B. D. E. F. G. K. L. M. N. S X* to *y* and *z* expressed in Equation 4. 5. 38. 22. 23. 24. 37. 45. 46. 47. 28. 34.

If we neglect not the *other root* of the 27th Equation, we shall find the like habitudes for the *third* Equicrural mentioned pag. 137.

27 ꝏ 2	48	$S = -12yy - 12yz + zz$ [A second value of *S*]
29 + 48	49	$3B + S = 24yy - 18yz + zz$
49 − 31	50	$3B - 3A + S = 24yy - 24yz - 2zz$
10, 50	51	$4x == 24yy - 24yz - 2zz$
51 ÷ 4	52	$X == 6yy - 6yz - \frac{1}{2}zz$ [A second value of *X*
4 + 52	53	$A + x == 6yy - 4yz + \frac{1}{2}zz$
5 − 52	54	$B - x == 6yy + 4yz + \frac{1}{2}zz$

These two Equations with the 11th. and 12th. will produce *second values* of *K and D*; and of *L. M. N* by them, which second values I will now call *T. V. P. Q. R.* (see pag. 137) For their sakes these *five new* Equations are here inserted.

$p = ?$	55	$t = A + x$
$q = ?$	56	$u = B - x$
$r = ?$	57	$p = TT + VV$
$t = ?$	58	$q = 2TV$
$u = ?$	59	$r = TT - VV$
55, 53	60	$T === 6yy - 4yz + \frac{1}{2}zz$
56, 54	61	$V === 6yy + 4yz + \frac{1}{2}zz$
60 ⊕ 2	62	$TT = 36y^4 - 48yyyz + 22yyzz - 4yz^3 + \frac{1}{4}z^4$
61 ⊕ 2	63	$VV = 36y^4 + 48yyyz + 22yyzz + 4yz^3 + \frac{1}{4}z^4$
60 * 61	64	$TV = 36y^4 \;....\; -10yyzz \;....\; +\frac{1}{4}z^4$

TT+.

62 + 63	65	$TT + VV = 72y^4 + 44yyzz + \frac{1}{2}z^4$
54 * 2	66	$2TV = 72y^4 - 20yyzz + \frac{1}{2}z^4$
62 − 63	67	$TT - VV = -96yyyz - 8yz^3$
57, 65	68	$P = 72yyyy + 44yyzz + \frac{1}{2}z^4$
58, 66	69	$Q = 72yyyy - 20yyzz + \frac{1}{2}z^4$
59, 67	70	$R = -96yyyz - 8yzzz$
68, 69, 70 48, 60, 61 52		We have now the habitudes of P, Q, R, S, T, V, X; to y and z. Which may be usefull if you would find the Equicrural $P, P, 2Q$ before the other two. (See Fig. XVII.) But if you have $E. F. G. L. M. N$ first; you may by them find P, Q, R (more easily than by Equation 68, 69, 70) by Equation 71, 72, 73.
	71	$R = -G - N$
	72	$Q = FG \div R = MN \div R$
	73	$P = E + F - Q = L + M - Q$. For
24 + 47	74	$G + N = 96yyyz + 8yz^3$
0 − 74	75	$-G - N = -96yyyz - 8yz^3$
70, 75	∴	$R = -G - N$. As was said in Equation 71
23	76	$F = 4yz$ multiplyed upon $+12yy + 4yz - zz$
24	77	$G = 12yy + zz$ upon $-12yy + 4yz + zz$
46	78	$M = 4yz$ upon $-12yy + 4yz + zz$
47	79	$N = 12yy + zz$ upon $+12yy + 4yz - zz$
69	80	$Q = -6yy - 2yz + \frac{1}{2}zz$ upon $-12yy + 4yz + zz$
70	81	$R = -8yz$ upon $12yy + zz$
	82	$FG = MN = QR$. (For 76 * 77 = 78 * 79 = 80 * 81)
82 ÷ R	∴	$Q = FG \div R = MN \div R$. as was said in Equation 72
22 + 23	83	$E + F = L + M = 144y^4 + 24yyzz + z^4$. (45 + 46)
83 − 69	84	$E + F - Q = L + M - Q = 72yyyy + 44yyzz + \frac{1}{2}z^4$
68, 84	∴	$P = E + F - Q = L + M - Q$. as was said in Equation 73
		Note concerning these factors of $F. G: M, N: Q. R$:
60 − 61	85	$-8yz = T - V$ $\left(\therefore 4yz = \frac{V-T}{\quad}\right)$
37 − 38	86	$12yy + 4yz - zz = K - D$. $\left(\therefore -6yy - 2yz + \frac{1}{2}zz = \frac{D-K}{2}\right)$
4 − 5	87	$-12yy + 4yz + zz = A - B$
4 + 5	88	$12yy + zz = A + B = K + D = T + V$. (37 + 38. 60 + 61)

Having found the way of deducing *this third* Equicrural ($P, P, 2Q$) from *the other two*, we now return *to them* , and to the search of those desired Remedyes against *inverted Repetion* and *confused Anticipation*. See pag. 142, lin. 3 and 15.

Two Examples of *inverted Repetion*.

$y.z$	$A.B$	$K.D$	$\div$	$a.b$	$k.d$	E	F	G	L	M	N
1. 3	15. 6	18. 3	3	5. 2	6. 1	29	20	21	37	12	35
1. 4	24. 4	20. 8	4	6. 1	5. 2	37	12	35	29	20	21
3.10	160. 48	168. 40	[illegible]	20.6	21.5	436	240	364	466	210	416
5.1[illegible]	504.120	480.144	24	21.5	20.6	466	210	416	436	24[illegible]	364

In these examples, from the lowest terms of the *ratio* $\frac{y}{z}$ (by Eq. 4, 5, 37, 38) are derived the values of $A.B.K.D$: whose *rationes* by a common divisor are reduced to their lowest terms, here called $a\,b.k\,d$. From which (by Equ. 6, 7, 8 : 13. 14. 15.) are derived the values of $E. F. G : L. M. N.$ Which ~~are~~ in the second pair of Permutators are all divisible by 2 : but here such bisection is needless. For *in either of those pairs* we see that *one ratio* of y to z makes $K > A$, and therefore $L > E$: but the *other ratio* of y to z makes $K < A$ and $L < E$. So that there will be *no permutation* when K is equal to A.

Scope		
	89	$A = K$
89, 4; 37	90	$zz + 2yz = 12yy + 2yz$
90 — 2yz	91	$zz = 12yy$
91 ω 2	92	$z = y * \sqrt{12}$
91 — 2yz	93	$z^2 - 2yz = 12yy - 2yz$
93, 38, 5	94	$D = B$
89, 94	95	$AA + BB = KK + DD$
89, 94	96	$2AB = 2KD$
89, 94	97	$AA - BB = KK - DD$
95, 6, 13	98	$E = L$
96, 7, 14	99	$F = M$
97, 8, 15	100	$G = N$

make $y = 1$; $z = \sqrt{12}$ ∴ $zz = 12$
∴ $A = K = 12 + \sqrt{48}$ $\sqrt{3} + 1 = a = k$
$B = D = 12 - \sqrt{48}$ $\sqrt{3} - 1 = b = d$
∴ $aa = kk = 4 + 2\sqrt{3}$ $ab = 2$
$bb = dd = 4 - 2\sqrt{3}$
∴ $aa + bb = kk + dd = 8$ 2
$2ab = 2kd = 4$ 1
$aa - bb = kk - dd = 4\sqrt{3}$ $\sqrt{3}$
∴ $e = l = 2$ ∴ $ee = ll = 4$
$f = m = 1$ $ff = mm = 1$
$g = n = \sqrt{3}$ $gg = nn = 3$

By a like process you may prove that $z < y * \sqrt{12}$ makes $A < K$, and therefore $a < k$; $d < b$; $E < L$: But $z > y * \sqrt{12}$ makes $A > K$ and therefore $a > k$; $d > b$; $E > L$.

In the next place it will be requisite that we compare these *rationes*

y together. Which may be done without confusion if we use letters of diffe-
$\bar{z}$ rent shapes thus.

1	$y > 0$	102	$\mathfrak{y} > 0$
2	$z > 2y$	103	$\mathfrak{z} > \mathfrak{y} * \sqrt{12}$
101	$z < y * \sqrt{12}$	104	$\mathfrak{z} < 6\mathfrak{y}$
4	$A = zz + 2yz$	105	$\mathfrak{A} = \mathfrak{z}\mathfrak{z} + 2\mathfrak{y}\mathfrak{z}$
5	$B = 12yy - 2yz$	106	$\mathfrak{B} = 12\mathfrak{y}\mathfrak{y} - 2\mathfrak{y}\mathfrak{z}$
37	$K = 12yy + 2yz$	107	$\mathfrak{K} = 12\mathfrak{y}\mathfrak{y} + 2\mathfrak{y}\mathfrak{z}$
38	$D = zz - 2yz$	108	$\mathfrak{D} = \mathfrak{z}\mathfrak{z} - 2\mathfrak{y}\mathfrak{z}$

In the following Table of five Examples $a.\ b.\ k.\ d.\ \mathfrak{a}.\ \mathfrak{b}.\ \mathfrak{k}.\ \mathfrak{d}$ are $A.B.K.D.\ \mathfrak{A}.\mathfrak{B}.\mathfrak{K}.\mathfrak{D}$ divided by their greatest common Divisors. Which Divisors are there set in two Columells marked thus ÷

Five examples of *Inverted Repetition.*

$y.\ z$	A	B	K	D	÷	a	b	k	d	$\mathfrak{y}.\ \mathfrak{z}$	$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{K}$	$\mathfrak{D}$	÷	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{k}$	$\mathfrak{d}$
1. 3	15	6	18	3	3	5	2	6	1	1. 4	24	4	20	8	4	6	1	5	2
3.10	160	48	168	40	8	20	6	21	5	5.18	504	120	480	144	24	21	5	20	6
7.18	576	336	840	72	24	24	14	35	3	3.14	280	24	192	112	8	35	3	24	14
5.12	264	180	420	24	12	22	15	35	2	1. 5	35	2	22	15	1	35	2	22	15
3. 8	112	60	156	16	4	28	15	39	4	2. 9	117	12	84	45	3	39	4	28	15

In all these, $a = \mathfrak{k}.\ b = \mathfrak{d}.\ k = \mathfrak{a}.\ d = \mathfrak{b}$. That is to say. These *Permutations* and *inverted repetitions* are seen in them *after* they have been divided by their greatest common Divisor and not *before*.

But before such division in all these examples $\frac{A}{B} = \frac{\mathfrak{K}}{\mathfrak{D}}$ and $\frac{K}{D} = \frac{\mathfrak{A}}{\mathfrak{B}}$ and therefore $A\mathfrak{D} = B\mathfrak{K}$ and $\mathfrak{A}D = \mathfrak{B}K$. Wherefore let these be the grounds of the following search for the habitude of $y.\ z.$ to $\mathfrak{y}.\ \mathfrak{z}$

	109	$A\mathfrak{D} + {}^{!}\,\mathfrak{K}$
	110	$\mathfrak{A}D = \mathfrak{B}K$
$4 * 108$	111	$A\mathfrak{D} = \mathfrak{z}\mathfrak{z}zz + 2\mathfrak{z}\mathfrak{z}yz - 2\mathfrak{y}\mathfrak{z}zz - 4\mathfrak{y}\mathfrak{z}yz$
$105 * 38$	112	$\mathfrak{A}D = \mathfrak{z}\mathfrak{z}zz - 2\mathfrak{z}\mathfrak{z}yz + 2\mathfrak{y}\mathfrak{z}zz - 4\mathfrak{y}\mathfrak{z}yz$
$5 * 107$	113	$B\mathfrak{K} = 144\mathfrak{y}\mathfrak{y}yy + 24\mathfrak{y}\mathfrak{z}yy - 24\mathfrak{y}\mathfrak{y}yz - 4\mathfrak{y}\mathfrak{z}yz$
$106 * 37$	114	$\mathfrak{B}K = 144\mathfrak{y}\mathfrak{y}yy - 24\mathfrak{y}\mathfrak{z}yy + 24\mathfrak{y}\mathfrak{y}yz - 4\mathfrak{y}\mathfrak{z}yz$
$111 - 112$	115	$A\mathfrak{D} - \mathfrak{A}D = 4\mathfrak{z}\mathfrak{z}yz - 4\mathfrak{y}\mathfrak{z}zz$
$113 - 114$	116	$B\mathfrak{K} - \mathfrak{B}K = 48\mathfrak{y}\mathfrak{z}yy - 48\mathfrak{y}\mathfrak{y}yz$
$109 - 110$	117	$A\mathfrak{D} - \mathfrak{A}D = B\mathfrak{K} - \mathfrak{B}K$
115,116,117	118	$4\mathfrak{z}\mathfrak{z}yz - 4\mathfrak{y}\mathfrak{z}zz = 48\mathfrak{y}\mathfrak{z}yy - 48\mathfrak{y}\mathfrak{y}yz$
$118 \div 4$	129	$\mathfrak{z}\mathfrak{z}yz - \mathfrak{y}\mathfrak{z}zz = 12\mathfrak{y}\mathfrak{z}yy - 12\mathfrak{y}\mathfrak{y}yz$ (Divide by $\mathfrak{z}y - \mathfrak{y}z$)
$119 \div$	120	$\mathfrak{z}z = 12\mathfrak{y}y$

120,	121	$z \,.\, 12y :: \mathfrak{p} \,.\, \mathfrak{z}$. or $\frac{z}{12y} = \frac{\mathfrak{p}}{\mathfrak{z}}$
120,	122	$\mathfrak{z} \,.\, 12\mathfrak{p} :: y \,.\, z$. or $\frac{\mathfrak{z}}{12y} = \frac{y}{z}$

Therefore having the *ratio* of z to y, you may find its correspondent permutator, the *ratio* of $\mathfrak{z}$ to $\mathfrak{p}$. Or having $\mathfrak{z} \,.\, \mathfrak{p}$. you may find $z \,.\, y$, by those two Analogyes in Equation 121, and 122.

Thus $\frac{y}{z} = \frac{1}{3} \therefore \frac{z}{12y} = \frac{3}{12} = \frac{1}{4} = \frac{\mathfrak{p}}{\mathfrak{z}}$ | $\frac{\mathfrak{p}}{\mathfrak{z}} = \frac{1}{4} \therefore \frac{\mathfrak{z}}{12\mathfrak{p}} = \frac{4}{12} = \frac{1}{3} = \frac{y}{z}$

Also $\frac{y}{z} = \frac{3}{10} \therefore \frac{z}{12y} = \frac{10}{36} = \frac{5}{18} = \frac{\mathfrak{p}}{\mathfrak{z}}$ | $\frac{\mathfrak{p}}{\mathfrak{z}} = \frac{5}{18} \therefore \frac{\mathfrak{z}}{12\mathfrak{p}} = \frac{18}{60} = \frac{3}{10} = \frac{y}{z}$

And therefore (as was said of **b.c.** pag. 142. lin. 1) every possible *ratio* of $\mathfrak{p}$ to $\mathfrak{z}$, hath a correspondent *ratio* of y to z. So that whatsoever value of *A. B K. D : E. F. G : L. M. N : P. Q. R* can be found by $z > y * \sqrt{12}$, the same may also be found by $z < y * \sqrt{12}$. Therefore in all the following work you may use only those values of z which be less than $y * \sqrt{12}$, without fear of losing one Triangle, or of falling into any *inverted repetition*.

To this end having found $\sqrt{12} = 3.46410, 16151$ &c. I multiply it by 1, 2, 3 &c. and set the products in the third Column of the adjoined Tablet, whose *use* is this.

y	$2y$	$y * \sqrt{12}$
1	2	3.464
2	4	6.928
3	6	10.392
4	8	13.856
5	10	17.320
6	12	20.784
7	14	24.248
8	16	27.712
9	18	31.176
10	20	34.641
11	22	38.105
12	24	41.569
13	26	45.033
&c	&c	&c.

Having made choice of any Integer to be the value of y, seek it in the first Column; the same line in the two following Columns gives you the limits of z.

Thus, making $y = 7$, z must be greater than 14, but less than 25; wherefore z may be any Integer between those limits. That is, z may be 15, 16. 17. 18. 19. 20. 21. 22. 23. 24. without danger of *inverted repetition*. And so $y = 11$ will admit z equal to any of the 16 Integers between 22 and 39.

But for the same reason $y = 1$ hath no other z but 3 and $y = 2$ hath no z but 5, and 6.

To

To such values of y and z placed orderly without interruption, you may fit the values of *A. B. K. D*, easily, by Equation 4. 5. 37, 38, Thus $A=zz+2yz$. $B=12yy-2yz$. $K=12yy+2yz$. $D=zz-2yz$ $y>0$ $z>2y \therefore z=2y+1$, $2y+2$ $2y+3$, &c.

z	zz	$2yz$	$\because A$	B	K	D
$2y$	$4yy$	$4yy$	$8yy$	$8yy$	$16yy$	0
$2y+1$	$4yy+4y+1$	$4yy+2y$	$8yy+6y+1$	$8yy-2y$	$16yy+2y$	$2y+1$
$2y+2$	$4yy+8y+4$	$4yy+4y$	$8yy+12y+4$	$8yy-4y$	$16yy+4y$	$4y+4$
$2y+3$	$4yy+12y+9$	$4yy+6y$	$8yy+18y+9$	$8yy-6y$	$16yy+6y$	$6y+9$
$2y+4$	$4yy+16y+16$	$4yy+8y$	$8yy+24y+16$	$8yy-8y$	$16yy+8y$	$8y+16$
$2y+5$	$4yy+20y+25$	$4yy+10y$	$8yy+30y+25$	$8yy-10y$	$16yy+10y$	$10y+25$
$2y+6$	$4yy+24y+36$	$4yy+12y$	$8yy+36y+36$	$8yy-12y$	$16yy+12y$	$12y+36$
$2y+7$	$4yy+28y+49$	$4yy+14y$	$8yy+42y+49$	$8yy-14y$	$16yy+14y$	$14y+49$
$2y+8$	$4yy+32y+64$	$4yy+16y$	$8yy+48y+64$	$8yy-16y$	$16yy+16y$	$16y+64$
&c.	&c.	&c.	&c.	&c.	&c.	&c.

Z	A	Differ.		B	dif	K	dif	D	Differ.	
$2y$	$8yy$	$6y+1$		$8yy$	$2y$	$16yy$	$2y$	0	$2y+2$	
$2y+1$	$8yy+6y+1$	$6y+3$	2	$8yy-2y$	$2y$	$16yy+2y$	$2y$	$2y+1$	$2y+3$	2
$2y+2$	$8yy+12y+4$	$6y+5$	2	$8yy-4y$	$2y$	$16yy+4y$	$2y$	$4y+4$	$2y+5$	2
$2y+3$	$8yy+18y+9$	$6y+7$	2	$8yy-6y$	$2y$	$16yy+6y$	$2y$	$6y+9$	$2y+7$	2
$2y+4$	$8yy+24y+16$	$6y+9$	2	$8yy-8y$	$2y$	$16yy+8y$	$2y$	$8y+16$	$2y+9$	2
$2y+5$	$8yy+30y+25$			$8yy-10y$		$16yy+10y$		$10y+25$		
&c.				&c.		&c.		&c.		

In these two patterns you see that when the values of z do increase by Unites, then $2y=$ the first difference in the Columns of B and K (additive for K, subtractive for B) But the second difference in the Columns of A and D is always $=2$ Unites additive. Wherefore suppose $y=1$, 2, 3 &c. and thence frame several values for z. Out of those values of y and z draw the leading Numbers and differences as in the adjoined tablet wherewith, by imitating the latter of these *two patterns*, we may fill a Table as in the next page.

y	$2y$	$6y$	$2y+1$	$6y+1$	yy	$8yy$	$16yy$
1	2	6	3	7	1	8	16
2	4	12	5	13	4	32	64
3	6	18	7	19	9	72	144
4	8	24	9	25	16	128	256
5	10	30	11	31	25	200	400
6	12	36	13	37	36	288	576
7	14	42	15	43	49	392	784

÷	y	z	A		B		K		D		÷	a	b	k	d
	1	2	8	7	8	2	16	2	0	2	8	1	1	2	0
	1	3	15		6		18		3		3	5	2	6	1
2	2	4	32	13	32	4	64	4	0	5	32	1	1	2	0
	2	5	45	15	28	4	68	4	5	7	1	45	20	68	5
2	2	6	60		24		72		12		12	5	2	6	1
3	3	6	72	19	72	6	144	6	0	7	72	1	1	2	0
	3	7	91	21	66	6	150	6	7	9	1	91	66	150	7
	3	8	112	23	60	6	156	6	16	11	4	28	15	39	4
3	3	9	135	25	54	6	162	6	27	13	27	5	2	6	1
	3	10	160		48		168		40		8	20	6	21	5
4	4	8	128	25	128	8	256	8	0	9	128	1	1	2	0
	4	9	153	27	120	8	264	8	9	11	3	51	40	88	3
2	4	10	180	29	112	8	272	8	20	13	4	45	28	68	5
	4	11	209	31	104	8	280	8	33	15	1	209	104	280	33
4	4	12	240	33	96	8	288	8	48	17	48	5	2	6	1
	4	13	273		88		296		65		1	273	88	296	65
5	5	10	200	31	200	10	400	10	0	11	200	1	1	2	0
	5	11	231	33	190	10	410	10	11	13	1	231	190	410	11
	5	12	264	35	180	10	420	10	24	15	12	22	15	35	2
	5	13	299	37	170	10	430	10	39	17	1	299	170	430	39
	5	14	336	39	160	10	440	10	56	19	8	42	20	55	7
5	5	15	375	41	150	10	450	10	75	21	75	5	2	6	1
	5	16	416	43	140	10	460	10	96	23	4	104	35	115	24
	5	17	459		130		470		119		1	459	130	470	119
6	6	12	288	37	288	12	576	12	0	13	288	1	1	2	0
	6	13	325	39	276	12	588	12	13	15	1	325	276	588	13
2	6	14	364	41	264	12	600	12	28	17	4	91	66	150	7
3	6	15	405	43	252	12	612	12	45	19	9	45	28	68	5
2	6	16	448	45	240	12	624	12	64	21	16	28	15	39	4
	6	17	493	47	228	12	636	12	85	23	1	493	228	636	85
6	6	18	540	49	216	12	648	12	108	25	108	5	2	6	1
	6	19	589	51	204	12	660	12	133	27	1	589	204	660	133
2	6	20	640		192		672		160		32	20	6	21	5

÷

÷	y	z	A		B		K		D		÷	a	b	k	d
7	7	14	392		392		784		0		392	1	1	2	0
	7	15	435	43	378	14	798	14	15	15	3	145	126	266	5
	7	16	480	45	364	14	812	14	32	17	4	120	91	203	8
	7	17	527	47	350	14	826	14	51	19	1	527	350	826	51
	7	18	576	49	336	14	840	14	72	21	24	24	14	35	3
	7	19	627	51	322	14	854	14	95	23	1	627	322	854	95
	7	20	680	53	308	14	868	14	120	25	4	170	77	217	30
7	7	21	735	55	294	14	882	14	147	27	147	5	2	6	1
	7	22	792	57	280	14	896	14	176	29	8	99	35	112	22
	7	23	851	59	266	14	910	14	207	31	1	851	266	910	207
	7	24	912	61	252	14	924	14	240	33	12	76	21	77	20

Thus adding or subtracting, you may continue this Table for any other value of *y*, without any *inverted repetition*. But you will meet with many *direct repetitions*, as in the lines, which have some number in the first columell. By leaving out *that Tables* first line and all its repeating lines you may thus make *a Second Table*.

y	z	A	B	K	D	÷	a	b	k	d
1	3	15	6	18	3	3	5	2	6	1
2	5	45	28	68	5	1	45	28	68	5
3	7	91	66	150	7	1	91	66	150	7
3	8	112	60	156	16	4	28	15	39	4
3	10	160	48	168	40	8	20	6	21	5
4	9	153	120	264	9	3	51	40	88	3
4	11	209	104	280	33	1	209	104	280	33
4	13	273	88	296	65	1	273	88	296	65
5	11	231	190	410	11	1	231	190	410	11
5	12	264	180	420	24	12	22	15	35	2
5	13	299	170	430	39	1	299	170	430	39
5	14	336	160	440	56	8	42	20	55	7
5	16	416	140	460	96	4	104	35	115	24
5	17	459	130	470	119	1	459	130	470	119
6	13	325	276	588	13	1	325	276	588	13
6	17	493	228	636	85	1	493	228	636	85
6	19	589	204	660	133	1	589	204	660	133
7	15	435	378	798	15	3	145	126	266	5
7	16	480	364	812	32	4	120	91	203	8
7	17	527	350	826	51	1	527	350	826	51
7	18	576	336	840	72	24	24	14	35	3
7	19	627	322	854	95	1	627	322	854	95
7	20	680	308	868	120	4	170	77	217	30
7	22	792	280	896	176	8	99	35	112	22
7	23	851	266	910	207	1	851	266	910	207
7	24	912	252	924	240	12	76	21	77	20
8	17	561	496	1040	17	1	561	496	1040	17
8	19	665	464	1072	57	1	665	464	1072	57
&	&	&c.	&c.	&c.	&c.	&	&c.	&c.	&c.	&c.

This

This second Table may also be made and continued thus. Let y be any Integer: $z > 2y : z < y * \surd 12$. Cast away every value of z, which doth not make up a *ratio* with y in its smallest terms (as $y = 2. z = 6 : y = 3. z = 9 : y = 4. z = 10$, or 12) Seek $z + 2y. z - 2y : 12y + 2z. 12y - 2z$. Multiply the two former by z: the two latter by y, the products will be *A.D K.B.* Find the greatest common divisor of *A. B. K. D*, and set it under the title ÷, set the Quotients under the Titles *a. b. k. d.*

Continue that Table as far as you will, you shall still see in it the same *disorder*, that you find in this beginning of it. When the Order in y and z is *natural*, the Order in the Columells *a. b. k. d.* is so confused that it seems uncapable of Redress; so as to be fitly subservient to our *scope*, which is *To exhibit an Orderly Enumeration of a considerable number of Answers to this 29th. Probleme*; and to that end, *To find every* ratio *of* y *to* z *that can afford such values of* a. b. k. d, *as may yield such* E. F. G : L. M. N *as being reduced to their lowest terms, may not leave the greatest of those six* (that is L) to be greater, than 100,000. Which limit was set pag. 142. lin. 19.

The disorder of the Numbers in the four last Columns depend upon the Columell next preceding (marked with ÷ wherein the greatest common *Divisors* of *A. B. K. D*, do stand as confusedly as if they had been drawn out of an Urn, like Lots. Their Quotients under *a. b. k. d*, must needs partake of that disorder, the Dividends standing orderly.

That Table shews but *six* different Divisors, namely, 24. 12. 8. 4. 3. 1; But as this 24 had not been in that Table, if I had not gone beyond $y = 6$: So *you may suspect* that by continuing that Table much farther, you shall discover some other common Divisors of *A. B. K. D* differing from those *six* and perhaps greater than 24.

But that *suspicion* will cease when you have considered that our y and z are Integers, and that z is greater than $2y$ (pag. 142) and therefore every z is $2y +$ some Integer. Call it h. Then, $z = 2y + h$.

But

But $A = zz + 2yz$. $B = 12yy - 2yz$. $K = 12yy + 2yz$. $D = zz - 2yz$

(pag. 149. lin. 3.)

When $z = 2y + b$, then $zz = 4yy + 4yh + hh$, and $2yz = 4yy + 2yh$

$$\begin{aligned} \therefore A &= 8yy + 6yh + hh \\ B &= 8yy - 2yh \\ K &= 16yy + 2yh \\ D &= \quad = \quad = 2yh + hh \end{aligned}$$

B and K may be divided by 2 and so might A, and D, if bb did not hinder. When b is an Odd number, hh is so: and then A and D will be so. Therefore 2 cannot be the common measure of all four, when $z - 2y$ is an *Odd* Number. When it is *even*, call it $2h$.

When $z = 2y + 2h$ then $zz = 4yy + 8yh + 4hh$, and $2yz = 4yy + 4yh$

$$\begin{aligned} \therefore A &= 8yy + 12yh + 4hh = 4a \therefore a = 2yy + 3yh + hh \\ B &= 8yy - 4yh = = = 4b \therefore b = 2yy - yh \\ K &= 16yy + 4yh = = 4k \therefore k = 4yy + yh \\ D &= = = 4yh + 4hh = 4d \therefore d = = yh + hh \end{aligned}$$

That is to say. When $z - 2y$ is an *even* number, then 4 may be the Common Divisor of A. B. K. D.

There is a *Second* sort of *Even* Numbers, whose *halves* are Even such a number we may express by $4h$.

When $z = 2y + 4h$, then $zz = 4yy + 16yh + 16hh$; and $2yz = 4yy + 8yh$

$$\begin{aligned} \therefore A &= 8yy + 24yh + 16hh = 8a \therefore a = yy + 3yh + 2hh \\ B &= 8yy - 8yh = = = 8b \therefore b = yy - yh \\ K &= 16yy + 8yh = = = 8k \therefore k = 2yy + yh \\ D &= = = 8yh + 16hh = 8d \therefore d = = yh + 2hh \end{aligned}$$

That is to say: when $z - 2y$ is the quadruple of an Integer then 8 may be the Common Divisor of A. B. K. D.

This gives you occasion to think of a *third* Sort of *Even Numbers* whose *quarters* are Even. They may be called $8h$. A *fourth* sort may be called $16h$. But these are useless here. For they cannot introduce any greater Divisor. With them A and D will end in $64hh$, $256hh$ &c: yet A and B must still begin with $8yy$. So that no power of 2, greater than 8, can be the greatest common Divisor.

Wherefore there are but these 3 Cases, $z - 2y = h$. or $= 2h$. or $= 4h$. And to these belong the 3 Numbers 1. 4. 8 as common Divisors of A. B. K. D. which were found by considering the limit $z > 2y$.

The greater limit was $z < \sqrt{12yy}$: That is $z < y * 3\frac{46411}{100000}$ (pag. 148) So that z may be *less* than $3y$, or *equal* to $3y$, or *greater* than $3y$.

$z = 3y$ in its smallest terms is $y = 1$. $z = 3$. Then $zz = 9$, $2yz = 6$ $\therefore A = 15$. $B = 6$. $K = 18$. $D = 3$. Here the Common Divisor is 3 and the Quotients are $a = 5$. $b = 2$. $k = 6$. $d = 1$ (as in pag. 146 and 147.)

$z < 3y$ is $z = 3y -$ Some Integer. Call it ϵ $\therefore z = 3y - \epsilon$. But remember that $z > 2y$. $\therefore 3y - \epsilon > 2y$ $\therefore \epsilon < y$. What you subtract from $3y$ must be less than y.

When $z == 3y - \epsilon$, then $zz == 9yy - 6y\epsilon + \epsilon\epsilon$, and $2yz == 6yy - 2y\epsilon$

$\therefore A = 15yy - 8y\epsilon + \epsilon\epsilon$		$A = 15yy + 8y\epsilon + \epsilon\epsilon$
$B = 6yy + 2y\epsilon$	So $z > 3y$	$B = 6yy - 2y\epsilon$
$K = 18yy - 2y\epsilon$	Or $z = 3y + \epsilon$ gives	$K = 18yy + 2y\epsilon$
$D = 3yy - 4y\epsilon + \epsilon\epsilon$		$D = 3yy + 4y\epsilon + \epsilon\epsilon$

Wherefore no common Divisor greater than 1, can be found to some entire values of ϵ. By the like operations, you may prove that the same is to be affirmed of $A. B. K. D$, when $z = 3y - 2\epsilon$, or $3y + 2\epsilon$, that is $-$ or $+$ some *even* Number. But if for ϵ, or 2ϵ you put 3ϵ, it will not be so.

For when $z = 3y - 3\epsilon$, then $zz = 9yy - 18y\epsilon + 9\epsilon\epsilon$, and $2yz = 6yy - 6y\epsilon$

$$\begin{aligned} \therefore A &= 15yy - 24y\epsilon + 9\epsilon\epsilon = 3a \therefore a = 5yy - 8y\epsilon + 3\epsilon\epsilon \\ B &= 6yy + 6y\epsilon == 3b \therefore b = 2yy + 2y\epsilon \\ K &= 18yy - 6y\epsilon == 3k \therefore k = 6yy - 2y\epsilon \\ D &= 3yy - 12y\epsilon + 9\epsilon\epsilon = 3d \therefore d = yy - 4y\epsilon + 3\epsilon\epsilon \end{aligned}$$

When $z = 3y + 3\epsilon$, then $zz = 9yy + 18y\epsilon + 9\epsilon\epsilon$, and $2yz = 6yy + 6y\epsilon$

$$\begin{aligned} \therefore A &= 15yy + 24y\epsilon + 9\epsilon\epsilon = 3a \therefore a = 5yy + 8y\epsilon + 3\epsilon\epsilon \\ B &= 6yy - 6y\epsilon == 3b \therefore b = 2yy - 2y\epsilon \\ K &= 18yy + 6y\epsilon == 3k \therefore k = 6yy + 2y\epsilon \\ D &= 3yy + 12y\epsilon + 9\epsilon\epsilon = 3d \therefore d = yy + 4y\epsilon + 3\epsilon\epsilon \end{aligned}$$

That is to say. When the difference between z and $3y$ is the triple of some Integer, then 3 may be the common Divisor of $A. B. K. D$, whether z be greater or less than $3y$.

(If $\epsilon = 0$, then $\epsilon\epsilon$ and $y\epsilon$ are 0: and therefore $a = 5yy$. $b = 2yy$, $k = 6yy$, $d = yy$. But then $z = 3y - 0$, or $z = 3y + 0$, that is $z = 3y$. $\therefore y$ may be 1, $\therefore yy = 1$ $\therefore a = 5$. $b = 2$. $k = 6$. $d = 1$. as above in the third line of this page.)

As

As the *lesser limit* helped us to 1, 4, 8 for common Divisors of *A. B. K. D*; So this *greater limit* gives us 1, 3. If these multiply those former, we shall have 3. 12. 24. And so we are again fallen upon the *six* Divisors (1. 3. 4. 8. 12. 24) Pag. 152. lin. 26.

The Reason of that *Multiplication* is this

1.) Whensoever $z=3y-3\epsilon$ or $3y+3\epsilon$, it makes *A. B. K. D.* divisible by 3. If the same z be also $2y+4h$ (as $y=7$. $z=18$. $y=17$. $z=54$) it makes them divisible by 8. And therefore their greatest common Divisor is 3 times 8, that is 24.

2.) But if the same z be (not $2y+4h$, but) $2y+2h$, it makes them divisible by 4; and therefore their greatest common Divisor is 3 times 4. that is 12. As when $y=5$. $z=12$. Or $y=7$. $z=24$.

3.) But if $z=3y-3\epsilon$ or $z=3y+3\epsilon$, be also $2y+$ some odd number, it makes them not divisible by any number greater than 1. So that their greatest common Divisor is 3 times 1 that is 3. As when $y=4$, $z=9$. Or when $y=10$, $z=33$.

4.) When z is not equal to $3y-$ or $+$ some tripled Integer then it makes not *A. B. K. D* divisible by any number greater than 1. So that their common Divisor is no other then was designed before by $z=2y+h$, $2y+2k$, $2y+4h$, as in pag. 153.

Wherefore now there is no place left for that suspicion (pag. 152 lin. 28) that *A. B. K. D* can have some other greatest common Divisors than one of those *six* 24. 12. 8. 4. 3. 1.

Let the *rationes* of y to z be distinguished by these *six* Divisors into *six Ranks*, whose *Leaders* may be found by these 3 last pages, supposing $h=1$. and $\epsilon=1$ or 0, and then saying

Greatest com. divis.		
24	$z=2y+4=3y-3 \therefore y=7 \therefore z=18$	I
12	$z=2y+2=3y-3 \therefore y=5 \therefore z=12$	II
8	$z=2y+4=3y+1 \therefore y=3 \therefore z=10$	III
4	$z=2y+2=3y-1 \therefore y=3 \therefore z=8$	IV
3	$z=2y+1=3y-0 \therefore y=1 \therefore z=3$	V
1	$z=2y+1=3y-1 \therefore y=2 \therefore z=5$	VI

The *last* of each Rank requires much more work to find it.

The *last ratio* of y to z may be *determined* in every one of the *six* Ranks because their Effects (*E. F. G. L. M. N*) are *limited*. When Division hath reduced them to their smallest terms (*e. f. g. l. m. n.*) then (*d*) the greatest of those *six* may not be greater than 100000, (pag. 152. lin. 19.) No number greater than 1 can divide *E. F. G. L. M. N.* if they come from y and z in the 2, 4, 5 or sixt Rank. When they come from y and z in the first or third Rank, then 2, and no other Number can be their common Divisor (See pag. 146 lin. 14. pag. 141 lin. 4. and pag. 153 lin. 18.) When the common Divisor is 8, then $z=2y+4h$ ∴ z is *even*. Then y must be *Odd*. If h be *even*, k and d are *even*, but a and b are *odd*. h being *Odd* makes k and d *odd*, but a and b *even*. So that whether h be *even* or *odd* *E. F. G. L. M. N* will be *even*. (See pag. 141. *l.* 3. and Equation 6, 7, 8, 13, 14, 15 pag. 142.) Wherefore when the greatest common Divisor of *A. B. K. D*, is 8, or [3 times 8, that is] 24 then $L \ngtr 200000$ that so $\frac{1}{2}L=l$, may be $\ngtr 100000$. As in the following Enquiry.

2	$z>2y$ These 9 belong to all six Ranks				These 4 belong only to (I) and (III)		
13	$kk+dd=L$	$2 * 2y$	125	$2yz>4yy$		129	$L=2l$
37	$12yy+2yz=K$	$125+12yy$	126	$12yy+2yz>16yy$	$124 * 2$	130	$2l \ngtr 200000$
123	$DD=0$	37,126	127	$K>16yy$	129,130	131	$L \ngtr 200000$
124	$l \ngtr 100000$	$123+KK$	128	$KK+DD>KK$	13, 131	132	$kk+dd \ngtr 200000$

(I)

	133	$K=24k$
	134	$D=24d$
133 ⊙ 2	135	$KK=576kk$
134 ⊙ 2	136	$DD=576dd$
135 + 136	137	$KK+DD=576kk+576dd$
132 * $\overline{576}$	138	$576kk+576dd \ngtr 115200000$
137,138	139	$KK+DD \ngtr 115200000$
128, 139	140	$KK \ngtr 115200000$
143 ω 2	141	$K \ngtr 10733\lfloor 12$
127, 141	142	$16yy \ngtr 10733\lfloor 12$
142 ÷ $\overline{16}$	143	$yy \ngtr 670\lfloor 82$
143 ω 2	144	$y \ngtr 25\frac{95}{100}$

(III)

	145	$K=8k$
	146	$D=8d$
145 ⊙ 2	147	$KK=64kk$
146 ⊙ 2	148	$DD=64dd$
147 + 148	149	$KK+DD=64kk+64dd$
132 * $\overline{64}$	150	$64kk+64dd \ngtr 12800000$
149, 150	151	$KK+DD \ngtr 12800000$
128, 151	152	$KK \ngtr 12800000$
152 ω 2	153	$K \ngtr 3577\lfloor 71$
127,153	154	$16yy \ngtr 3577\lfloor 71$
154 ÷ $\overline{16}$	155	$yy \ngtr 223\lfloor 61$
155 ω 2	156	$y \ngtr 14\frac{95}{100}$

These *three* Collations belong to the other 4 Ranks, namely II. IV. V. VI.

157	$L-l$	124 · 157	158	L ∆ 100000	13, 158	159	$kk+dd$ ∆ 100000	

		(II)			(V)
	160	$K=12k$		184	$K=3k$
	161	$D=12d$		185	$D=3d$
160 ⊚ 2	162	$KK=144kk$	184 ⊚ 2	186	$KK=9kk$
161 ⊚ 2	163	$DD=144dd$	185 ⊚ 2	187	$DD=9dd$
162 + 163	164	$KK+DD=144kk+144DD$	186 + 187	188	$KK+DD=9kk+9dd$
159 ✱ $\overline{144}$	165	$144kk+144DD$ ∆ 14400000	159 ✱ $\overline{9}$	189	$9kk+9dd$ ∆ 900000
164, 165	166	$KK+DD$ ∆ 14400000	188, 189	190	$KK+DD$ ∆ 900000
128, 166	167	KK ∆ 14400000	128, 190	191	KK ∆ 900000
167 ω 2	168	K ∆ 3794,73 ...	191 ω 2	192	K ∆ 948,684
127, 168	169	$16yy$ ∆ 3794,73 ...	127, 192	193	$16yy$ ∆ 948,684
159 ÷ $\overline{16}$	170	yy ∆ 237,171	193 ÷ $\overline{16}$	194	yy ∆ 59,292
170 ω 2	171	y ∆ $15\frac{4+}{10}$	194 ω 2	195	y ∆ $>\frac{7}{10}+$

		(IV)			(VI)
	172	$K=4k$		196	$K=k$
	173	$D=4d$		197	$D=d$
172 ⊚ 2	174	$KK=16kk$	196 ⊚ 2	198	$KK=kk$
173 ⊚ 2	175	$DD=16dd$	197 ⊚ 2	199	$DD=dd$
174 + 175	176	$KK+DD=16kk+16dd$	198 + 199	200	$KK+DD=kk+dd$
159 ✱ $\overline{16}$	177	$16kk+16dd$ ∆ 1600000	159 ✱ $\overline{1}$	201	$kk+dd$ ∆ 100000
176, 177	178	$KK+DD$ ∆ 1600000	200, 201	202	$KK+DD$ ∆ 100000
128, 178	179	KK ∆ 1600000	128, 202	203	KK ∆ 100000
179 ω 2	180	K ∆ 1264,91 ...	203 ω 2	204	K ∆ 316,227 ...
127, 180	181	$16yy$ ∆ 1264,91	127, 204	205	$16yy$ ∆ 316,227 ...
181 ÷ $\overline{16}$	182	yy ∆ 79,057 ...	205 ÷ $\overline{16}$	206	yy ∆ 19,764 ...
182 ω 2	183	y ∆ $8\frac{891}{1000}$	206 ω 2	207	y ∆ $4\frac{445}{10}+$

y is always an Integer. Therefore omit the fractions and say y ∆ 25. 15. 14. 8. 7. 4
When the greatest common Divisor of $A.\ B.\ K.\ D$ is 24. 12. 8. 4. 3. 1

Yet I expressed those values of *K*. *yy* and *y*, *with fractions* because in the *Integers alone* you cannot see the Analogys which appear in them when the values of those roots have fractions adhering to them. Thus

Greatest com. divisor.		*K*	*yy*		*y*
24	3	1073312	67082	$\sqrt{3}=1.73205$	2590
8	1	357771	22361	$\sqrt{1}=1.00000$	1495
12		3794733	237171	$\sqrt{12}=3.46410$	15.400
4		1264911	79057	$\sqrt{4}=2.00000$	8.891
3		948683	59292	$\sqrt{3}=1.73250$	7.700
1		316227	19764	$\sqrt{1}=1.00000$	4.445

So that having *y* either for 24, or for 8 the other might have been found by analogy: and having *y* for any of the other *four*, Analogy would find *y* for the remaining *three*.

The close of pag. 155 said that *y* is not less than	7	5	3	3	1	2
pag. 157, lin. last saith that *y* is not greater than	25	15	14	8	7	4
when the greatest common Divisor of *A*. *B*. *K*. *D* is	24	12	8	4	3	1

So that where the greatest common Divisor is 1 *y* may be 2, 3, or 4. But there is not the same reason for the rest. Some of their intermedial values must be ~~required~~, for these causes.

First *y* and *z* must be Integers expressing a *ratio* in its smallest terms, and therefore they must not be both Even Numbers. For their *halves*, will be smaller Integers in the same *ratio*. This will cost all the Even values of *y* out of Rank I. II. III. IIII. because in them *z* is always Even. as being $2y+2h$ seeing the Divisors 24. 12, 8. 4 are all divisible by 4. See pag. 153 lin. 9.

Secondly *y* and *z* must not both be divisible by 3 for their *tierces* will be smaller Integers in the same *ratio*. This will cast out all Triples, (as 3, 6, 9 &c.) out of Rank I. II. V. because in them *z* is always divisible by 3 as being $3y-3$ or $3y+3$ seeing the Divisors 24, 12, 3 are all divisible by 3. See pag. 154 lin. 18 to l. 23.

For both these causes, all values of *y* dividable by 6 will be cast out of the two first Ranks.

Thirdly, no Number can be admitted for a value of *y* if it can have

have no z joined with it. Thus in Rank *V* (where the greatest common Divisor of *A. B. K. D* is 3, and therefore $z = 3y3 - 3e$ or $3y + 3e$) neither 2 nor 5 can be a value of y For

When $y = 2$, then between the limits of z (pag. 148) are only 5 and 6; which are both rejected. 5, because it is no tripled Integer; and 6, because it is an even Number as well as 2.

When $y = 5$, then between the limits of z (pag. 148) are only 11, 12, 13, 14, 15, 16, 17. Whereof none but 12 and 15 are triples. These I reject, because I had them before: 5, 12 in the Rank II. 5, 15 (that is 1, 3) in Rank *V*.

These may conveniently be exhibited all at one view, thus

Greatest com. divis.	Rank	Values of y, rejected	Remaining values of y
24	I	8. 9. 10. 12. 14. 15. 16. 18. 20. 21. 22. 24	7. 11 13. 17. 19. 23. 25
12	II	6. 8. 9. 10. 12. 14. 15	5. 7. 11. 13
8	III	4. 6. 8. 10. 12. 14	3. 5. 7. 9. 11. 13
4	IV	4. 6. 8.	3. 5. 7
3	V	2. 3. 5. 6	1. 4. 7
1	VI	None rejected	2. 3. 4 (p. 158. l. 18)

So that *few* values of y do fit our *scope*, which (pag. 152. lin. 19) was *To have all those Equicrurals belonging to this XXIX Probleme, whose greatest side (l) exceeds not* 100000. To this Scope these are unserviceable. *First* all Integers greater than 25

Secondly, of numbers *less* than 25, these XII. 6. 8, 10. 12. 14. 15. 16. 18. 20. 21. 22. 24.

Of the remaining XIII [namely. 1. 2. 3. 4. 5 7. 9. 11. 13. 17. 19. 23. 25] all are useless, save those few in each Rank, that are expressed in the last column of the Tablet next preceding.

We may now proceed, and seek values of z fit to be joined with these values of y

First, I add some numbers of the Tablet pag. 148, to continue it, for 4 other values of y with their limits for z. Then out of that Table and this continuation of it,

y	$2y$	$y * \sqrt{12}$
17	34	58.888
19	38	65.817
23	46	79.674
25	50	86.602

we

we may gather that, when y is	1	2. 3. 4	5. 7. 9	11.13.17	19.23.25
then z must be { greater than	2	4. 6. 8	10.14.18	22.26.34	38.46.50
{ less than	4	7.11.14	18.25.32	39.46.59	66.80.87

To find z for Rank I; wherein $K=24k$

Pag. 155, lin. 7 tells you that $z=2y+4b=3y-3s$ or $3y+3s$

Pag. 159 lin. 13 tells you that y may be 7. 11. 13. 17. 19. 23. or 25

then $2y$ will be 14.22.26. 34. 38. 46. or 50

∴ Add to these such a ($4b$) quadruple of some Integer, as that the sum may be triple to some Integer. So they will be of *two* sorts

y	7	13	19	25
$2y$	14	26	38	50
$2y+4$	18	30	42	54
$2y+16$	30	42	54	66
$2y+28$	42	54	56	78

y	11	17	23
$2y$	22	34	46
$2y+8$	30	42	54
$2y+20$	42	54	66
$2y+32$	54	66	78

12 is the difference in *both* sorts. Under the pricks stand Values greater than lin. 3 permits.

To find z for Rank II, wherein $K=12k$

Pag. 155 lin. 10 says, that $z=2y+2b=3y-3s$ or $3y+3s$

Pag. 159 lin. 14 says y may be 5. 7. 11. or 13.

∴ To their doubles add such a ($2b$) double of some Odd number as that the sum may be triple to some Integer. *Two* sorts.

y	5	11
$2y$	10	22
$2y+2$	12	24
$2y+14$	24	36

y	7	13
$2y$	14	26
$2y+10$	24	36
$2y+22$	36	48

In *both* sorts 12 is the difference line the 3d of this page excludes those which are here under the pricks.

To find z, for Rank III, wherein $K=8k$.

Pag. 155 lin. 17 says, $z=2y+4b=$ to no tripled Integer.

(See pag. 153 lin. 23.) Pag. 159 line 15, y may be 3. 5. 7. 9. 11 or 13.

∴ To their doubles add such a ($4b$) Quadruple of some Integer as that the sum may not be triple to any Integer.

y	3	5	7	9	11	13
$2y$	6	10	14	18	22	26
$2y+4$	10	14	18	22	26	30
$2y+8$	14	18	22	26	30	34
$2y+12$	18	22	26	30	34	38
$2y+16$	22	26	30	34	38	42

The difference is always 4

Put out 18. 30. 42. They are triples there will remain above the pricked lines.

y	3	5	7	9	9	11	11	11	13	13
z	10	14	22	22	26	26	34	38	34	38

To find z, for Rank IV; wherein $K=4k$

Pag. 155, lin. 17. $z=2y+2h=3y$—or+ some Integer, which is not dividable by 3 (see pag. 153. l. 9.) Pag. 159, l. 16. y may be 3, 5, or 7 ∵ To 6, 10, or 14 add such doubled odd numbers, as may be equal to $3y$—or + some Integer which is not the triple of an Integer. Of these *nine* values of z, *five* are excluded. *Three* by pag. 160. l. 3. *Two* are of Rank II. $y=5.\ z=12.$ and $y=7.\ z=24.$ So there remain 4. $y=3.\ z=8:\ y=5.\ z=16:\ y=7.\ z=16.$ or 20.

y	3	5	7
$2y$	6	10	14
$2y+2$	8	12	16
$2y+6$	12	16	20
$2y+10$	16	20	24

To find z for Rank V; wherein $K=3k$

Pag. 155. lin. 13. $z=2y+h=3y-3k$ or $3y+3k$. k is odd. Pag. 159. l. 17. y may be 1. 4. 7 ∵ To 2. 8. 14. add such odd numbers as will permit the difference between z and $3y$ to be divided by 3. Pag. 160. lin. 3. excludes 4 of these 12. *Foure* more that is 11. 13. 17. 19. are excluded because 12—11. 13—12. 21—17. 21—19. are not dividable by 3 without fraction. $y=7.\ z=21$ repeats $y=1.\ z=3.$ *Three* remain. $y=1.\ z=3:\ y=4.\ z=9:\ y=7.\ z=15.$

y	1	4	7
$2y$	2	8	14
$2y+1$	3	9	15
$2y+3$	5	11	17
$2y+5$	7	13	19
$2y+7$	9	15	21

To find z for Rank VI; wherein $K=k$

Pag. 155. lin. 17. $z=2y+h=$ some Integer not divisible by 3. Pag. 159. lin. 18. y may be 2, 3, or 4 ∵ To 4. 6. or 8 add some odd Number. You shall find *nine* values of z whereof pag. 160 excludes *three*. Rank V claims *two* more; namely $y=3.\ z=9$ (which is but $y=1.\ z=3$) and $y=4.\ z=9.$ So that only *four* remain. $y=2.\ z=5:\ y=3.\ z=7.\ y=4.\ z=11$, or 13.

y	2	3	4
$2y$	4	6	8
$2y+1$	5	7	9
$2y+3$	7	9	11
$2y+5$	9	11	13

Therefore, the different *rationes* of y to z fit for our purpose, are these 38

y 7. 11. 13. 13. 17. 17. 19. 19. 23. 23. 23. 25. 5. 7. 11. 11. 13. 3. 5
z 18. 30. 30. 42. 42. 54. 42. 54. 54. 66. 78. 54. 12. 24. 24. 36. 36. 10. 14

y 7. 9. 9. 11. 11. 11. 13. 13. 3. 5. 7. 7. 1. 4. 7. 2. 3. 4. 4
z 22. 22. 26. 26. 34. 38. 34. 38. 8. 16. 16. 20. 3. 9. 15. 5. 7. 11. 13

Pag. 160. lin. 3 says, when $y=25$, then z must be *less than* 87, or else you will fall upon *inverted repetition.* Here I find z must be *less than* 65, or else l will be greater than 100 000 (against pag. 152. lin. 18) Therefore I have here omitted $z=66$, or 78, though found pag. 160.

	208	$y=25$
37, 141	209	$12yy+2yz < 10733\frac{1}{2}$
208	210	$12yy=75000$
209—210	211	$2yz < 3233\frac{1}{2}$
208 * 2	212	$2y=50$
211 ÷ 212	213	$z < 64\,6624$

By those 39 values of y and of z you may find $A.\ B.\ K.\ D.$ thus:

y	7	13	19	23	5	11	5	9	11	3	7	7	4	*y*
z	18	42	42	66	12	36	14	26	38	8	20	15	11	*z*
zz	324	1764	1764	4356	144	1296	196	676	1444	64	400	225	121	
2 *yz*	252	1092	1596	3036	120	792	140	468	836	48	280	210	88	
Summ	576	2856	3360	7392	264	2088	336	1144	2280	112	680	435	209	*A*
Differ.	72	672	168	1320	24	504	56	208	608	16	120	15	33	*D*
12 *yy*	588	2028	4332	6348	300	1452	300	972	1452	108	588	588	192	
2 *yz*	252	1092	1596	3036	120	792	140	468	836	48	280	210	88	
Summ	840	3120	5928	9384	420	2244	440	1440	2288	156	868	798	280	*K*
Differ.	336	936	2736	3312	180	660	160	504	616	60	308	378	104	*B*
y	11	17	19	23	7	13	7	11	13	5	1	2	4	*y*
z	30	42	54	78	24	36	22	26	34	16	3	5	13	*z*
zz	900	1764	2916	6084	576	1296	484	676	1156	256	9	25	169	
2 *yz*	660	1428	2052	3588	336	936	308	572	884	160	6	20	104	
Summ	1560	3192	4968	9672	912	2232	792	1248	2040	416	15	45	273	*A*
Differ.	240	336	864	2496	240	360	176	104	272	96	3	5	65	*D*
12 *yy*	1452	3468	4332	6348	588	2028	588	1452	2028	300	12	48	192	
2 *yz*	660	1428	2052	3588	336	936	308	572	884	160	6	20	104	
Summ	2112	4896	6384	9936	924	2964	896	2024	2912	460	18	68	296	*K*
Differ.	792	2040	2280	2760	252	1092	280	880	1144	140	6	28	88	*B*
y	13	17	23	25	11	3	9	11	13	7	4	5	.	*y*
z	30	54	54	54	24	10	22	34	38	16	9	7	.	*z*
zz	900	2916	2916	2916	576	100	484	1156	1444	256	81	49	.	
2 *yz*	780	1836	2484	2700	528	60	396	748	988	224	72	42	.	
Summ	1680	4752	5400	5616	1104	160	880	1904	2432	480	153	91	.	*A*
Differ.	120	1080	432	216	48	40	88	408	456	32	9	7	.	*D*
12 *yy*	2028	3468	9348	7500	1452	108	972	1452	2028	588	192	108	.	
2 *yz*	780	1836	2484	2700	528	60	396	748	988	224	72	42	.	
Summ	2808	5304	8832	10200	1980	168	1268	2200	3016	812	264	150	.	*K*
Differ.	1248	1632	3864	4800	924	48	576	70	1040	364	120	66	.	*B*

$y.z$	A	B	K	D	÷	a	b	k	d
7. 18	576	336	840	72	24	24	14	35	3
11. 30	1560	792	2112	240	24	65	33	88	10
13. 30	1680	1248	2808	120	24	70	52	117	5
13. 42	2856	936	3120	672	24	119	39	130	28
17. 42	3192	2040	4896	336	24	133	85	204	14
17. 54	4752	1632	5304	1080	24	198	68	221	45
19. 42	3360	2736	5928	168	24	140	114	247	7
19. 54	4968	2280	6384	864	24	207	95	266	36
23. 54	5400	3864	8832	432	24	225	161	368	18
23. 66	7392	3312	9384	1320	24	308	138	391	55
23. 78	9672	2760	9936	2496	24	403	115	414	104
25. 54	5616	4800	10200	216	24	234	200	425	9
5. 12	264	180	420	24	12	22	15	35	2
7. 24	912	252	924	240	12	76	21	77	20
11. 24	1104	924	1980	48	12	92	77	165	4
11. 36	2088	660	2244	504	12	174	55	187	42
13. 36	2232	1092	2964	360	12	186	91	247	30
3. 10	160	48	168	40	8	20	6	21	5
5. 14	336	160	440	56	8	42	20	55	7
7. 22	792	280	896	176	8	99	35	112	22
9. 22	880	576	1368	88	8	110	72	171	11
9. 26	1144	504	1440	208	8	143	63	180	26
11. 26	1248	880	2024	104	8	156	110	253	13
11. 34	1904	704	2200	408	8	238	88	275	51
11. 38	2280	616	2288	608	8	285	77	286	76
13. 34	2040	1144	2912	272	8	255	143	364	34
13. 38	2432	1040	3016	456	8	304	130	377	57
3. 8	112	60	156	16	4	28	15	39	4

$y.z$	A	B	K	D	÷	a	b	k	d
5. 16	416	140	460	95	4	104	35	115	24
7. 16	480	364	812	32	4	120	91	203	8
7. 20	680	308	868	120	4	170	77	217	30
1. 3	15	6	18	3	3	5	2	6	1
4. 9	153	120	264	9	3	51	40	88	3
7. 15	435	378	798	15	3	145	126	266	5
2. 5	45	28	68	5	1	45	28	68	5
3. 7	91	66	150	7	1	91	66	150	7
4. 11	209	104	280	33	1	209	104	280	33
4. 13	273	88	296	65	1	273	88	295	65

The five first Columns are tranſcribed out of pag. 162. the *ſixt* hath the *Diviſors* belonging to each Rank. (ſee pag. 160, 161) Their *quotients* fill up the 4 laſt Columns. Of which Quotients I make *two Claſſes*, becauſe thoſe which were found by dividing by 24 or 8 will give *E. F. G. L. M. N.* in *even* numbers, and therefore diviſible by 2 : but *the reſt* will expreſs the *rationes* of *E. F. G. L. M. N.* in their ſmalleſt terms.

In each Claſſis I range the values of k, according to their natural order making $d.\ a.\ b$ comply with k though not ſo orderly. From which (by Equat. 13. 14. 15. 6. 7. 8. pag. 142) are derived *L. M. N. E. F. G.* pag. 164. 165.

k	d	kk	dd	kk+dd=L	kk--dd=N	l	m=kd	n
21	5	441	25	466	416	233	105	208
35	3	1225	9	1234	1216	617	105	608
55	7	3025	49	3074	2976	1537	385	1438
88	10	7744	100	7844	7644	3922	880	3822
112	22	12544	484	13028	12060	6514	2464	6030
117	5	13689	25	13714	13664	6857	585	6832
130	28	16900	784	17684	16116	8842	3640	8058
171	11	29241	121	29362	29120	14681	1881	14560
180	26	32400	676	33076	31724	16538	4680	15862
204	14	41616	196	41812	41420	20906	2856	20710
221	45	48841	2025	50866	46816	25433	9945	23408
247	7	61009	49	61058	60960	30529	1729	30480
253	13	64009	169	64178	63840	32089	3289	31920
265	36	70756	1296	72052	69460	36026	9576	34730
275	51	75625	2601	78226	73024	39113	14025	36512
286	76	81796	5776	87572	76020	43786	21736	38010
364	34	132496	1156	133652	131340	66826	12376	65670
368	18	135424	324	135748	135100	67874	6624	67550
377	57	142129	3249	145378	138880	72689	21489	69440
391	55	152881	3025	155906	149856	77953	21505	74928
425	9	180625	81	180706	180544	90353	3825	90272
414	104	171396	10816	182212	160580	91106	43056	80290

k	d	kk	dd	kd	L=l	M=2kd	N=n
6	1	36	1	6	37	12	35
35	2	1225	4	70	1229	140	1221
39	4	1521	16	156	1537	312	1505
68	5	4624	25	340	4649	680	4599
77	20	5929	400	1540	6329	3080	5529
88	3	7744	9	264	7753	528	7735
115	24	13225	576	2760	13801	5520	12649
150	7	22500	49	1050	22549	2100	22451
165	4	27225	16	660	27241	1320	27209
187	42	34969	1764	7854	36733	15708	33205
203	8	41209	64	1624	41273	3248	41145
217	30	47089	900	6510	47989	13020	46189
247	30	61009	900	7410	61909	14820	60109
266	5	70756	25	1330	70781	2660	70731
280	33	78400	1089	9240	79489	18480	77311
296	65	87616	4225	19240	91841	38480	83391

Read these *two* Pages as *one* Table, thus, when *k* is 21, then *d* is 5, *l* is 233, *m* is 105, *n* is 208, *a* is 20, *b* is 6, *e* is 218, *f* is 120, *g* is 182. Also, when *k* is 6, then *d* is 1, *l* is 37, *m* is 12, *n* is 35, *a* is 35, *b* is 2, *e* is 29, *f* is 20, *g* is 21.

a	b	aa	bb	aa+bb=E	aa—bb=G	e	f=ab	g
20	6	400	36	436	364	218	120	182
24	14	576	196	772	380	386	336	190
42	20	1764	400	2164	1364	1082	840	682
65	33	4225	1089	5314	3136	2657	2145	1568
99	35	9801	1225	11026	8576	5513	3465	4288
70	52	9400	2704	7604	2196	3802	3640	1098
119	39	14161	1521	15682	12640	7841	4641	6320
110	72	12100	5184	17284	6916	8542	7920	3458
143	63	20449	3969	24418	16480	12209	9009	8240
133	85	17689	7225	24914	10464	12457	11305	5232
198	68	39204	4624	43828	34580	21914	13464	17290
140	114	19600	12996	32596	6604	16208	15960	3302
155	110	24336	12100	36436	12236	18218	17160	6118
207	95	42849	9025	51874	33824	25937	19665	16912
238	88	56644	7744	64388	48900	32194	20944	24450
285	77	81225	5929	87154	75296	43577	21945	37648
255	143	65025	20449	85474	44576	42737	36465	22288
225	161	50625	25921	76546	24704	38273	36225	12352
304	130	92416	16900	109316	75516	54658	39520	37758
308	138	94864	19044	113908	75820	56954	42504	37910
234	200	54756	40000	94756	14756	47378	46800	7378
403	115	162409	13225	175634	149184	87817	46345	74592

a	b	aa	bb	ab	E=e	F=2ab	G=g
5	2	25	4	10	29	20	21
22	15	484	225	330	709	660	259
28	15	784	225	420	1009	840	559
45	28	2025	784	1260	2809	2520	1241
76	21	5776	441	1596	6217	3192	5335
51	40	2601	1600	2040	4201	4080	1001
104	35	10815	1225	3640	12041	7280	9591
91	66	8281	4356	6006	12537	12012	3925
92	77	8464	5929	7084	14393	14168	2535
174	55	30276	3025	9570	33301	19140	27251
120	91	14400	8281	10920	22681	21840	6119
170	77	28900	5929	13090	34829	26180	22971
185	91	34596	8281	16926	42877	33852	26315
145	126	21025	15876	18270	36901	36540	5149
209	104	43681	10816	21736	54497	43472	32865
273	88	74529	7744	24024	82273	48048	66785

In page 166. I fill the Column of *l* with its 38. numbers in their natural order, out of both these Classes together. The other Columns must have theirs, as those of *l* will permit.

	e	*f*	*g*	*a*	*b*	*l*	*m*	*n*	*k*	*d*
1	29	20	21	5	2	37	12	35	6	1
* 2	218	120	182	20	6	233	105	208	21	5
* 3	386	336	190	24	14	617	105	608	35	3
4	709	660	259	22	15	1229	140	1221	35	2
5	1009	840	551	28	15	1537	312	1505	39	4
* 6	1082	840	682	42	20	1537	385	1488	55	7
* 7	2657	2145	1568	65	33	3922	880	3822	88	10
8	2809	2520	1241	45	28	4649	680	4599	68	5
9	6217	3192	5335	76	21	6329	3080	5529	77	20
* 10	5513	3465	4288	99	35	6514	2464	6030	112	22
* 11	3802	3640	1098	70	52	6857	585	6832	117	5
12	4201	4080	1001	51	40	7753	528	7735	88	3
* 13	7841	4641	6320	119	39	8842	3640	8058	130	28
14	12041	7280	9591	104	35	13801	5520	12649	115	24
* 15	8642	7920	3458	110	72	14681	1881	14560	171	11
* 16	12209	9009	8240	143	63	16538	4680	15862	180	26
* 17	12457	11305	5232	133	85	20906	2856	20710	204	14
18	12637	12012	3925	91	66	22549	2100	22451	150	7
* 19	21914	13464	17290	198	68	25433	9945	23408	221	45
20	14393	14168	2535	92	77	27241	1320	27209	165	4
* 21	16298	15960	3302	140	114	30529	1729	30480	247	7
* 22	18218	17160	6118	156	110	32089	3289	31920	253	13
* 23	25937	19665	16912	207	95	36026	9576	34730	266	36
24	33301	19140	27251	174	55	36733	15708	33205	187	42
* 25	32194	20944	24450	238	88	39113	14025	36512	275	51
26	22681	21840	6119	120	91	41273	3248	41145	203	8
* 27	43577	21945	37648	285	77	43786	21736	38010	286	76
28	34829	26180	22971	170	77	47989	13020	46189	217	30
29	42877	33852	26315	186	91	61909	14820	60109	247	30
* 30	42737	36465	22288	255	143	66826	12376	65670	364	34
* 31	38273	36225	12352	225	161	67874	6624	67550	368	18
32	36901	36540	5149	145	126	70781	2660	70731	266	5
* 33	54658	39520	37758	304	130	72689	21489	69440	377	57
* 34	56954	42504	37910	308	138	77953	21505	74928	391	55
35	54497	43472	32865	209	104	79489	18480	77311	280	33
* 36	47378	46800	7378	234	200	90353	3825	90272	425	9
* 37	87817	46345	74592	403	115	91106	43056	80290	414	104
38	82273	48048	66785	273	82	91841	38480	83391	296	65

	p	q	r	t	u	h	i	o	w	y. z
1	56.5	7.5	56	15	1	3	5	4	7	1. 3
* 2	394	56	390	14	1	7	8	15	26	3. 10
* 3	802	80	798	20	1	5	16	21	38	7. 18
4	1484.5	115.5	1480	77	3	7	33	20	37	5. 12
5	2076.5	227.5	2064	91	5	13	35	24	43	3. 8
* 6	2186	261	2170	33	2	11	24	35	62	5. 14
* 7	5426	624	5390	52	3	16	39	55	98	11 30
8	5864.5	535.5	5840	153	7	17	63	40	72	2. 5
9	10976.5	1567.5	10864	209	15	55	57	56	97	7. 24
* 10	10418	1440	10318	72	5	32	45	77	134	7. 22
* 11	7946	504	7930	63	2	9	56	65	122	13.30
12	8748.5	467.5	8736	187	5	11	85	48	91	4. 9
* 13	14522.	2040	14378	85	6	40	51	91	158	13.42
14	22460 5	3139 5	22240	299	21	69	91	80	139	5. 16
* 15	18082	1520	18018	95	4	19	80	99	182	9. 22
* 16	24298	3080	24102	110	7	40	77	117	206	9. 26
* 17	26042	2280	25942	114	5	24	95	119	218	17.42
18	26436.5	1787.5	26375	325	11	25	143	84	157	3. 7
* 19	41098	5720	40698	143	10	65	88	153	266	17.54
20	29768.5	1207.5	29744	245	7	15	161	88	169	11.24
* 21	33818	1560	33782	130	3	13	120	133	254	19.42
* 22	38138	2760	38038	138	5	23	120	143	266	11.26
* 23	52042	6440	51642	161	10	56	115	171	302	19.54
24	61068.5	8627.5	60456	493	35	119	145	132	229	11.36
* 25	61538	8400	60962	175	12	75	112	187	326	11.34
26	47348.5	2827.5	47264	435	13	29	195	112	211	7. 16
* 27	76442	10920	75658	195	14	104	105	209	362	11.38
28	69704.5	8695.5	69160	527	33	93	187	140	247	7. 20
29	87036.5	10307.5	86424	589	35	95	217	156	277	13.36
* 30	88442	9240	87958	210	11	56	165	221	398	13.34
* 31	80098	5600	79902	200	7	32	175	207	386	23.54
32	75920.5	2479 5	75880	551	9	19	261	140	271	7. 15
* 33	108098	13920	107198	232	15	87	160	247	434	13.38
* 34	113738	14280	112838	238	15	85	168	253	446	23.66
35	110936.5	12967.5	110176	665	39	105	247	176	313	4. 11
* 36	97714	3536	97650	221	4	17	208	225	434	25.54
* 37	156482	22320	154882	279	20	144	155	299	518	23.78
38	151688.5	21367.5	150176	777	55	185	231	208	361	4. 13

In the two preceding Pages you have some Solutions of
Probl. XXIX. proposed pag. 131. which was declared capable
Pag. 138. *l*. 8 of *innumerable* Answers. And therefore I prescribed a *Limit* [No
P. 142. *l*. 19 side greater than 100, 000.] Pag. 152, I required that the
l. 14 Enumeration of them should be *orderly*. pag. 159. I declared that
l. 20 I would have that Enumeration *Complete*, giving *All* the answers
that do not exceed 160, 000. in their greatest side. Now, the
first Columell of pag. 166 tells you that there are 38 such pairs
of Equicrurall Triangles; as the first Columell of pag. 167. tells
l. 10 you, that there are *as many Adherents*. I had told you (pag. 138.)
that *each* of those paires had a *third* Equicrural, which in all wayes
of Inquisition would offer it self, pressing to be taken notice of with
its two *cognata*.

These *three* Equicrurals are expressed
in the XVII Scheme thus,

	I	II	III
Legs	*e. e*	*l. l*	*p. p*
Base	2 *f*	2 *m*	2 *q*
Height	*g*	*n*	*r*
Area	*f g*	*m n*	*q r*

Of these 38 Answers, the *first* is that
P. 136. *l*. 5 which Monsieur *Des Cartes* gave for
he had thought of giving no sign that
an answer. But the *Adherent*, $p=56\frac{1}{2}$. $q=7\frac{1}{2}$. $r=56$.

At first sight it appears, they are all Integers except 16 values
of *p*. and as many of *q*, which could not be made Integers, but by
doubling all the Numbers in the Columns of *e. f. g. m. n. k*; contrary to our intention of having them expressed in as small Integers as may be. The *rationes* of *a* to *b*, or of *k* to *d*, or of *t* to *u*
cannot be expressed in smaller terms. The same may be said of
e. f. g. l. m. n. for no common divisor can be found for them all
six. The remaining properties required in them are these
$ee=ff+gg$. $ll=mm+nn$. $pp=qq+rr$. $fg=mn=pq$. $2e+2f=$
Pag. 132. $2l+2m=2p-2q$ ∵ $e+f=l+m=p-q$.

The remaining properties required in them are these
Pag. 132 $ee=ff+gg$. $ll=mm+nn$. $pp=qq+rr$. $fg=nm=qr$.
Pag. 137 $2e+2f=2l+2m=2p-2q$ ∵ $e+f=l+m=p-q$.

For if the numbers, under *e. f. g : l. m. n : p. q. r :* in
pag. 166 and 167, have these properties; they are just
answers satisfying this XXIX. Probleme as it was proposed
Pag. 131 at *Paris* : and as it hath been further clogged with other
requisites by *mee*.

That in all the 38 lines of those two Pages $e+f=l+m=p-q$, Addition and subtraction alone will assure you; And the Summe or Difference so found will be $=\mathfrak{w}\mathfrak{w}$ when $\mathfrak{w}$ is odd: but it will be twice the square of $\frac{1}{2}\mathfrak{w}$, when $\mathfrak{w}$ is even.

He, that would try, whether fg, mn, qr be equal to one another, needs not multiply f upon g, m upon n, q upon r and compare their products: but let him find these smaller products $\mathfrak{h}\mathfrak{i}$, $\mathfrak{h}\mathfrak{o}$, $\mathfrak{h}\mathfrak{w}$, $\mathfrak{i}\mathfrak{o}$, $\mathfrak{i}\mathfrak{w}$, $\mathfrak{o}\mathfrak{w}$, and compare them with f, g, m, n, q, r according to the Equations in *this* or *the following* page. He will then grant, that $\mathfrak{i}\mathfrak{o}\mathfrak{h}\mathfrak{w}=\mathfrak{h}\mathfrak{o}\mathfrak{i}\mathfrak{w}=\frac{1}{2}\mathfrak{h}\mathfrak{i}*2\mathfrak{o}\mathfrak{w}=\mathfrak{h}\mathfrak{i}\mathfrak{o}\mathfrak{w}$; That is, that $fg=mn=qr$.

And so without seeking the square of every Number in the Columels of e, f, g; l, m, n; p, q, r; you may be assured that in every line of those two Pages, $ee=ff+gg$. $ll=mm+nn$. $pp=qq+rr$. If in the examining of these nine columels you find that the habitudes of e, f, g to a and b; of l, m, n, to k and d; of p, q, r to t and u be according to the Equations of Pag. 170.

Pag. 170 and 171 are useful not only in the Examination of the pages 166 and 167, and for discovering the reasons of divers Concinnityes, which you will find in the Numbers of those pages; but also to shew several wayes of filling up any defect in those columns, or extending that Table as much further as you will.

Mutual habitudes of $a.\ b.\ d.\ e.\ f.\ g.\ \mathfrak{h}.\ \mathfrak{i}.\ k.\ l.\ m.\ n.\ \mathfrak{o}.\ p.\ q.\ r.\ t.\ u.\ \mathfrak{w}$
in *any one* of the 38 lines of pag. 166 and 167.

$f=\mathfrak{i}\mathfrak{o}$. $g=r-n=\mathfrak{h}\mathfrak{w}$. $m=\mathfrak{h}\mathfrak{o}$. $n=r-g=\mathfrak{i}\mathfrak{w}$. $2q=f+2m-e$
$2q=2f+m-l$. $r=g+n=\mathfrak{h}\mathfrak{w}+\mathfrak{i}\mathfrak{w}$. $2p=l+2f+3m=e+2m+3f$

$$b=\frac{m}{\mathfrak{o}}=\frac{g}{\mathfrak{w}}.\ t=\frac{n}{\mathfrak{w}}=\frac{f}{\mathfrak{o}}.\ \mathfrak{o}=a-d=k-b=\frac{f}{\mathfrak{i}}=\frac{m}{\mathfrak{h}}$$

$$w=a+b=k+d==\frac{g}{\mathfrak{h}}.\ e+f=l+m=p-q$$

$$ee=ff+gg.\ ll=mm+nn.\ pp=qq+rr.\ fg=mn=qr=\mathfrak{h}\mathfrak{i}\mathfrak{o}\mathfrak{w}$$

In pag. 170, you have more Equations in two Columns. The *former* of them belongs to the *Upper Classis* pag. 164, 165, and the *starred* lines in pag. 166, 167. The *other* column is for the *rest*.

Mutual habitudes of *a. b. d. e. f. g.* $\mathfrak{h}$. $\mathfrak{i}$. *k. l. m. n* $\mathfrak{o}$. *p. q. r. t. u.* $\mathfrak{w}$.

In the 22 lines marked with *

$e=\frac{aa+bb}{2}=2\mathfrak{hh}+\mathfrak{io}=\frac{1}{2}\mathfrak{ww}-\mathfrak{io}$

$f=ab=\mathfrak{io}$

$g=\frac{aa-bb}{2}=\mathfrak{hw}$

$l=\frac{kk+dd}{2}=2\mathfrak{ii}+\mathfrak{ho}=\frac{1}{2}\mathfrak{ww}-\mathfrak{ho}$

$m=kd=\mathfrak{ho}$

$n=\frac{kk-dd}{2}=\mathfrak{iw}$

$p=2tt+2uu=\frac{1}{2}\mathfrak{ww}+\mathfrak{hi}=2\mathfrak{oo}-\mathfrak{hi}$

$q=4tu=\mathfrak{hi}$

$r=2tt-2uu=\mathfrak{ow}=\mathfrak{hw}+\mathfrak{iw}$

$\mathfrak{h}=\frac{a-b}{2}=\frac{kd}{k-b}=\frac{q}{\mathfrak{i}}$

$\mathfrak{i}=\frac{k-d}{2}=\frac{ab}{a-d}=\frac{q}{\mathfrak{h}}$

$\mathfrak{o}=t+u=\mathfrak{h}+\mathfrak{i}=\frac{r}{\mathfrak{w}}$

$\mathfrak{w}=2t-2u=\frac{r}{\mathfrak{o}}$

$aa=e+g$ $\quad bb=e-g$

$kk=l+n$ $\quad dd=l-n$

$4tt=p+r$ $\quad 4uu=p-r$

$2\mathfrak{hh}=e-f=\frac{aa-2ab+bb}{2}$

$2\mathfrak{ii}=l-m=\frac{kk-2kd+dd}{2}$

$2\mathfrak{oo}=p+q=2tt+4tu+2uu$

$\frac{1}{2}\mathfrak{ww}=e+f=\frac{aa+2ab+bb}{2}$

$\frac{1}{2}\mathfrak{ww}=l+m=\frac{kk+2kd+dd}{2}$

$\frac{1}{2}\mathfrak{ww}=p-q=2tt-4tu+2uu$

In the 16 lines not so marked.

$e=aa+bb=\mathfrak{hh}+\mathfrak{io}=\mathfrak{ww}-\mathfrak{io}$

$f=2ab=\mathfrak{io}$

$g=aa-bb=\mathfrak{hw}$

$l=kk+dd=\mathfrak{ii}+\mathfrak{ho}=\mathfrak{ww}-\mathfrak{ho}$

$m=2kd=\mathfrak{ho}$

$n=kk-dd=\mathfrak{iw}$

$p=\frac{tt+uu}{4}=\mathfrak{ww}+\frac{1}{2}\mathfrak{hi}=4\mathfrak{oo}-\frac{1}{2}\mathfrak{hi}$

$q=\frac{1}{2}tu=\frac{1}{2}\mathfrak{hi}$

$r=\frac{tt-uu}{4}=2\mathfrak{ow}=\mathfrak{hw}+\mathfrak{iw}$

$\mathfrak{h}=a-b=\frac{2kd}{k-b}=\frac{2q}{\mathfrak{i}}$

$\mathfrak{i}=k-d=\frac{2ab}{a-d}=\frac{2q}{\mathfrak{h}}$

$\mathfrak{o}=\frac{t+u}{4}=\frac{r}{2\mathfrak{w}}=\frac{\mathfrak{h}+\mathfrak{i}}{2}$

$\mathfrak{w}=\frac{t-u}{2}=\frac{r}{2\mathfrak{o}}$

$2aa=e+g$ $\quad 2bb=e-g$

$2kk=l+n$ $\quad 2dd=l-n$

$\frac{1}{2}tt=p+r$ $\quad \frac{1}{2}uu=p-r$

$\mathfrak{hh}=e-f=aa-2ab+bb$

$\mathfrak{ii}=l-m=kk-2kd+dd$

$4\mathfrak{oo}=p+q=\frac{tt+2tu+uu}{4}$

$\mathfrak{ww}=e+f=aa+2ab+bb$

$\mathfrak{ww}=l+m=kk+2kd+dd$

$\mathfrak{ww}=p-q=\frac{tt-2tu+uu}{4}$

But to express all these by their habitudes to y and z, it is not sufficient to distinguish them thus into *two Classes*: *Six Ranks* must be made of them, as in the next Page.

The

The *rationes* of *y* to *z* in 6 ranks.

I	II	III	IV	V	VI
y. z	y. z	y. z	y. z	y. z	y. z
7.18	5.12	3.10	3.8	1.3	2.5
11.30	7.24	5.14	5.16	4.9	3.7
13.30	11.24	7.22	7.16	7.15	4.11
13.42	11.35	9.22	7.20		4.13
17.42	13.36	9.26	*l*, being not greater than 100 thousand permits no more but *these* 38 *rationes* of *y* to *z*.		
17.54		11.26			
19.42		11.34			
19.54		11.38			
23.54		13.34			
23.66		13.38			
23.78					
25.54					

Habitudes of *y* and *z* to *a.b.k.d.t.u.h.i.o.w.*

	is equal to	divided by					
a	$zz+2yz$	24	12	8	4	3	1
b	$12yy-2yz$	24	12	8	4	3	1
k	$12yy+2yz$	24	12	8	4	3	1
d	$zz-2yz$	24	12	8	4	3	1
t	$8yz+12yy+zz$	96	12	32	4	3	1
u	$8yz-12yy-zz$	96	12	32	4	3	1
h	$4yz-12yy+zz$	48	12	16	4	3	1
i	$4yz+12yy-zz$	48	12	16	4	3	1
o	$4yz$	24	12	8	4	3	1
w	.... $12yy+zz$	24	12	8	4	3	1
	In rank	I	II	III	IV	V	VI

The Habitudes of *y* & *z* to *e. f. g. l. m. n. p. q. r.*

	yyyy	yyyz	yyzz	yzzz	zzzz	divided by					
e =	+144	−48	+8	+4	+1	1152	144	128	16	9	1
f =	....	+48	+16	−4	.....	1152	144	128	16	9	1
g =	−144	+48	....	+4	+1	1152	144	128	16	9	1
l =	+144	+48	+8	−4	+1	1152	144	128	16	9	1
m =	.....	−48	+16	+4	.....	1152	144	128	16	9	1
n =	+144	+48	....	+4	−1	1152	144	128	16	9	1
p =	+144	.....	+88	.....	+1	2304	288	256	32	18	2
q =	−144	.....	+40	.....	−1	2304	288	256	32	18	2
r =	.....	+192	....	+16	.....	2304	288	256	32	18	2
f =	....	+12	+4	−1	.....	288	36	32	4	...	...
m =	....	−12	+4	+1	.....	288	36	32	4	...	...
r =	...	+12	.....	+1	.....	144	18	16	2	...	...
					In Rank	I	II	III	IV	V	VI

That is to say; When the *ratio* of *y* to *z* falls in the first rank.

then $a=\frac{zz+2yz}{24}$. $b=\frac{12yy-2yz}{24}$. $k=\frac{12yy+2yz}{24}$. $d=\frac{zz-2yz}{24}$

$t=\frac{8yz+12yy+zz}{96}$. $u=\frac{8yz-12yy-zz}{96}$. $h=\frac{4yz-12yy+zz}{48}$

$i=\frac{4yz+12yy-zz}{48}$. $o=\frac{4yz}{24}=\frac{1}{6}yz$. $w=\frac{12yy+zz}{24}$

$e=\frac{144yyyy-48yyyz+8yyzz+4yzzz+zzzz}{1152}$. and so for the rest.

These habitudes of $a.\ b.\ k.\ e.$ &c. to y and z in pag. 171 differ from those in page 142. 143. 144, because after those pages I resolved *to make them all arise in the smallest integer terms that would fit their* rationes. Which forced me to consider their greatest common Divisors from page 150 to this place.

What *new* ways of solving *this Problem* may be discovered by the help of these Habitudes, I leave to greater leisure. But I would not forget to tell you that they will be useful when *other conditions* are adjoyned to those of page 131. Whereby this 29th. Problem may produce *innumerable varieties of new* Problems; Some of them more limited than here, but some of them as unlimited as this.

As for Example; If it be a question supposing only a *circle inscribed* into the Equicrurals of this Problem, or *described about* them. The Number of Answers will be as many as before. And the *rationes* of the Numbers of pag. 166 and 167 will be the same.

The Diameters of those Circles will be to those lines $e.\ f.\ g$, &c. as some entire number is to some entire number, because the said lines are so to one another. For $ee \div 2g$ is the Radius of the circle *described about* the Equicrural $e.e.2f$; and so $ll \div 2n$, $pp \div 2r$ for the two other Triangles. But the *inscribed* circles, require but *two* different Semidiameters for the *three* Equicrurals. For each line of page 166 gives two Triangles of equal Perimeter and Area; therefore the Radius of the Circle inscribed into *one* of them [$e.\ e.\ 2f$] is equal to the Radius of the Circle inscribed into *the other* [$l.l.\ 2m$]. This Radius common to both is $\frac{fg}{e+f} = \frac{mn}{l+m} = \frac{qr}{p \cdot q}$. But the Circle in the third Triangle[$p.p.2q$]hath $\frac{qr}{p+q}$ for its Radius. And therefore is shorter than the former. For it is to it as $p-q$ to $p+q$.

In the lines *marked with* ✳, as $\frac{1}{2}\mathfrak{w}$ to $\mathfrak{b}$, so f to the *longer* Radius, or As $\frac{1}{2}\mathfrak{w}$ to $\mathfrak{t}$, so m to the *longer Radius*. But as $\mathfrak{o}$ to $\frac{1}{2}\mathfrak{w}$, so q to the *shorter* Radius. In the lines *not so marked*, As $\mathfrak{w}$ to $\mathfrak{b}$, so f to the *longer* Radius; or as $\mathfrak{w}$ to $\mathfrak{t}$, so m to the *longer* Radius; but as $4\mathfrak{o}$ to $\mathfrak{w}$, so $2q$ to the shorter Radius.

Wherefore, in the lines *marked with* ✳, the *longer* Radius will be to the *shorter* as $\mathfrak{oo}$ to the square of $\frac{1}{2}\mathfrak{w}$. But in the lines not *so marked* the longer Radius will be to the shorter as $4\mathfrak{oo}$ to $\mathfrak{ww}$; which is the *same ratio* : But I prescribe this distinction, that the *ratio* may emerge in its smallest terms.

For

For Example.

In the *first* line of pag. 166, 7. 3 :: 20. 60 ÷ 7. or 7. 5 :: 12. 60 ÷ 7 But 16, 7 :: 15. 105 ÷ 16. Here the longer Radius $\frac{60}{7}$ is to the shorter $\frac{105}{16}$ as 64 to 49. In the second line [*marked with* ✱] 13. 7 :: 120. $\frac{840}{13}$. or 13. 8 :: 105. $\frac{840}{13}$. But 15. 13 :: 56. $\frac{728}{15}$. Here the longer Radius $\frac{840}{13}$ is to the shorter $\frac{728}{15}$ as 225 to 169.

If now *this question* were proposed to you:

Find Eleven *commensurable right lines; whereof*
The first *may be the Radius of a Circle.*
The second *the Tangent of an arch of that circle.*
The third *the Tangent* } *of the doubled complement of that arch.*
The fourth *the Secant* }
The fift *the Tangent of* a third *arch of that Circle.*
The sixt *the Tangent* } *of the doubled complement of that* third *arch.*
The seventh *the Secant* }
The eight *a Radius of* another *Circle.*
The ninth *the Tangent of an arch of* this other *circle.*
The tenth *the Tangent* } *of the doubled complement of this* fift *arch.*
The eleventh *the Secant.* }
The tenth line *is equal to the third with twice the second.*
as also to the sixt with twice the fift.
Also these three Rectangles *are equal to one another:*
made of the second line upon the first and fourth.
of the fift upon the first and seventh.
of the ninth upon the eighth and eleventh.

It requires no great skill to discover that this Question is but our XXIX Problem disguised: and that the answers are innumerable. Wherefore, from the first line of Pag. 166, 167, you may borrow *e. f. g. l. m. n. p. q. r.* and from this page two Semidiameters $\frac{60}{7}$ and $\frac{105}{16}$. Then multiply these eleven numbers by 7 times 16, that is 112, to take away the fractions: You shall have integers

Integers fit for your purpose. So you may reply *thus*

One Answer to the Question of pag. 173.

The XI right lines.		The six Arches.		Proof.			
1	960						
2	2240	Grad.		1008	1392	2960	5537
3	1008	I	66.8014..	2240	960	960	735
4	1392	compl	23.1985...	2240	2352	3920	6272
5	1344	II	46.3971...	5488	2352=112*21		
6	2800	III	54.4623...	2800	2240=112*20		
7	2960	compl	35.5376...	1344	3920=112*35		
8	735	IV	71.0753...	1344	1344=112*12		
9	840	V	48.8140...	5488	6272=112*56		
10	5488	compl	41.1859		840=112*7⅓		
11	5537	VI	82.2718..	21*20=35*12=56*7⅓			

Page 166, and 166 will yeild 36 other Answers.

You have seen how from the last column of those two pages, all the numbers of the other columns are deduced. If instead of those 38 values of *y* and *z*, you take p and 3 out of this pages last column, you may *from them* also deduce the same numbers, to fill up those pages, 166, 167. Pag. 148 lin. 5. shews how they are found. And pag. 147 shews the Inversion that they will make, calling those numbers a. b. k. d. which in pag. 166, are called *k. d. a. b.* But *this page* shews you, that, as in deducing *a. b. k. d.* from *y z*, you had six Ranks of those values of *y z* [see pag. 155] because of their six common Divisors [see pag. 163.]: So in seeking *a. b. k. d.* by p and 3 you shall have the *same* Divisors, but not in the *same* lines. For wheresoever, working with *y. z* I did divide by 24. 8. 12. 4. 3. 1. there p. 3 will make you divide by 8. 24. 1. 3. 4. 12. which correspondence between those Divisors is here expressed in the first Columel under the Title ÷

÷	y. z	p. 3	÷
3	1. 3	1. 4	4
8	3.10	5.18	24
24	7.18	3.14	8
12	5.12	1. 5	1
4	3. 8	2. 9	3
8	5.14	7.30	24
24	11.30	5.22	8
1	2. 5	5.24	12
12	7.24	2. 7	1
8	7.22	11.42	24
24	13.30	5.26	8
3	4. 9	3.16	4
24	13.42	7.26	8
4	5 16	4.15	3
8	9.22	11 54	24
8	9 26	13.54	24
24	17.42	7.34	8
1	3. 7	7.36	12
24	17.54	9.34	8
12	11.24	2.11	1
24	19.42	7.38	8
8	11.26	13.66	24
24	19.54	9.38	8
12	11.36	3.11	1
8	11.34	17.66	24
4	7.16	4.21	3
8	11.38	19.66	24
4	7.20	5.21	3
12	13.36	3.13	1
8	13.34	17.78	24
24	23.54	9.46	8
3	7.15	5.28	4
8	13.38	19.78	24
24	23.66	11.46	8
2	4.11	11.48	12
24	25.54	9.50	8
24	23.78	13.46	8
1	4.13	13.48	12

Probl. XXX. *To solve the foregoing Problem after another manner.*

$e=?$	1	$ee=ff+gg$
$f=?$	2	$ll=mm+nn$
$g=?$	3	$e+f=l+m$
$l=?$	4	$fg=mn$
$m=?$	5	(*)
$n=?$	6	(*)

See Figure XVII.

Let every one of these *six* [*e. f. g. l. m. n.*] be to each of the other *five*, as some entire number is to some entire number.

(See page 132 of this Book.)

$a=?$	7	Let $e=aa+bb$
$b=?$	8	Let $f=aa-bb$
1, 7, 8	9	$\because g=2ab$
$k=?$	10	Let $l=kk+dd$
$d=?$	11	Let $m=kk-dd$
2, 10, 11	12	$\because n=2kd$
7+8	13	$e+f=2aa$
10+11	14	$l+m=2kk$
3, 13, 14	15	$2aa=2kk$
15 ÷ 2	16	$aa=kk$
16 ɯ 2	17	$a=k$
16 + dd	18	$kk+dd=aa+dd$
16 − dd	19	$kk-dd=aa-dd$
17 * 2d	20	$2kd=2ad$
10, 18	21	$l=aa+dd$
11, 19	22	$m=aa-dd$
12, 20	23	$n=2ad$
4 ÷ 2a	24	$fg\div 2a=mn\div 2a$
8 * 9 ÷ 2a	25	$fg\div 2a=aab-bbb$
22 * 23 ÷ 2a	26	$mn\div 2a=aad-ddd$
24, 25, 26	27	$aab-bbb=\cancel{ddd}-ddd$
Fig. XVII.	28	$l>e$ aad
28, 21, 7	29	$aa+dd>aa+bb$
29 − aa	30	$dd>bb$
30 ɯ 2	31	$d>b$

31. 27 + −	32	$ddd-bbb=aad-aab$
32 ÷ d − b	33	$aa=dd+db+bb$
	34	$db>0$
34 + dd &c.	35	$dd+2db+bb>dd+db+bb$
35, 33	36	$dd+2db+bb>aa$
36 ɯ 2	37	$d+b>a$
$c=?$	38	Let $d+b-c=a$
38 ⊙ 2	39	$aa=dd+2db+bb+cc-2dc-2bc$
39 − 33	40	$0=db-2dc-2bc+cc$
40 + −	41	$2bc-cc=db-2dc$
41 ÷	42	$d=\dfrac{2bc-cc}{b-2c}$
	43	$b=\dfrac{bb-2bc}{b-2c}$
	44	$c=\dfrac{bc-2cc}{b-2c}$
42, 43	45	$d+b=\dfrac{bb-cc}{b-2c}$
45 − 44	46	$d+b-c=\dfrac{bb-bc+cc}{b-2c}$
38, 46	47	$a=\dfrac{bb-bc+cc}{b-2c}$

86. *Now*

47	84	$\mathfrak{a} > \mathfrak{b}\mathfrak{b} - \mathfrak{b}\mathfrak{c} + \mathfrak{c}\mathfrak{c}$
43		$\mathfrak{b} \lesseqgtr \mathfrak{b}\mathfrak{b} - 2\mathfrak{b}\mathfrak{c}$
44		$\mathfrak{c} \leqq \mathfrak{b}\mathfrak{c} - 2\mathfrak{c}\mathfrak{c}$
42		$\mathfrak{d} \leq 2\mathfrak{b}\mathfrak{c} - \mathfrak{c}\mathfrak{c}$
48, z=?	49	Let $\mathfrak{a} = zz - zy + yy$
48, y=?	50	Let $\mathfrak{b} = zz - 2zy$
48, 49, 50	51	$\because\ \mathfrak{d} = 2zy - yy$
	52	$\mathfrak{b} > 0$
50, 52,	53	$zz - 2zy > 0$
53 + 2zy	54	$zz - 2zy$
54 ÷ $\overline{z}$	55	$z > 2y$
55 ÷ $\overline{2}$	55	$y < \frac{1}{2} z$
	57	When $\frac{z}{y} < = > 2 + \sqrt{3}$
		then $\mathfrak{b} < = > \mathfrak{d}$

57	58	Suppose $\mathfrak{d} = \mathfrak{b}$
58, 50, 51	59	$2zy - yy = zz - 2zy$
59 + —	60	$4zy - yy - zz = 0$
3zz — 60	61	$4zz - 4zy + yy = 3zz$
61 ω 2	62	$2z - y = +$ or $- \sqrt{3zz}$
52, 58	63	$\mathfrak{d} > 0$
63, 51	64	$2zy - yy > 0$
64 ÷ y	65	$2z - y > 0$
52, 65	66	$2z - y = + \sqrt{3zz}$ only
66 + —	67	$2z - \sqrt{3zz} = y$
67	68	$y\ z\ \mathrm{Z}^{2-\sqrt{3}}_{1}\ \mathrm{Z}^{1}_{2+\sqrt{3}}$
	69	$\sqrt{3} = 1.73205,08075$
$\overline{2}$ — 69	70	$2 - \sqrt{3} = 0.26794,91924\cdots$
$\overline{2}$ + 69	71	$2 + \sqrt{3} = 3.73205,08075\cdots$
68, 58	72	
	73	$\frac{z}{y} = \frac{2+\sqrt{3}}{1}$ when $\mathfrak{d} = \mathfrak{b}$
71, 72		$\frac{z}{y} = 3.73205,08075\cdots$ when $\mathfrak{d} = \mathfrak{b}$

57	74	ſuppoſe $\frac{z}{y} < 2 + \sqrt{3}$
74 * y	75	$z < 2y + \sqrt{3yy}$
75 — 2y	76	$z - 2y < \sqrt{3yy}$
76 ⊚ 2	77	$zz - 4zy + 4yy < 3yy$
77 — 3yy	78	$zz - 4zy + yy < 0$
78 + —	79	$zz - 2zy < 2zy - yy$
79, 50, 51	80	$\mathfrak{b} < \mathfrak{d}$
74, 80	81	when $\frac{z}{y} < 2 + \sqrt{3}$ then $\mathfrak{b} < \mathfrak{d}$

In like manner you may ſhew that when $\frac{z}{y}$ is $> 2 + \sqrt{3}$, then $\mathfrak{b} > \mathfrak{d}$, as was ſaid in the 57th of this page.

57, 68, 73	82	If y and z be entire numbers $\mathfrak{b}$ is not equal to $\mathfrak{d}$.
	83	To make $\mathfrak{b} < \mathfrak{d}$, take

$y = 1.\ 2.\ 3.\ 4.\ 5.\ 6.\ 7.\ 8.\ 9.\ 10.\ 11$
$z \ngtr 3.\ 7.\ 11.\ 14.\ 18.\ 22.\ 26.\ 29.\ 33.\ 37.\ 41$

	84	To make $\mathfrak{b} > \mathfrak{d}$, take

$y = 1.\ 2.\ 3.\ 4.\ 5.\ 6.\ 7.\ 8,$ &c.
$z \nless 4.\ 8.\ 12.\ 15.\ 19.\ 23.\ 27.\ 30,$ &c.

	85	Theſe Limits are taken out of this Tablet.

*	$2+\sqrt{3}$
1	3.73205 . . .
2	7.46410 . . .
3	11.19615 . . .
4	14.92820 . . .
5	18.66025 . . .
6	22.39230 . . .
7	26.12435 . . .
8	29.85640 . . .
9	33.58845 . . .
10	37.32050 ...
11	41.05255 ....
12	44.78460 ....
13	48.51666 ....

86. *Now* our *Scope* is this; To find all the integer values of y and z, which will permit $\mathfrak{b}$ to be greater than e, but less than $\mathfrak{d}$; and l to be less than 100000: Supposing y. z. $\mathfrak{a}$. $\mathfrak{b}$. $\mathfrak{d}$. e. f. g. l. m. n. to be the smallest Integers fit to express their *rationes* to one another: and their Habitudes to be as in pag. 175, 176. That is,

49	$\mathfrak{A}=zz+yy-zy$	7	$E=\mathfrak{aa}+\mathfrak{bb}$	21	$L=\mathfrak{aa}+\mathfrak{dd}$
50	$\mathfrak{B}=zz-2zy$	8	$F=\mathfrak{aa}-\mathfrak{bb}$	22	$M=\mathfrak{aa}-\mathfrak{dd}$
51	$\mathfrak{D}=2zy-yy$	9	$G=2\mathfrak{ab}$	23	$N=2\mathfrak{ad}$
55	$Z>2y$	75	$Z<2y+\sqrt{3yy}$		

87. The *ratio* of two *even* numbers is reducible to smaller terms by Bisection; therefore y, z cannot be *both even*. But either they are *both odd*; or *one* of them is *Odd* and the *other* Even.

88. If y and z be *both odd*, they will cause $\mathfrak{a}$. $\mathfrak{b}$. $\mathfrak{d}$. to be *Odd*: Wherefore $\mathfrak{aa}+\mathfrak{bb}$, $\mathfrak{aa}-\mathfrak{bb}$, $\mathfrak{aa}+\mathfrak{dd}$, $\mathfrak{aa}-\mathfrak{dd}$ will be *even* numbers, as well as $2\mathfrak{ab}$, $2\mathfrak{ad}$. That is to say, E. F. G. L. M. N. will be all *even* numbers: So that [e. f. g. l. m. n.] their halfs will be Integers.

89. y, even; z. odd will make e. f. even; but l. m. odd.

90. y. odd; z. even will make e. f. odd; but l. m. even. So that in neither of these two cases E. F. G. L. M. N. can be all divided by z: Nor indeed by any other common Divisor, if the *rationes* of $\mathfrak{a}$. $\mathfrak{b}$. $\mathfrak{d}$. be expressed in their smallest terms.

55	91	Let $z=2y+b \therefore b=z-2y$.	$b<\sqrt{3yy}$ (See 75)
21⊙2	92	$zz=4yy+4yb+bb$	
$91 * y$	93	$zy=2yy+by$	
$93 * 2$	94	$2zy=4yy+2yb$	
$92-93+yy$	95	$zz+yy-zy=3yy+3yb+bb$	
$92-94$	96	$zz-2zy=2yb+bb$	
$94-yy$	97	$2zy-yy=3yy+2yb$	
49, 95	98	$\mathfrak{A}=3yy+3yb+bb=3yy+9yx+9xx$	If $b=3x$.
50, 96	99	$\mathfrak{B}=2yb+bb=6yx+9xx$	
51, 97	100	$\mathfrak{D}=3yy+2yb=3yy+6yx\ldots.$	
$98\div 3$	101	$\mathfrak{A}\div 3=\mathfrak{a}=yy+3yx+3xx$	therefore
$99\div 3$	102	$\mathfrak{B}\div 3=\mathfrak{b}=2yx+3xx$	
$100\div 3$	103	$\mathfrak{D}\div 3=\mathfrak{d}=yy+2yx\ldots.$	

When $\frac{1}{3}b=x$ is an Integer, then $\mathfrak{A}$. $\mathfrak{B}$. $\mathfrak{D}$. may be divided by 3.

Hence arise *six different* sorts of these values of y and z.

1	y, even;	z odd;	*h* odd, no triple.	$\mathfrak{A} = {}^{a}$.	$L = l$	($h = z - 27$)
2	y, odd;	z even;	*h* even, no triple.	$\mathfrak{A} = {}^{a}$.	$L = l$	
3	y, even;	z odd;	*h* a tripled odd.	$\mathfrak{A} = 3^{a}$.	$L = l$	
4	y, odd;	z even;	*h* a tripled even.	$\mathfrak{A} = 3^{a}$.	$L = l$	
5	y, odd;	z odd;	*h* odd, no triple.	$\mathfrak{A} = {}^{a}$.	$L = 2l$	
6	y, odd;	z odd;	*h* a tripled odd.	$\mathfrak{A} = {}^{a}$.	$L = 2l$	

1 *Sort.*	2 *Sort.*	3 *Sort.*	4 *Sort.*	5 *Sort.*	6 *Sort.*
y. z. *h.*	y. z. *h*	y. z. *h.*	y. z. *h.*	y. z. *h.*	y. z. *h.*
2. 5. 1	3. 8. 2	2. 7. 3	5.16. 6	1. 3. 1	5.13. 3
4. 9. 1	3.10. 4	4.11. 3	7.20. 6	3. 7. 1	7.17. 3
4.13. 5	5.12. 2	8.19. 3	7.26. 12	3.11. 5	7.23. 9
6.13. 1	5.14. 4	8.25. 9	11.28. 6	5.11. 1	11.25. 3
6.17. 5	5.18. 8	10.23. 3		5.17. 7	11.31. 9
8.17. 1	7.16. 2	10.29. 9		7.15. 1	11.37. 15
				7.19. 5	13.29. 3
				9.19. 1	13.35. 9

These, and none but these values of y and z will fit *our Scope*. For *only these* will be left, when the Conditions of our *Scope* (pag. 177. lin. 1.) have caused al the *rest* to be *cast out* of these 6 Tablets.

1 *Sort.*

y	2	4	6	8	10
2y	4	8	12	16	20
2y+ 1	5	9	13	17	21
2y+ 5	9	13	17	21	25
2y+ 7	11	15	19	23	27

2 *Sort.*

y	3	5	7	9
2y	6	10	14	18
2y+ 2	8	12	16	20
2y+ 4	10	14	18	22
2y+ 8	14	18	22	26
2y+10	16	20	24	28

3 *Sort.*

y	2	4	8	10	14
2y	4	8	16	20	28
2y+ 3	7	11	19	23	31
2y+ 9	13	17	25	29	37
2y+15	19	23	31	35	43
2y+21	25	29	37	41	49

4 *Sort.*

y	5	7	11	13
2y	10	14	22	26
2y+ 6	16	20	28	32
2y+12	22	26	34	38
2y+18	28	32	40	44

5 *Sort.*

y	1	3	5	7	9	11
2y	2	6	10	14	18	22
2y+ 1	3	7	11	15	19	23
2y+ 5	7	11	15	19	23	27
2y+ 7	9	13	17	21	25	29

6 *Sort.*

y	5	7	11	13	17
2y	10	14	22	26	34
2y+ 3	13	17	25	29	37
2y+ 9	19	23	31	35	43
2y+15	25	29	37	41	49
2y+21	31	35	43	47	55

45+56=101. Of 101, but 38 are left ∵ 63 are cast away in the *third* Table.

y. z	☾
1 Sort.	
2. 5	
2. 9	2
2.11	2
4. 9	
4.13	
4.15	2
6.13	
6.17	
6.19	3
8.17	
8.21	3
8.23	3
10.21	3
10.25	1
10.27	3
2 Sort.	
3. 8	
3.10	
3.14	2
3.16	2
5.12	
5.14	
5.18	
5.20	1
7.16	
7.18	3
7.22	3
7.24	3
9.20	3
9.22	3
9.26	3
9.28	3
3 Sort.	
2. 7	
2.12	2
2.19	2
2.25	2
4.11	
4.17	2
4.23	2
4.29	2
8.19	
8.25	
8.31	2
8.37	2
10.23	
10.29	
10.35	1
10.41	2
14.31	3
14.37	3
14.43	3
14.49	3
4 Sort.	
5.16	
5.22	2
5.28	2
7.20	
7.26	
7.32	2
11.28	
11.34	3
11.40	3
13.32	3
13.38	3
13.44	3
5 Sort.	
1. 3	
1. 7	2
1. 9	2
3. 7	
3.11	
3.13	2
5.11	
5.15	1
5.17	
7.15	
7.19	
7.21	1
9.19	
9.23	3
9.25	3
11.23	3
11.27	3
11.29	3
6 Sort.	
5.13	
5.19	
5.25	
5.31	
7.17	
7.23	
7.29	2
7:35	1
11.25	
11.31	
11.37	
11.43	2
13.29	
13.35	
13.41	3
13.47	3
17.37	3
17.43	3
17.49	3
17.55	3

All the numbers of the *second* Table are repeated in *this third Table*. Some values of y and z have not 1, 2, or 3 set against them under ☾: you will find all those in the same order in the *first* Table, *pag*. 178. The *rest* are rejected for one or more of these *three causes*.

1 Some of them express not their *ratio* in the smallest terms, 10.25::2.5 5.20::1.4.

2 24 of the remaining values of z are greater than the limits prescribed § .83.*p*.176.

3 Many of them will make l greater than 100000 [A limit prescribed *p*.177.*l*.3.]

You have all in this *fourth* Table.

1 *Sort.*	2 *Sort.*	3 *Sort.*	4 *Sort.*	5 *Sort.*	6 *Sort.*
y. z.	y. z	y. z	y. z	y. z	y. z
6.19	7.18	14.31	13.32	9.23	17.37
8.21	9.20	14.37	11.34	11.23	13.41
10.21	7.22	14.43	13.38	9.25	17.43
8.23	9.22		11.40	11.27	13.47
80.27	7.24		13.44	11.29	17.49
	9.26				17.55
	9.28				

In some of these the Order is not the same that they had in the *third* Table. For here in every sort the values of z are set in their natural order increasing downwards. And when one value of z is *twice* written; the lesser y is matched with the uppermost. The *same* order is observed also in *pag*. 180; that so both here and there any pair of values of y and z may make l greater than that l which belongs to the pair over them, if it be of the *same* sort. This *adjoyned Tablet* shews the value of l, which belongs to the *Leader* or upmost of each sort in the *fourth* Table. Their *Followers* will make l much greater than these.

y. z	l
6.19	116953
7.18	102218
14.31	108257
13.32	115922
9.23	136649
17.37	110989

	y. z	zz	yy	zy	2zy	zz+yy	𝔄	𝔅	𝔇	a	b	d	b+d
First Sort.	2. 5	25	4	10	20	29	19	5	16	19	5	16	21
	4. 9	81	15	36	72	97	61	9	56	61	9	56	65
	4.13	169	16	52	104	185	133	65	88	133	65	88	153
	6.13	169	36	78	156	205	127	13	120	127	13	120	133
	6.17	289	36	102	204	325	223	85	168	223	85	168	253
	8.17	289	64	136	272	353	217	17	208	217	17	208	225
Second Sort.	3. 8	64	9	24	48	73	49	16	39	49	16	39	55
	3.10	100	9	30	60	109	79	40	51	79	40	51	91
	5.12	144	25	60	120	169	109	24	95	109	24	95	119
	5.14	196	25	70	140	221	151	56	115	151	56	115	171
	7.16	256	49	112	224	305	193	32	175	193	32	175	207
	5.18	324	25	90	180	349	259	144	155	259	144	155	299
Third Sort.	2. 7	49	4	14	28	53	39	21	24	13	7	8	15
	4.11	121	16	44	88	137	93	33	72	31	11	24	35
	8.19	361	64	152	304	425	273	57	240	91	19	80	99
	10.23	529	100	230	460	629	399	69	360	133	23	120	143
	8.25	625	64	200	400	689	489	225	336	163	75	112	187
	10.29	841	100	290	580	941	651	261	480	217	87	160	247
Fourth Sort.	5.16	256	25	80	160	281	201	96	135	67	32	45	77
	7.20	400	49	140	280	449	309	120	231	103	40	77	117
	7.26	676	49	182	364	725	543	312	315	181	104	105	209
	11.28	784	121	308	616	905	597	168	495	199	56	165	221
Fifth Sort.	1. 3	9	1	3	6	10	7	3	5	7	3	5	8
	3. 7	49	5	21	42	58	37	7	33	37	7	33	40
	3.11	121	9	33	66	130	97	55	57	97	55	57	112
	5.11	121	25	55	110	146	91	11	85	91	11	85	96
	7.15	225	49	105	210	274	169	15	161	159	15	161	176
	5.17	289	25	85	170	314	229	119	145	229	119	145	264
	7.19	361	49	133	266	410	277	95	217	277	95	217	312
	9.19	361	81	171	342	442	271	19	261	271	19	261	280
Sixth Sort.	5.13	169	25	65	130	194	129	39	105	43	13	35	48
	7.17	289	49	119	238	338	219	51	189	73	17	63	80
	7.23	529	49	161	322	578	417	207	273	139	69	91	160
	11.25	625	121	275	550	746	471	75	429	157	25	143	168
	13.29	841	169	377	754	1010	633	87	585	211	29	195	224
	11.31	961	121	341	682	1082	741	279	561	247	93	187	280
	13.35	1225	169	455	910	1394	939	315	741	313	105	247	352
	11.37	1369	121	407	814	1490	1083	555	693	361	185	231	416

In pag. 180, for the first columel, see pag. 178. lin. 10, &c. and pag. 179. lin. 29. The five next Columels have Titles taken from pag. 177. Equat. 49, 50, 51, to the end that Subtraction might fill the columels A. B. D. Whole numbers without any alteration are repeated in the columels a. b. d. in the *first*, *second and fifth sort*, because they have no common Divisor to reduce them to smaller. But in the *third*, *fourth*, and *sixth sort*, $\frac{z - 2y}{3}$, or $\frac{1}{3}h$ is an Integer, [1, 2, 3, 4, or 5] and a. b. d. are found by dividing A. B. D. by 3.

Another value of b or d belonging to every one of those pairs of y and z.

Pag. 175	33	$aa = dd + db + bb$
$33 * 4$	104	$4dd + 4db + 4bb = 4aa$
$104 - 3bb$	105	$4dd + 4db + 4bb = 4aa - 3bb$
105 ω 2	106	$2d + b = +$ or $- \sqrt{4aa - 3bb}$
$106 - b$	107	$2d = + \sqrt{4aa - 3bb} - b$
$106 - b$	108	$2d = - \sqrt{4aa - 3bb} - b$

So that d hath a *Negative value.*

$t = ?$	109	Let this *value* be called $-t$
108, 109	110	$2t = + \sqrt{4aa - 3bb} + b$
$107 + 2b$	111	$2d + 2b = + \sqrt{4aa - 3bb} + b$
110, 111	112	$2t = 2d + 2b$
$112 \div 2$	113	$t = d + b$
$50 + 51$	114	$d + b = zz - yy$
113, 114	115	$t = zz - yy$
Pag. 176.	55	$z > 2y$
$55 * y$	116	$zy > 2yy$
$116 + zz$	117	$zz + zy > zz + 2yy$
117 — —	118	$zz - yy > zz + yy - zy$
118, 115, 49	119	$t > a$
119 ② 2	120	$tt > aa$
120, $q = ?$	121	Let $tt - aa = q$.

$p = ?$	122	Let $tt + aa = p$.
$r = ?$	123	Let $pp = qq + rr$
121, 122, 123	124	$2ta = r$.
$122 - 121$	125	$p - q = 2aa$
$121 * 124$	126	$qr = 2ttta - 2taaa$
125, 13, 14	127	$p - q = e + f = l + m$
126,	128	$qr = fg = mn$

So that we have now *another* rectangled Triangle [*p. q. r.*] half a *third Equicrura* [*p. p. 2q.*] belonging to this Problem.

The 128*th*. may be proved thus;

$113 - d$	129	$t - d = b$
$129 * 113$	130	$tt - td = db + bb$
$130 + dd$	131	$tt - td + dd = dd + db + bb$
131, 33	132	$tt - td + dd = aa$
$132 * t + d$	133	$ttt + ddd = taa + daa$
133 — —	134	$ttt - taa = daa - ddd$
$129 \div 2a$	135	$ttt - taa = qr \div 2a$
135, 26	136	$qr \div 2a = mn \div 2a$
$136 * 2a$	127	$qr = mn$ as in the 128*th*.

113. The last Columel of Pag. 180 hath all the values of t. Thence being transferred into pag. 184, they help the finding of *p. q. r.* As *e. f. g.* are deduced out of a and b in pag. 182; and *l. m. n.* out of a and d in pag. 183.

In Pag. 182. 183. 184. the numbers a. b. d. t. may keep the order of page 180. So the 16 lowest lines will have $g, n, r, = ab, ad, ta$: When the *rest* have $g, n, r = 2ab, 2ad, 2ta$.

a	b	aa	bb	ab	aa+bb = e	aa-bb = f	2ab = g
19	5	361	25	95	386	336	190
61	9	3721	81	549	3802	3640	1098
133	65	17689	4225	8645	21914	13464	17290
127	13	16129	169	1651	16298	15960	3302
223	85	49729	7225	18955	56954	42504	37910
217	17	47089	289	3689	47378	46800	7378
49	16	2401	256	784	2657	2145	1568
79	40	6241	1600	3160	7841	4641	6320
109	24	11881	576	2616	12457	11305	5232
151	56	22801	3136	8456	25937	19665	16912
193	32	37249	1024	6176	38273	36225	12352
259	144	67081	20736	37296	87817	46345	74592
13	7	169	49	91	218	120	182
31	11	961	121	341	1082	840	682
91	19	8281	361	1729	8642	7920	3458
133	23	17689	529	3059	18218	17160	6118
163	75	26569	5625	12225	32194	20944	24450
217	87	47089	7569	18879	54658	39520	37758
67	32	4489	1024	2144	5513	3465	4288
103	40	10609	1600	4120	12209	9009	8240
181	104	32761	10816	18824	43577	21945	37648
199	56	39601	3136	11144	42737	36465	22288

a	b	aa	bb	aa+bb = E	aa-bb = F	e	f	ab = g
7	3	49	9	58	40	29	20	21
37	7	1369	49	1418	1320	709	660	259
97	55	9409	3025	12434	6384	6217	3192	5335
91	11	8281	121	8402	8160	4201	4080	1001
169	15	28561	225	28786	28336	14393	14168	2535
229	119	52441	14161	66602	38280	33301	19140	27251
277	95	76729	9025	85754	67704	42877	33852	26315
271	19	73441	361	73802	73080	36901	36540	5149
43	13	1849	169	2018	1680	1009	840	559
73	17	5329	289	5618	5040	2809	2540	1241
139	69	19321	4761	24082	14560	12041	7280	9591
157	25	24649	625	25274	24024	12637	12012	3925
211	29	44521	841	45362	43680	22681	21840	6119
247	93	61009	8649	69658	52360	34829	26180	22971
313	105	97969	11025	108994	86944	54497	43472	32865
361	185	130321	34225	164546	96096	82273	48048	66785

a	d	aa	dd	ad	aa+dd=L	aa-dd=m	2ad=n
19	16	361	256	304	617	105	608
61	56	3721	3136	3416	6857	585	6832
133	88	17689	7744	11704	25433	9945	23408
127	120	16129	14400	15240	30529	1729	30480
223	168	49729	28224	37464	77953	21505	74928
217	208	47089	43264	45136	90353	3825	90272
49	39	2401	1521	1911	3922	880	3822
79	51	6241	2601	4029	8842	3640	8058
109	95	11881	9025	10355	20906	2856	20710
151	115	22801	13225	17365	36026	9576	34730
193	175	37249	30625	33775	67874	6624	67550
259	155	67081	34025	40145	91106	43056	80290
13	8	169	64	104	233	105	208
31	24	961	576	744	1537	385	1488
91	80	8281	6400	7280	14681	1881	14560
133	120	17689	14400	15950	32089	3289	31920
163	112	26569	12544	18256	39113	14025	36512
217	160	47089	25600	34720	72689	21489	69440
67	45	4489	2025	3015	6514	2464	6030
103	77	10609	5929	7931	16538	4680	15862
181	105	32761	11025	9005	437[illegible]6	21736	38010
199	165	39601	27225	32835	66826	12376	65670

a	d	aa	dd	aa+dd=L	aa-dd=M	l	m	ad=n
7	5	49	25	74	24	73	12	35
37	33	1369	1089	2458	280	1229	140	1221
97	57	9409	3249	12658	6160	6329	3080	5529
91	85	8281	7225	15506	1056	7753	528	7735
169	161	28561	25921	54482	2640	7241	1320	27209
229	145	52441	21025	73466	31416	36733	15708	33205
277	217	79729	47089	123818	29640	61909	14820	60109
271	261	73441	68121	141562	5320	70781	2660	70731
43	35	1849	1225	3074	624	1537	312	1505
73	63	5329	3969	9298	1360	4649	680	4599
139	91	19321	8281	27602	11040	13801	5520	12649
157	143	24649	20449	45098	4200	22549	2100	22451
211	195	44521	38025	82545	6496	41273	3248	41145
247	187	61009	34969	95978	26040	47989	13020	46189
313	247	97969	61009	158978	36960	79489	18480	77311
361	231	130321	53361	183682	76960	91841	38480	83391

t	a	tt	aa	ta	tt+aa=p	tt-aa=q	2a=r
21	19	441	361	399	802	80	798
65	61	4225	3721	3965	7946	504	7930
153	133	23409	17689	20349	41098	5720	40698
133	127	17689	16129	16891	33818	1560	33782
253	223	64009	49729	56419	113738	14280	112838
225	217	50625	47089	48825	97714	3536	97650
55	49	3025	2401	2595	5426	624	5390
91	79	8281	6241	7189	14522	2024	14378
119	109	14161	11881	12971	26042	2280	25942
171	151	29241	22801	25821	52042	6440	51642
207	193	42849	37249	39951	80098	5600	79902
299	259	89401	67081	77441	156482	22320	154882
15	13	225	169	195	394	56	390
3	31	1225	961	1085	2186	264	2170
99	91	9801	8281	9009	18082	1520	18018
143	133	20449	17689	19019	38138	2760	38038
187	163	34969	26569	30481	61538	8400	60962
247	217	61009	47089	53599	108098	13920	107198
77	67	5929	4489	5159	10418	1440	10318
117	103	13689	10609	12051	24298	3080	24102
209	181	43681	32761	37829	76442	10920	75658
221	199	48841	39601	43979	88442	9240	87958

t	a	tt	aa	tt+aa=P	tt-aa=Q	p	q	ta=r
8	7	64	49	113	15	56.5	7.5	56
40	37	1600	1369	2969	231	1484.5	115.5	1480
112	97	12544	9409	21953	3135	10976.5	1567.5	10864
96	91	9216	8281	17497	935	8748.5	467.5	8736
176	169	30976	28561	59537	2415	29768.5	1207.5	29744
264	229	69696	52441	122137	17255	61068.5	8627.5	60456
312	277	97344	76729	174073	20615	87036.5	10307.5	86424
280	271	78400	73441	151841	4959	75920.5	2479.5	75880
48	43	2304	1849	4153	455	2076.5	227.5	2064
80	73	6400	5329	11729	1071	5864.5	535.5	5840
160	139	25600	19321	44921	6279	22460.5	3139.5	22240
168	157	28224	24649	52873	3575	26436.5	1787.5	26376
224	211	50176	44521	94697	5655	47348.5	2827.5	47264
280	247	78400	61009	139409	17391	69704.5	8695.5	69160
352	313	123904	97969	221873	25935	110936.5	12967.5	110176
116	361	173056	130321	303377	42735	151688.5	21367.5	150176

	y. z	+	a	b	d	t
1	1. 3	1	7	3	5	8
2	2. 7	3	13	7	8	15
3	2. 5	1	19	5	16	21
4	3. 7	1	37	7	33	40
5	5.13	3	43	13	35	48
6	4.11	3	31	11	24	35
7	3. 8	1	49	16	39	55
8	7.17	3	73	17	63	80
9	3.11	1	97	55	57	112
10	5.16	3	67	32	45	77
11	4. 9	1	61	9	56	65
12	5.11	1	91	11	85	96
13	3.10	1	79	40	51	91
14	7.23	3	139	69	91	160
15	8.19	3	91	19	80	99
16	7.20	3	103	40	77	117
17	5.12	1	109	24	95	119
18	11. 5	3	157	25	143	168
19	4.13	1	133	65	88	153
20	7.15	1	169	15	161	176
21	6.13	1	127	13	120	133
22	10.23	3	133	23	120	143
23	5.14	1	151	56	115	171
24	5.17	1	229	119	145	264
25	8.25	3	163	75	112	187
26	13.29	3	211	29	195	224
27	7.26	3	181	104	105	209
28	11.31	3	247	93	187	280
29	7.19	1	277	95	217	312
30	11.28	3	199	56	165	221
31	7.16	1	193	32	175	207
32	9.19	1	271	19	261	280
33	10.29	3	217	87	160	247
34	6.17	1	223	85	168	253
35	13.35	3	313	105	247	352
36	8.17	1	217	17	208	225
37	5.18	1	259	144	155	299
38	11.37	3	361	185	231	416

If you make nine columels for *e. f g: l. m. n. p. q. r.* And if you fill them all out of pag. 182, 183, 184, in this manner; first fill the Columel of *l* with its 38 numbers in their natural order, and let the numbers of the other eight (for *e. f.* &c.) stand in such order as those of *l* will permit; you shall make a Table not at all differing in those nine columels, from pag. 166, 167.

But those two pages have also columels marked *a,b:k,d:t,n:y,z*. In stead of which, *this way*, having no *k* or *n*, requires but such a Table as *this* here adjoined: whose numbers are taken out of pag. 182,183,184,180.

And so *I have solved the* 29th. *Problem after another manner*, as was required pag. 175.

Another way to find these 38 Answers to this 30th. Problem.

Having y z, you may find **Y Z**: that is, having any *ratio* of y to z which **b** to be less than **d**, (as in pag. 180) you may find its *inverted correspondent*, that is, Another *ratio* of y to z, which working by the same Equations, may give the same **a** and **t**, but may invert the values of **b** and **d**, calling that **b** which the *former ratio* had called **d**, and that **d** which the former had called **b**.

Thus y.z	A. B. D. C.	÷	a. b. d. t
1.3	7. 3. 5.	1	7. 3. 5. 8
1.5	21.15. 9.24	3	7. 5. 3. 8
2.7	36.21.24.45	3	13. 7. 8.15
2.4	13. 8. 7.15	1	13. 8. 7.15

That is to say, these equalities are seen in them *after* they have been divided by their greatest common divisor. *Before* such division, it is but equality of *rationes*, thus; 3.5::9.15. 21.24::7.8 And therefore according to Equa 50.51.pag.177

As zz—2zy, to 2zy—yy : So $2\mathfrak{Z}\mathfrak{Y}-\mathfrak{Y}\mathfrak{Y}$, to $\mathfrak{Z}\mathfrak{Z}-2\mathfrak{Z}\mathfrak{Y}$
which may be resolved into these two Analogisms

As z to y ::So $2\mathfrak{Z}-\mathfrak{Y}$, to $\mathfrak{Z}-2\mathfrak{Y}$	Or z . $2\mathfrak{Z}-\mathfrak{Y}$ ::	y . $\mathfrak{Z}-2\mathfrak{Y}$
As z—2y, to 2z—y::So $\mathfrak{Y}$, to $\mathfrak{Z}$	z—2y . $\mathfrak{Y}$	::2z—y . $\mathfrak{Z}$

Wherefore, if I make $\mathfrak{Y}=z-2y$, I must make $\mathfrak{Z}=2z-y$
and, if I make $y=\mathfrak{Z}-2\mathfrak{Y}$, I must make $z=2\mathfrak{Z}-\mathfrak{Y}$
So the whole process of seeking $\mathfrak{Y}$ and $\mathfrak{Z}$, by the help of y and z, is this,

From y z of the first, second, and fifth sort, pag. 180

y	2. 4. 4. 6. 6. 8	3. 3. 5. 5. 7. 5	1. 3. 3. 5. 7. 5. 7. 9
z	5. 9.13.13.17.17	8.10.12.14.16.18	3. 7.11.11.15.17.19.19
2y	4. 8. 8.12.12.16	6. 6.10.10.14.10	2. 6. 6.10.14.10.14.18
2z	10.18.26.26.34.34	16.20.24.28.32.36	6.14.22.22.30.34.38.38
$z-2y=\mathfrak{Y}$	1. 1. 5. 1. 5. 1	2. 4. 2. 4. 2. 8	1. 1. 5. 1. 1. 7. 5. 1
$2z-y=\mathfrak{Z}$	8.14.22.20.28.26	13.17.19.23.25.31	5.11.19.17.23.29.31.29

From y,z of the 3d. 4th. and 6th. sort, pag. 180. $z-2y=3\mathfrak{Y}$. $2z-y=3\mathfrak{Z}$

y	2. 4. 8.10. 8.10	5. 7. 7.11	5. 7. 7. 11.13.11.13.11
z	7.11.19.23.25.29	16.20.26.28	13.17.23.25.29.31.35.37
2y	4. 8.16.20.16.20	10.14.14.22	10.14.14.22.26.22.26.22
2z	14.22.38.46.50.58	32.40.52.56	26.34.46.50.58.62.70.74
z—2y	3. 3. 3. 3. 9. 9	6. 6.12. 6	3. 3. 9. 3. 3. 9. 9.15
2z—y	12.18.30.36.42.48	27.33.45.45	21.27.39.39.45.51.57.63
$\mathfrak{Y}$	1. 1. 1. 1. 3. 3	2. 2. 4. 2	1. 1. 3. 1. 1. 3. 3. 5
$\mathfrak{Z}$	4. 6.10.12.14.16	9.11.15.15	7. 9.13.13.15.17.19.21

$\mathfrak{a}\cdot\mathfrak{b}.\mathfrak{d}.\mathfrak{t}=\mathfrak{b}+\mathfrak{d}$, derived from $\mathfrak{Y},\mathfrak{Z}$. In each sort two Examples.

Sort	y z	zz	y y	z y	2zy	zz+yy	$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{D}$	÷	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{d}$	$\mathfrak{b}+\mathfrak{d}$	Sort
1	1. 8	64	1	8	16	65	57	48	15	3	19	16	5	21	4
	1.14	196	1	14	28	197	182	168	27	3	61	56	9	65	
2	2.13	9	4	20	52	173	147	117	48	3	49	39	16	55	3
	4.17	289	16	68	136	305	237	153	120	3	79	51	40	91	
5	1. 5	25	1	5	10	26	21	15	9	3	7	5	3	8	6
	1.11	121	1	11	22	122	111	99	21	3	37	33	7	40	
3	1. 4	16	1	4	8	17	13	8	7	1	13	8	7	15	2
	1. 6	36	1	6	12	37	31	24	11	1	31	24	11	35	
4	2. 9	81	4	18	36	85	67	45	32	1	67	45	32	77	1
	2.11	121	4	22	44	125	103	77	40	1	103	77	40	177	
6	1. 7	49	1	7	14	50	43	35	13	1	43	35	13	48	5
	1. 9	81	1	9	18	82	73	63	17	71	73	63	17	80	

	y. z.	÷	sort	$\mathfrak{Y}.\mathfrak{Z}$	÷	sort
1	1. 3	1	5	1. 5	3	6
2	2. 7	3	3	1. 4	1	2
3	2. 5	1	1	1. 8	3	4
4	3. 7	1	5	1.11	3	6
5	5.13	3	6	1. 7	1	5
6	4.11	3	3	1. 6	1	2
7	3. 8	1	2	2.13	3	3
8	7.17	3	6	1. 9	1	5
9	3.11	1	5	5.19	3	6
10	5.16	3	4	2. 9	1	1
11	4. 9	1	1	1.14	3	4
12	5.11	1	5	1.17	3	6
13	3.10	1	2	4.17	3	3
14	7.23	3	6	3.13	1	5
15	8.19	3	3	1.10	1	2
16	7.20	3	4	2.11	1	1
17	5.12	1	2	2.19	3	3
18	11.25	3	6	1.13	1	5
19	4.13	1	1	5.22	3	4
20	7.15	1	5	1.23	3	6
21	6.13	1	1	1.20	3	4
22	10.23	3	3	1.12	1	2
23	5.14	1	2	4.23	3	3
24	5.17	1	5	7.29	3	6
25	8.25	3	3	3.14	1	2
26	13.29	3	6	1.15	1	5
27	7.26	3	4	4.15	1	1
28	11.31	3	6	3.17	1	5
29	7.19	1	5	5.31	3	6
30	11.28	3	4	2.15	1	1
31	7.16	1	2	2.25	3	3
32	9.19	1	5	1.29	3	6
33	10.29	3	3	3.16	1	2
34	6.17	1	1	5.28	3	4
35	13.35	3	6	3.19	1	5
36	8.17	1	1	1.26	3	4
37	5.18	1	2	8.31	3	3
38	11.37	3	6	5.21	1	5

As the Table in pag. 174 did compare the 38 values of y z with as many of $\mathfrak{y}\,\mathfrak{z}$: So this adjoined Table compares as many of y z with their inverting correspondents $\mathfrak{Y}\,\mathfrak{Z}$ found pag. 186.

The two columels marked ÷ have no numbers but 1 or 3: and those interchangeably; Where the former columel hath 1, the latter hath 3; and where *that* hath 3, *this* hath 1. Their use is to shew what is the greatest common Divisor of $\mathfrak{A}.\mathfrak{B}.\mathfrak{D}.\mathfrak{C}$ to make $\mathfrak{a}, \mathfrak{b}, \mathfrak{d}, \mathfrak{c}$ as small integers as may be. The other two columels shew you to which of the *six sorts* (pag. 178) the adjoyned *ratio* doth belong.

In these you see a perpetual correspondence of those sorts, so that when the *ratio* y z is of the 1.2.3.4.5.6 sort, then the *ratio* $\mathfrak{Y}.\mathfrak{Z}$ is of the 4.3.2.1.6.5 sort.

a.b.k.d.t.u (of pag. 166.) and $\mathfrak{a}.\mathfrak{b}.\mathfrak{d}.\mathfrak{c}$. (of pag. 185) compared together.

In the 22 lines marked with ✱ pag. 166, 167

$a+b=2\mathfrak{a}$	$k+d=2\mathfrak{a}$	$t+u=\mathfrak{c}$
$a-b=2\mathfrak{b}$	$k-d=2\mathfrak{d}$	$t-u=\mathfrak{a}$
$\mathfrak{a}+\mathfrak{b}=a$	$\mathfrak{a}+\mathfrak{d}=k$	$\mathfrak{c}+\mathfrak{a}=2t$
$\mathfrak{a}-\mathfrak{b}=b$	$\mathfrak{a}-\mathfrak{d}=d$	$\mathfrak{c}-\mathfrak{a}=2u$
As $20+6=2*13$	$21+5=2*13$	$14+1=15$
$20-6=2*7$	$21-5=2*8$	$14-1=13$
$13+7=20$	$13+8=21$	$15+13=2*14$
$13-7=6$	$13-8=5$	$15-13=2*1$

In the 16 lines not so marked, pag. 166, 167

$a+b=\mathfrak{a}$	$k+d=\mathfrak{a}$	$t+u=2t$
$a-b=\mathfrak{b}$	$k-d=\mathfrak{d}$	$t-u=2a$
$\mathfrak{a}+\mathfrak{b}=2a$	$\mathfrak{a}+\mathfrak{d}=2k$	$t+\mathfrak{a}=t$
$\mathfrak{a}-\mathfrak{b}=2b$	$\mathfrak{a}-\mathfrak{d}=2d$	$t-\mathfrak{a}=u$
As $5+2=7$	$6+1=7$	$15+1=2*8$
$5-2=3$	$6-1=5$	$15-1=2*7$
$7+3=2*5$	$7+5=2*6$	$8+7=15$
$7-3=2*2$	$7-5=2*1$	$8-7=1$

The numbers $a.b.k.d.t.u.$ $\mathfrak{a}.\mathfrak{b}.\mathfrak{d}.\mathfrak{t}.$ compared with the Numbers $e.f.g.l.m.n.p.q.r.$

Pag. 142. 175 e — $aa+bb$ — $\mathfrak{aa}+\mathfrak{bb}$ f — $2ab$ — $\mathfrak{aa}-\mathfrak{bb}$ g — $aa-bb$ — $2\mathfrak{ab}$	∵ As $e+g$ is to f	so $2aa$ to $2ab$	so a to b	And so $\mathfrak{aa}+2\mathfrak{ab}+\mathfrak{bb}$ is to $\mathfrak{aa}-\mathfrak{bb}$	so $\mathfrak{a}+\mathfrak{b}$ to $\mathfrak{a}-\mathfrak{b}$
	As $e+f$ is to g	so $aa+2ab+bb$ is to $aa-bb$	so $a+b$ to $a-b$	And so $2\mathfrak{aa}$ to $\mathfrak{ab}$	so $\mathfrak{a}$ to $\mathfrak{b}$
Pag. 142. 175 l — $kk+dd$ — $\mathfrak{aa}+\mathfrak{dd}$ m — $2kd$ — $\mathfrak{aa}-\mathfrak{dd}$ n — $kk-dd$ — $2\mathfrak{ad}$	∵ As $l+n$ is to m	so $2kk$ to $2kd$	so k to d	And so $\mathfrak{aa}+2\mathfrak{ad}+\mathfrak{dd}$ is to $\mathfrak{aa}-\mathfrak{dd}$	so $\mathfrak{a}+\mathfrak{d}$ to $\mathfrak{a}-\mathfrak{d}$
	As $l+m$ is to n	so $kk+2kd+dd$ is to $kk-dd$	so $k+d$ to $k-d$	And so $2\mathfrak{aa}$ to $2\mathfrak{ad}$	so $\mathfrak{a}$ to $\mathfrak{d}$
Pag. 144. 181. p — $tt+uu$ — $\mathfrak{tt}+\mathfrak{aa}$ q — $2tu$ — $\mathfrak{tt}-\mathfrak{aa}$ r — $tt-uu$ — $2\mathfrak{ta}$	∵ As $p+r$ is to q	so $2tt$ to $2tu$	so t to u	And so $\mathfrak{tt}+2\mathfrak{ta}+\mathfrak{aa}$ is to $\mathfrak{tt}-\mathfrak{aa}$	so $\mathfrak{t}+\mathfrak{a}$ to $\mathfrak{t}-\mathfrak{a}$
	As $p+q$ is to r	so $tt+2tu+uu$ is to $tt-uu$	so $t+u$ to $t-u$	And so $2\mathfrak{tt}$ to $2\mathfrak{ta}$	so $\mathfrak{t}$ to $\mathfrak{a}$

And therefore	$\mathfrak{a}+\mathfrak{b}.\mathfrak{a}-\mathfrak{b}::a.b$	$\mathfrak{a}+\mathfrak{d}.\mathfrak{a}-\mathfrak{d}::k.d$	$\mathfrak{t}+\mathfrak{a}.\mathfrak{t}-\mathfrak{a}::t.u$
	$a+b.a-b::\mathfrak{a}.\mathfrak{b}$	$k+d.k-d::\mathfrak{a}.\mathfrak{d}$	$t+u.t-u::\mathfrak{t}.\mathfrak{a}$

Also $a.b::e+g.f$
$\mathfrak{a}.\mathfrak{b}::e+f.g$
$k.d::l+n.m$
$\mathfrak{a}.\mathfrak{d}::l+m.n$
$t.u::p+r.q$
$\mathfrak{t}.\mathfrak{a}::p+q.r$

But e is the Hypotenuse of a rectangled triangle (see fig. XVII.) If its base f be produced beyond the Vertex of that acute angle, till it become equal to $e+f$, and then a new Hypotenuse be drawn; I say *the new triangle* whose base is $e+f$ and perpendicular is g, shall be like the triangle whose base is $\mathfrak{a}$, and perpendicular is $\mathfrak{b}$. In *this new triangle*, the acute at the base, is the half of the acute at the base f of the Triangle $e.f.g$. There is the same reason for the other acute of the same triangle $e.f.g$; as also for the two other Triangles of Figure XVII, that is $l.m.n$ and $p.q.r$

For *example*, take the first line of pag. 166, 167. In which the *acutes* are *these*,

	Sides		Opposite acutes	Sides		Opposite acutes	Sides		Opposite acutes
(1)	f	40	43.602818...	m	24	18.924644...	q	15	7.628149....
(2)	g	42	46.397181...	n	70	71.075355..	r	112	82.371850....
(3)	a	5	68.198590...	k	6	80.537677...	t	15	86.185925....
(4)	b	2	21.801409...	d	1	9.462322...	u	1	3.814074....
(5)	$\mathfrak{a}$	7	66.801409...	$\mathfrak{a}$	7	54.462322..	$\mathfrak{t}$	8	48.814074....
(6)	$\mathfrak{b}$	3	23.198590...	$\mathfrak{d}$	5	35.537677..	$\mathfrak{a}$	7	41.185925....

In each of these, 90 Grades $=(1)+(2)=(3)+(4)=(5)+(6)$
45 Gr. $=(4)+(6)=(3)-(6)=(5)-(4)$. The 4th. is half the first, The (5) = half (2). So that it is easie, having *any one* of these six acutes, to find the other five.

For the agreement between these angles, and those of pag. 174. See Euclid III.20. and IV.4.

Probl.

Probl. XXXI. To find *two* unlike Equicrural Triangles, equal to one another in Perimeter and in Area.

$e=?$	1	$ee=ff+gg$	Let theſe two Triangles be e, e, $2f$ and l, l, $2m$
$f=?$	2	$ll=mm+nn$	therefore, their Perimeters are $2e+2f=2l+2m$
$g=?$	3	$e+f=l+m$	Let their Altitudes be g, n ∵ their Area's are $fg=mn$
$l=?$	4	$fg=mn$	(See Figure XVII.)
$m=?$	5	(*)	Here the commenſurability of the ſides and perpendiculars
$n=?$	6	(*)	is not deſired, as in Problem 29 and 30 pag. 131 and 175

Let	7	$e+f=S$
3,7	8	$l+m=S$
4 ⊙ 2	9	$ffgg=mmnn$
2 — mm	10	$ll-mm=nn$
8 — m	11	$l=S-m$
11 ⊙ 2	12	$ll=SS-2Sm+mm$
12 — mm	13	$ll-mm=SS-2Sm$
10, 13	14	$nn=SS-2Sm$
14 * mm	15	$mmnn=SSmm-2Sm^3$
15, 9	16	$SSmm-2Sm^3=ffgg$
7 — $2f$	17	$e-f=S-2f$
17 * 7	18	$ee-ff=SS-2Sf$

1 — ff	19	$ee-ff=gg$
19, 18	20	$gg=SS-2Sf$
20 * ff	21	$ffgg=SSff-2Sfff$
21 — 16	22	$2Smmm-Ssmm-2sfff+ssff=0$
22 ÷ S	23	$2mmm-Smm-2fff+Sff=0$

∵ m hath three values, l and n will alſo have each of them three values.

16, 21	24	$m=f$ (one of the three values
24 — f	25	$m-f=0$ of m is equal to f)
23 ÷ 25	26	$2mm+2fm-Sm+2ff-Sf=0$
26 * 8	27	$16mm+16fm\ 8Sm+16Sff-8Sf=0$
27 + —	28	$16mm+16fm-8Sm=8Sf-16ff$

28 + —	29	$16mm+16fm-8Sm+4ff-4fS+SS=SS+4fS-12ff$
Let	30	$SS+4Sf-12ff=xx$
29, 30	31	$16mm+16fm-8Sm+4ff-4fS+SS=XX$
31 ω 2	32	$4m+2f-S=X$ or $-X$
32 + —	33	$4m=S-2f+X$. Let $\frac{1}{4}$ of this keep the name of M
32 + —	34	$4m=S-2f-X$. Let this leſſer value of m be called Q

and let the greateſt value of l be called P: of n be called R.
and therefore

2, 34	35	$pp=qq+rr$
8, 34	36	$p+q=S$
34,	37	$qr=mn$
1 Scope	38	$f=m$
3 — 38	39	$e=l$
4 ÷ 38	40	$g=n$
38 * 4	41	$4f=4m$
41, 33	42	$4f=S-2f+X$

42 + —	43	$6f-S=X$
43 ⊙ 2	44	$6ff-12Sf+SS=XX$
44 — 3	45	$48ff-16Sf=0$
45 ÷ $16f$	46	$f-S=0$
46 + S	47	$3f=S$
7, 47	48	$e+f=3f$
48 — f	49	$e=2f$
49 ⊙ 2	50	$ee=4ff$

50 — ff	51	$ee-ff=3ff$
19, 51	52	$gg=3ff$
52 ω 2	53	$g=f*\sqrt{3}$
39, 49	54	$l=2f$
40, 53	55	$n=f*\sqrt{3}$

$6f - 47$	56	$6f - S = 3f$
43 , 56	57	$X = 3f$
$47 - 2f$	58	$S - 2f = f$
$58 - 57$	59	$S - 2f - X = -2f$
34 , 59	60	$4q = -2f$
$60 \div 4$	61	$q = -\frac{1}{2}f$
36 , 47	62	$p + q = 3f$
$62 - 61$	63	$p = \frac{7}{2}f$
$38 * 55$	64	$mn = ff * \sqrt{3}$
37 , 64	65	$qr = ff * \sqrt{3}$
$65 \div 61$	66	$r = -f * \sqrt{12}$
	67	

Therefore

39 , 54	As 2	: So is $e = l$
38 , 49	to 1	: to $f = m$
40 , 55	to $\sqrt{3}$	: to $g = n$
47 , 57	to 3	: to $S = X$
63	to $\frac{7}{2}$	: to p
61	to $-\frac{1}{2}$	: to q
66	to $-\sqrt{12}$:	to r

2d. ſcope	68	$m = q$
$37 \div 68$	69	$n = r$
36 , 8	70	$p + q = l + m$
$70 - 68$	71	$p = l$
$68 * 4$	72	$4m = 4q$
$72 - 4q$	73	$4m - 4q = 0$
$33 - 34$	74	$4m - 4q = 2X$
73 , 74	75	$2X = 0$
$75 \div 2$	76	$X = 0$
76 ⊙ 2	77	$XX = 0$
30 , 77	78	$SS + 4Sf - 12ff = 0$
$78 + 16ff$	79	$SS + 4Sf + 4ff = 16ff$
79 ω 2	80	$S + 2f = 4f$

$80 - 2f$	81	$S = 2f$
7 , 81	82	$e + f = 2f$
$82 - f$	83	$e = f$
83 ⊙ 2	84	$ee = ff$
$84 - ff$	85	$ee - ff = 0$
19 , 85	86	$gg = 0$
86 ω 2	87	$G = 0$
$81 - 2f$	88	$S - 2f = 0$
$88 + 76$	89	$S - 2f + X = 0$
33 , 89	90	$4m = 0$
$90 \div 4$	91	$M = 0$
$8 - 91$	92	$L = S$
92 , 81	93	$L = 2f$
91 ⊙ 2	64	$MM = 0$
$LL - 94$	95	$LL - MM = LL$
10 , 95	96	$NN = LL$
96 ω 2	97	$N = L$
97 , 93	98	$N = 2f$
71 , 93	99	$P = 2f$
68 , 91	100	$O = 0$
69 , 98	101	$R = 2f$
	102	

Therefore

83	As 1	: So e
	to 1	: to f
87	to 0	: to g
93, 99, 81	to 2	: to $l = p = S$
91, 100, 76	to 0	: to $m = q = X$
98, 101	to 2	: to $n = r$

$2\sqrt{3} = 3.46410,161$ &c. ∵ $4\sqrt{3} = 6.92820,323$ &c.

The Tablet of Limits.

4	e	3
2	f	3
$2\sqrt{3}$	g	0
4	l	6
2	m	0
$2\sqrt{3}$	n	6
7	p	6
-1	q	0
$-4\sqrt{3}$	r	6
6	S	6
[illegible]	X	0
16	ee	9
4	ff	9
12	gg	0
16	ll	36
4	mm	0
12	nn	36
49	pp	36
1	qq	0
48	rr	36
36	SS	36
36	XX	0
$4\sqrt{3}$	fg	0
$4\sqrt{3}$	mn	0
$4\sqrt{3}$	qr	0

From Equat. 67 and 102 , I drew this *Tablet of Limits*, for the Equations which may be aſſumed in ſtead of Equ. 5 and 6, which are wanting pag. 189. As 6 to the aſſumed Perimeter; ſo are 4 and 3 to the limits between which e muſt be taken: and ſo are 2 and 3 to the limits between which f muſt be taken: and ſo are 4 and 6 to the limits of l, &c.

But

But you need not regard theſe limits if you take ſome known Equicrural for *one* of thoſe required, and ſeek the other thus:

Let the *given* ſides be 29.29.40∴Perimeter=98. Height=21. Area=420.

Of the ſought Equicrural let the three ſides be *h.b.2b* (ſee pag. 65 & Fig. 1.)

$b=?$	1	$bb+cc=hh$	
$c=?$	2	$h+b=49$	
$h=?$	3	$bc=420$	
$2-b$	4	$h=49-b$	
$4 \odot 2$	5	$hh=2401-98b+bb$	
1 , 5	6	$bb+cc=2401-98b+bb$	
$6-bb$	7	$cc=2401-98b$	
$7 \div \overline{49}$	8	$cc \div 49=49-2b$	
$3 \div \overline{7}$	9	$bc \div 7=60$	
$9 \odot 2$	10	$bbcc \div 49=3600$	
$8 * bb$	11	$bbcc \div 49=49bb-2b^3$	
$10-11$	12	$2bbb-49bb+3600=0$	
line 3	13	$b=20$	
$13-\overline{20}$	14	$b-20=0$	
$12 \div 14$	15	$2bb-9b-180=0$	
$15+\overline{180}$	16	$2bb-9b=180$	
$16 * 8$	17	$16bb-72b=1440$	
$17+\overline{81}$	18	$16bb-72b+81=1521$	
$18\, \omega\, 2$	19	$4b-9=39$. Or -39	
$19+\overline{9}$	20	$4b=48$	Or, $4b=-30$
$21 \div \overline{4}$	21	$B=12$	Or, $B=-7\frac{1}{2}$
$2-21$	22	$H=37$	Or, $H=56\frac{1}{2}$
$3 \div 21$	23	$C=35$	Or, $C=-56$

Or thus. *C* firſt.

$2-b$	4	$h=49-b$	
$4 \odot 2$	5	$hh=2401-98b+bb$	
$1-5$	6	$cc-2401+98b=0$	
$6 * c$	7	$ccc-2401c+98bc=0$	
$3 * \overline{98}$	8	$98bc=41160$	
7 , 8	9	$ccc-2401c+41160=0$	
line 3	10	$c=21$	
$10-\overline{21}$	11	$c-21=0$	
$9 \div 11$	12	$cc+21c-1960=0$	
$12+\overline{1960}$	13	$cc+21c=1960=\frac{7840}{4}$	
13 + fract	14	$cc+21c+\frac{441}{4}=\frac{8281}{4}$	
$14\, \omega\, 2$	15	$c+\frac{21}{2}=\frac{91}{2}$. Or $-\frac{91}{2}$	
$15-\frac{21}{2}$	16	$C=\frac{70}{2}$	Or $C=-\frac{112}{2}$
16	17	$C=35$	Or $C=-56$
$3 \div 17$	18	$B=12$	Or $B=-7\frac{1}{2}$
$2-18$	19	$H=37$	Or $H=56\frac{1}{2}$

Or thus *h* firſt

$2-h$	4	$b=49-h$	
$4 \odot 2$	5	$bb=2401-98h+hh$	
$1-5$	6	$cc=98h-2401$	
$6 \div \overline{49}$	7	$cc \div 49=2h-49$	
$5 * 7$	8	$bbcc=2h^3-245hh+9604h-117649$ over $\overline{49}$	
$3 \div \overline{7}$	9	$bc \div 7=60$	
$9 \odot 2$	10	$bbcc \div 49=3600$	
$8-10$	11	$2h^3-245hh+9604h-121249=0$	
line 3	12	$h=29$	
$12-\overline{29}$	13	$h-29=0$	
$11 \div 13$	14	$2hh-187h+4181=0$	
$14-$	15	$2hh-187h=-4181$	
$15 * \overline{8}$	16	$16hh-1496h=-33448$	
$16+$	17	$16hh-1496h+34969=1521$	
$17\, \omega\, 2$	18	$4h-187=-39$, or $+39$	
$13+\overline{187}$	19	$4h=148$	Or $4h=226$
$19 \div \overline{4}$	20	$H=37$	Or $H=56\frac{1}{2}$
$2-20$	21	$B=12$	Or $B=-7\frac{1}{2}$
$3 \div 21$	22	$C=35$	Or $C=-56$

The

The three Cubical Equations [$2bbb$ &c. ccc &c. $2hhh$ &c.] in pag. 191 may be found by the help of *three more-general Equations*, found thus.

$h==?$	1	$bb+cc=hh$
$b==?$	2	$h+b=S$ Semiperimeter
$c==?$	3	$bc=A$ Area
$2-b$	4	$h=S-b$
4 ⊚ 2	5	$hh=SS-2Sb+bb$
1, 5	6	$bb+cc=SS-2Sb+bb$
$6-bb$	7	$cc=SS-2Sb$
$7 * bb$	8	$bbcc=SSbb-2Sbbb$
3 ⊚ 2	9	$bbcc=AA$
9 — 8	10	$2Sbbb-SSbb+AA=0$
10 ÷ S	11	$2bbb-Sbb+\frac{AA}{S}=0$
$2 * c$	12	$hc+bc=Sc$
12 — 3	13	$hc=Sc-A$
13 ⊚ 2	14	$hhcc=SScc-2SAc+AA$
14 — 9	15	$hhcc-bbcc=SScc-2SAc$
$1-bb$	16	$hh-bb=cc$
$16 * cc$	17	$hhcc-bbcc=cccc$
17, 15	18	$cccc=SScc-2SAc$
$18 \div c$	19	$ccc=SSc-2SA$
19 +	20	$ccc-SSc+2SA=0$
$2-h$	21	$b=S-h$
21 ⊚ 2	22	$bb=SS-2Sh+hh$
1 — 22	23	$cc=2Sh-SS$
22 * 23	24	$bbcc=2Sh^3-5SShh+4S^3h-S^4$
24 — 9	25	$2Sh^3-5SShh+4S^3h-S^4-AA=0$
25 ÷ S	26	$2h^3-5Shh+4SSh-S^3-\frac{AA}{S}=0$

To Equat 11, 20, 26, put these following

$S=49 \therefore SS=2401 \therefore SSS=117649$

$A=420 \therefore SA=20580 \therefore AA \div S=3600$

then $2bbb-49bb+3600=0$

$ccc-2401c+41160=0$

$2hhh-245hh+9604h-121249=0$

as in page 191

The three roots of each of these three Equations may be found, as there

$B=20.$ or $B=12.$ or $B=-7\frac{1}{2}$

$C=21.$ $C=35.$ $C=-56$

$H=29.$ $H=37.$ $H=56\frac{1}{2}$

By these 4 last pages you see that this question is but a *Cubical* Problem: but by clogging it with the condition of commensurability (pag. 131.) we were forced to arise to Biquadratic Equations, pag. 143, 171.

You see also in these more general ways of Inquisition, a third Equicrural offers it self to be considered. So that though the Proposer speak but of *Pairs* of Equicrurals, the Answerer returns *ternions*. See pag. 138. lin. 10. and pag. 168. lin. 11.

I might adjoin more Uses of these three Equations, For the solving of this 31 Problem by *Delineation*: as also For exhibiting in long numbers an *orderly Set of as many ternions of Triangles, as shall be desired*, by the help of such Tables as are used by Writers of Canonical Trigonometry. But those precepts and examples require more time than I can now spare. Other affairs constrain me to break off and here to make

AN END.

This is *the Table* mentioned pag. 34. lin. 8. It fills 50 pages. Its firſt page calls it *a Table of Incompoſit numbers leſs than* 100000, But it contains far more *compoſit* numbers, than *incompoſit*; For it doth not only give an *Orderly enumeration of all odd numbers* which are not *compoſit*: but alſo it ſhews that none of the reſt are ſo. To every other odd number there expreſſed, the Table ſets ſome *incompoſit* that will *divide it without fraction.*

Each page hath 21 columels, whereof the *firſt* is filled with 40 odd numbers ſtanding in their natural order. The *following twenty* columels are diſtinguiſhed on their Tops, by numbers in their natural order 0. 1. 2. 3. to 998. 999. Theſe *Top numbers* are *hundreds*; the 40 *marginal* numbers are Unites *adhering to the Centuryes.* A line runing from any marginal croſs the page, ſhews, in any column, the *place* of the number made up of the Top-number and that marginal. In every ſuch *place* of concourſe you ſhall either find *p*, or ſome *incompoſit* leſs than 317. *p* ſhews the number to be a *prime or incompoſit* (See Euclid. VII. def. 11 & 13) If any number leſs than 100, 000, do end in 1.3.7 or 9, you may find its place in one of thoſe 50 pages, and then ſee whether it be a *prime* or no: If it be compoſit, you will there find its *leaſt Diviſor*. Thus in pag. 1. where the line marked with the marginal 67, croſſeth the columel whoſe Top-number is 16; there you find *p*, that is, ~~1767~~ is a prime. Where the ſame line croſſeth the next columel, you find 3. That is 1667 is no prime, 3 is the *leaſt Diviſor of it.* So in pag. 25, you ſee 49031, 49033, 49037 are primes; but 49039 is a Compoſit; 19 is its ſmalleſt Diviſor. 1667 1767

It may be of great uſe ſometimes to have *a complete and orderly enumeration of all incompoſits between* 0, *and* 100000, *without any mixture of Compoſits*; thus 1. 2. 3. 5. 7. 11. 13. *&c.* leaving out 9, 21 and all other compoſits. 2 and 5 are primes though they be left out of the long Table, becauſe no other incompoſit ends ſo. 2 and 5 being duely placed, all the reſt of the primes are taken out of the long Table as they there ſtand *marked with p*, from 7 in the firſt page to 99991 in the end of the 50*th* page.

If to *each of theſe primes* you ſet the Briggian Logarithm, you may find the Logarithms for *all the reſt of the numbers* in the firſt 100 Chiliads, by addition of the Logarithms of their incompoſit Factors.

The Reſolving of a number into all its incompoſit Factors [as 4620 into 2. 2. 3. 5. 7. 11] is altogether neceſſary, for the determining *how many Diviſors* that number hath, and *which they be*; As in page 194. 195.

(29)
1.abcdef
a. bcdef
b. acdef
c. abdef
d. abcef
e. abcdf
f. abcde
ab. cdef
ac. bdef
ad. bcef
ae. bcdf
af. bcde
bc. adef
bd. acef
be. acdf
bf. acde
cd. abef
ce. abdf
cf. abde
de. abcf
df. abce
ef. abcd
abc. def
abd. cef
abe. cdf
abf. cde
acd. bef
ace. bdf
acf. bde
ade. bcf
adf. bce
aef. bcd

(28)
1.aabcde
1. abcde
b. aacde
c. aabde
d. aabce
e. aabcd
aa. bcde
ab. acde
ac. abde
ad. abce
ae. abcd
bc. aade
bd. aace
be. aacd
cd. aabe
ce. aabd
de. aabc
aab. cde
aac. bde
aad. bce
aae. bcd
abc. ade
abd. ace
abe. acd

(27)
1.aabbcd
a. abbcd
b. aabcd
c. aabbd
d. aabbc
aa. bbcd
ab. abcd
ac. abbd
ad. abbc
bb. aacd
bc. aabd
bd. aabc
cd. aabb
aab. bcd
aac. bbd
aad. bbc
abb. acd
abc. abd

(26)
1.aaabcd
a. aabcd
b. aaacd
c. aaabd
d. aaabc
aa. abcd
ab. aacd
ac. aabd
ad. aabc
bc. aaad
bd. aaac
cd. aaab
aaa. bcd
aab. acd
aac. abd
aad. abc

(25)
1.aabbcc
a. abbcc
b. aabcc
c. aabbc
aa. bbcc
ab. abcc
ac. abbc
bb. aacc
bc. aabc
cc. aabb
aab. bcc
aac. bbc
abb. acc
abc. abc

(24)
1.aaabbc
a. aabbc
b. aaabc
c. aaabb
aa. abbc
ab. aabc
ac. aabb
bb. aaac
bc. aaab
aaa. bbc
aab. abc
aac. abb

(23)
1.aaaabc
a. aaabc
b. aaaac
c. aaaab
aa. aabc
ab. aaac
ac. aaab
bc. aaaa
aaa. abc
aab. aac

(22)
1.aaabbb
a. aabbb
b. aaabb
aa. abbb
ab. aabb
bb. aabb
aaa. bbb
aab. abb

(21)
1.aaaabb
a. aaabb
b. aaaab
aa. aabb
ab. aaab
bb. aaaa
aaa. abb
aab. aab

(20)
1.aaaaab
a. aaaab
b. aaaaa
aa. aaab
ab. aaaa
aaa. aab

(19)
1.aaaaaa
a. aaaaa
aa. aaaa
aaa. aaa

(18)
1. abcde
a. bcde
b. acde
c. abde
d. abce
e. abcd
ab. cde
ac. bde
ad. bce
ae. bcd
bc. ade
bd. ace
be. acd
cd. abe
ce. abd
de. abc

(17)
1.aabcd
a. abcd
b. aacd
c. aabd
d. aabc
aa. bcd
ab. acd
ac. abd
ad. abc
bc. aad
bd. aac
cd. aab

(16)
1.aabbc
a. abbc
b. aabc
c. aabb
aa. bbc
ab. abc
ac. abb
bb. aac
bc. aab

(15)
1. aaabc
a. aabc
b. aaac
c. aaab
aa. abc
ab. aac
ac. aab
bc. aaa

(14
1.aaabb
a. aabb
b. aaab
aa. abb
ab. aab
bb. aaa

(13)
1. aaaab
a. aaab
b. aaaa
aa. aab
ab. aaa

(12)
1.aaaaa
a. aaaa
aa. aaa

(11)
1. abcd
a. bcd
b. acd
c. abd
d. abc
ab. cd
ac. bd
ad. bc

(10)
1.aabc
a. abc
b. aac
c. aab
aa. bc
ab. ac

(9)
1.aabb
a. abb
b. aab
aa. bb
ab. ab

(8)
1.aaab
a. aab
b. aaa
aa. ab

(7)
1.aaaa
a. aaa
aa. aa

(6)
1. abc
a. bc
b. ac
c. ab

(5)
1.aab
a. ab
b. aa

(4)
1.aaa
a. aa

(3)
1. ab
a. b

(2)
1. aa
a. a

(1)
1. a

(29)	(28)	9. 140	15.60	9. 24	21.110	10.12	(9)
1.30030	1.4620	15. 84	25.36	8. 27	33. 70	15. 8	1.36
2.15015	2.2310	21. 60	12.75	12. 18	35. 66	(14)	2.18
3.10010	3.1540	35. 36	20.45	(21)	55. 42	1.72	3.12
5.6006	5.924	12.105	18.50	1.144	77. 30	2.36	4. 9
7.4290	7.660	20. 63	30.30	2. 27	(17)	3.24	6. 6
11.2730	11.420	28. 45	(24)	3. 48	1.420	4.18	(8)
13.2310	4.1155	18. 70	1.360	4. 36	2.210	6.12	1.24
6.5005	6.770	30. 42	2.180	6. 24	3.140	9. 8	2.12
10.3003	10.462	(20)	3.120	9. 16	5. 84	(13)	3. 8
14.2145	14.330	1.840	5.72	8. 18	7. 60	1.48	4. 6
22.1365	22.210	2.420	4.90	12. 12	4.105	2.24	(7)
26.1155	15.308	3.280	6.60	(20)	6. 70	3.16	1.16
15.2002	21.220	5.168	10.36	1. 96	10. 42	4.12	2. 8
21.1430	33.140	7.120	9.40	2. 48	14. 30	6. 8	4. 4
33. 910	35.132	4.210	15.24	3. 32	15. 28	(12)	(6)
39. 770	55. 84	6.140	8.45	4. 24	21. 20	1.32	1.30
35. 858	77. 60	10. 84	12.30	6. 16	35. 12	2.16	2.15
55. 546	12.385	14. 60	20.18	9. 12	(16)	4. 8	3.10
65. 462	20.231	15. 56	(23)	(19)	1.180	(11)	5. 6
77. 390	28.165	21. 40	1.240	1. 64	2. 90	1.210	(5)
91. 330	44.105	35. 24	2.120	2. 32	3. 60	2.105	1.12
143.210	30.154	8. 105	3. 80	4. 16	5. 36	3.70	2. 6
30.1001	42.110	12. 70	5. 48	8. 8	4. 45	5.42	3. 4
42.715	66. 70	20. 42	4. 60	(18)	6. 30	7.30	(4)
66.455	(27)	28. 30	6.40	1.2310	10. 18	6.35	1. 8
78.385	1.1260	(25)	10.24	2.1155	9. 20	10.21	2. 4
70.429	2.630	1.900	15.16	3. 770	15. 12	14.15	(3)
110.273	3.420	2.450	8.30	5. 462	(15)	(10)	1. 6
130.231	5.252	3.300	12.20	7. 330	1.120	1.60	2. 3
154.195	7.180	5.180	(22)	11.210	2. 60	2.30	(2)
182.165	4.315	4.225	1.216	6.385	3. 40	3.20	1. 4
286.105	6.210	6.150	2.108	10.231	5. 24	5.12	2. 2
	10.126	10. 90	3.72	14.165	4. 30	4.15	(1)
	14. 90	9.100	4.54	22.105	6. 20	6.10	1. 2
			3.66	15.154			

Sort	Forme	Div.
29	abcdef	64
28	aabcde	48
27	aabbcd	36
26	aaabcd	32
25	aabbcc	27
24	aaabbc	24
23	aaaabc	20
22	aabbb	16
21	aaabb	15
20	aaaaab	12
19	aaaaaa	7
18	abcde	32
17	aabcd	24
16	aabbc	18
15	aaabc	16
14	aaabb	12
13	aaaab	10
12	aaaaa	6
11	abcd	16
14	aabc	12
9	aabb	9
8	aaab	8
7	aaaa	5
6	abc	8
5	aab	6
4	aaa	4
3	ab	4
2	aa	3
1	a	2

That is: The 29th sort hath 64 Divisors; the 18th hath but 32 &c.

Every *Aliquot part* of a Number is one of the just Divisors of it. The greatest Divisor being equal to the *whole* Dividend, must not be called a *Part*: Wherefore, substract 1 from every number in the last columel of page 195, you shall have the number of *aliquot* parts belonging to every one of those 29 sorts.

Having the least Divisor of any number of the long Table to find all its other incomposit Co-efficients.

If that Divisor end in 1 or 9, and have a black stroke *under* it in the Dividends place in the long table; or if the Divisor end in 3 or 7, and have such a stroke *over* it in the Dividends place; the Divident is the *square of an incomposit*, and the Quotient is given, for it is equal to the Divisor.

If the least Divisor have no such stroke by it, let it divide the proposed number, the Quotient shall be the greatest *aliquot part* of that Dividend: Seek that Quotient in the same long Table; if it be there *marked with p*, your inquiry is at an end; The Dividend is of the form AB. If it be *not so marked*, By the Prime there found, divide your *first* Quotient, Deal with the *second* Quotient as you had done with the *first*, repeating such Divisions, till the Quotient be incomposit. Thus 53191 is found in page 27, with its smallest Divisor 43. 53191 divided by 43 gives 1237. Pag. 1 says This 1237 is a prime. Inquire no farther.

But desiring the incomposit factors of 93611. I find it in pag. 47 of the long Table, with 7 for its least Divisor. The Quotient 13373 is found in pag. 7 with its least Divisor 43. This 43 gives a second quotient 311. Pag. 1 says, This 311 is an incomposit. So the prime Co-efficients of 93611 are 7. 43. 311.(Hence infer that 53191 is to 93611, as 1237 to 2177 = 7 * 311.

If you divide *any odd* number by *all the primes in order*, beginning with 3, The first Divisor that finds a Quotient without fraction, is the least Divisor that the Dividend can have. 239 is the least number that measures 1 111 111. Try 3, 7, 11, *&c.* No prime can divide 1 111 111 till you come to 239. If no such Divisor find an Integer Quotient, before the Quotient is less than the Divisor, pronounce your Dividend to be incomposit, and that last Divisor to be greater than the Dividends square root. Frequent occasion of Dividing by Incomposits calls for a *Tariffa* of as many *primes* as shall be needful. For resolving of numbers less than 100000 it sufficeth if it be extended to 313, As in the next page.

All Incomposits, less than √ 10000, *multiplyed by* 2, 3, 4, 5, 6, 7, 8, 9.

√100,000

1	2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73
2	4	6	10	14	22	26	34	38	46	58	62	74	82	86	94	106	118	122	134	142	146
3	6	9	15	21	33	39	51	57	69	87	93	111	123	129	141	159	177	183	201	213	219
4	8	12	20	28	44	52	68	76	92	116	124	148	164	172	188	212	236	244	268	284	292
5	10	15	25	35	55	65	85	95	115	145	155	185	205	215	235	265	295	305	335	355	365
6	12	18	30	42	66	78	102	114	138	174	186	222	246	258	282	318	354	366	402	426	438
7	14	21	35	49	77	91	119	133	161	203	217	259	287	301	329	371	413	427	469	497	511
8	16	24	40	56	88	104	136	152	184	232	248	296	328	344	376	424	472	488	536	568	584
9	18	27	45	63	99	117	153	171	207	261	279	333	369	387	423	477	531	549	603	939	657

1	79	83	89	97	101	103	107	109	113	127	131	137	139	149	151	157
2	158	166	178	194	202	206	214	218	226	254	262	274	278	298	302	314
3	237	249	267	291	303	309	321	327	339	381	393	411	417	447	453	471
4	316	332	256	388	404	412	428	436	452	508	524	548	556	596	604	628
5	395	415	445	485	505	515	535	545	565	635	655	685	695	745	755	785
6	474	498	534	582	606	618	642	654	678	762	786	822	834	894	906	942
7	553	581	623	679	707	721	749	763	791	889	917	959	973	1043	1057	1099
8	632	664	712	776	808	824	856	872	904	1016	1048	1096	1112	1192	1208	1256
9	711	747	801	871	909	927	963	981	1017	1143	1179	1233	1251	1341	1359	1413

1	163	167	173	179	181	191	193	197	199	211	223	227	229	233
2	326	334	346	358	362	382	386	394	398	422	446	454	458	466
3	489	501	519	537	543	573	579	591	597	633	669	681	687	699
4	652	668	692	716	724	764	772	788	796	844	892	908	916	932
5	815	835	865	895	905	955	965	985	995	1055	1115	1135	1145	1165
6	978	1002	1038	1074	1086	1146	1158	1182	1194	1266	1338	1362	1374	1398
7	1141	1169	1211	1253	1267	1337	1351	1379	1393	1477	1561	1589	1603	1631
8	1304	1336	1384	1432	1448	1528	1544	1576	1592	1688	1784	1816	1832	1864
9	1467	1503	1557	1611	1629	1719	1737	1773	1791	1899	2007	2043	1061	2097

1	239	241	251	257	263	269	271	277	281	283	293	307	311	313
2	478	482	502	514	526	538	542	554	562	566	586	614	622	626
3	717	723	753	771	789	807	813	831	843	849	879	921	933	939
4	956	964	1004	1028	1052	1076	1084	1108	1124	1132	1172	1228	1244	1252
5	1195	1205	1255	1285	1315	1345	1355	1385	1405	1415	1465	1535	1555	1565
6	1434	1446	1506	1542	1578	1614	1626	1662	1686	1698	1758	1842	1866	1878
7	1673	1687	1757	1799	1841	1883	1897	1939	1967	1981	2051	2149	2177	2191
8	1912	1928	2008	2056	2104	2152	2168	2216	2248	2264	2344	2456	2488	2504
9	2151	2169	2259	2313	2367	2421	2439	2493	2529	2547	2637	2763	2799	2817

P. Guildin sayes; 149 is divisible by 7, 229 by 31. *Schooten* leaves 809 out of his *Catalog. of Incomposits. Rhonius* makes 1209 and 1673 incomposits, and says 11833 is divisible by 19. But *this Table* sayes more truly, that 149.229.809.11833 are Incomposits: and that 1209 is divisible by 3 and 1673 by 7. Yet trust it not, before you have amended *these faults* in it.

Pa.	Numb.	For.	Set.	Pa.	Numb.	For.	Set.	Pa.	Numb.	For.	Set.	Pa.	Numb.	For.	Set.
5	9211	19	61	21	40277	13	p		60779	63	163		72381	p	3
	9799	40	41		40591	3	p		61779	p	3		72383	3	p
6	10199	p	7		40593	p	3	32	62011	3	p		72557	73	37
	10813	13	11		40597	3	p		62013	p	3		72601	97	79
7	13201	23	43	23	44659	11	17		62017	3	p		73023	p	3
9	17563	3	7		45353	p	7		62019	p	3		73051	7	11
	17981	41	p		45837	p	3		63839	71	p		73481	179	197
10	18903	7	3	24	466..	476	466		63883	191	193		73493	p	7
	18907	3	7		46089	7	3	33	641..	541	641		73913	p	7
	18909	7	3		46457	3	p		659..	569	659	40	78199	p	11
12	23203	3	p		47201	11	7		64237	61	p	41	80333	67	11
	23381	193	103		47577	7	3		64693	3	p		80663	p	11
13	24011	3	13		47579	3	7		64973	23	43	42	83123	103	101
	25093	13	23		47663	p	7		65959	17	71	43	84311	57	59
	25873	23	p	25	48601	53	7	34	66239	19	p	44	86699	281	181
14	27233	31	113	27	53361	7	3		66561	7	3	46	91189	p	7
	27517	p	7		53791	3	p		66563	3	7		91707	p	3
15	28201	3	p	28	54507	7	3		66567	p	3		91793	23	17
	28203	p	3		54509	3	7		66569	3	p	47	92701	3	7
	29599	blank	p		54589	71	79		66761	191	101		92703	7	3
17	32297	71	p	29	56323	157	151		66951	2	3		92773	163	113
	33259	97	79	30	58123	11	13	35	68809	53	13		93101	151	157
	33591	7	3		58181	71	73	36	70313	167	p		93161	52	59
	33593	3	7		58301	137	173		70981	167	p	48	95371	281	283
18	34089	7	3		58901	blank	p		71113	3	7		95797	p	13
	34209	23	3		59901	7	3		71603	p	7	49	97903	3	13
	35089	3	p		59909	137	139		71983	167	p	50	98099	26	263
20	39263	p	7	31	60079	63	73	37	72357	7	3		98551	39	139
	39589	p	11		60293	7	p		72359	3	7		99443	17	77

Tho.

Dr Wallis says that he has examin'd the following Table very accurately, by the same method & with the same trouble with which it was at first computed; & gives the following Table of Errours to be added to those in the opposite page.

Page	Number	For	Set
3	5579	p	7
5	9287	19	37
8	14873	73	107
11	20983	3	p
16	30167	71	97
	31001	29	29
17	33409	47	p
19	37533	13	7
21	40049	19	29
	40599	p	3
	40759	3	p
	41581	41	43
24	46199	73	p
27	53941	13	17
28	54449	71	p

Page	Number	For	Set
28	55609	3	p
31	60701	01	101
	60799	63	163
33	64499	13	p
	65479	3	p
34	67993	1	p
38	75653	151	p
41	80561	17	13
43	85909	137	p
44	86993	79	p
47	93719	7	p
48	94769	41	97
49	96109	3	13
	97487	3	13

Page 7, in the margin, after 43, for 37, set 47

The Dr adds that when the errours noted in these two pages are corrected, he believes that the Table will be perfectly exact & without fault. See Wallis's works Vol: 2. page 510.

In the following Table, all those errours are corrected by which any number is mark'd as a prime, that is really composite.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
01	1. P	P	3	7	P	3	P	P	3	17	7	3	P	P	3	19	P	3	P	P
03	P	P	7	3	13	P	3	19	11	3	17	P	3	P	23	3	7	13	3	11
07	P	P	3	P	11	3	P	7	3	P	19	3	17	P	3	11	P	3	13	P
09	3	P	11	3	P	P	3	P	P	3	P	P	3	7	P	3	P	P	3	23
11	P	3	P	P	3	7	13	3	P	P	3	11	7	3	17	P	3	29	P	3
13	P	P	3	P	7	3	P	23	3	11	P	3	P	13	3	17	P	3	7	P
17	P	3	7	P	3	11	P	3	19	7	3	P	P	3	13	37	3	17	23	3
19	P	7	3	11	P	3	P	P	3	P	P	3	23	P	3	7	P	3	17	19
21	3	11	13	3	P	P	3	7	P	3	P	19	3	P	7	3	P	P	3	17
23	P	3	P	17	3	P	7	3	P	13	3	P	P	3	P	P	3	P	P	3
27	3	P	P	3	7	17	3	P	P	3	13	7	3	P	P	3	P	11	3	41
29	P	3	P	7	3	23	17	3	P	P	3	P	P	3	P	11	3	7	31	3
31	P	P	3	P	P	3	P	17	3	7	P	3	P	11	3	P	7	3	P	P
33	3	7	P	3	P	13	3	P	7	3	P	11	3	31	P	3	23	P	3	P
37	P	P	3	P	19	3	7	11	3	P	17	3	P	7	3	29	P	3	11	13
39	3	P	P	3	P	7	3	P	P	3	P	17	3	13	P	3	11	37	3	7
41	P	3	P	11	3	P	P	3	29	P	3	7	17	3	11	23	3	P	7	3
43	P	11	3	7	P	3	P	P	3	23	7	3	11	17	3	P	31	3	19	29
47	P	3	13	P	3	P	P	3	7	P	3	31	29	3	P	7	3	P	P	3
49	7	P	3	P	P	3	11	7	3	13	P	3	P	19	3	P	17	3	43	P
51	3	P	P	3	11	19	3	P	23	3	P	P	3	7	P	3	13	17	3	P
53	P	3	11	P	3	7	P	3	P	P	3	P	7	3	P	P	3	P	17	3
57	3	P	P	3	P	P	3	P	P	3	7	13	3	23	31	3	P	7	[illegible]	19
59	P	3	7	P	3	13	P	3	P	7	3	19	P	3	P	P	3	P	11	3
61	P	7	3	19	P	3	P	P	3	31	P	3	13	P	3	7	11	3	P	37
63	3	P	P	3	P	P	3	7	P	3	P	P	3	29	7	3	P	41	3	13
67	P	P	3	P	P	3	23	13	3	P	11	3	7	P	3	P	P	3	P	7
69	3	13	P	3	7	P	3	P	11	3	P	7	3	37	13	3	P	29	3	11
71	P	5	P	7	3	P	11	3	13	P	3	P	31	3	P	P	3	7	P	3
73	P	P	3	P	11	3	P	P	3	7	29	3	19	P	3	11	7	3	P	P
77	7	3	P	13	3	P	P	3	P	P	3	11	P	3	7	19	3	P	P	3
79	P	P	3	P	P	3	7	19	3	11	13	3	P	7	3	P	23	3	P	P
81	3	P	P	3	13	7	3	11	P	3	23	P	3	P	P	3	41	13	3	7
83	P	3	P	P	3	11	P	3	P	P	3	7	P	3	P	P	3	P	7	3
87	3	11	7	3	P	P	3	P	P	3	P	P	3	19	P	3	7	P	3	P
89	P	3	17	P	3	19	13	3	7	23	3	29	P	3	P	7	3	P	P	3
91	7	P	3	17	P	3	P	7	3	P	P	3	P	13	3	37	19	3	31	11
93	3	P	P	3	17	P	3	13	19	3	P	P	3	7	P	3	P	11	3	P
97	P	P	3	P	7	3	17	P	3	P	P	3	P	11	3	P	P	3	7	P
99	3	P	13	3	P	P	3	17	29	3	7	11	3	P	P	3	P	7	3	P

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
01	3	11	31	3	7	41	3	37	P	3	P	7	3	P	19	3	13	P	3	47
03	P	3	P	7	3	P	19	3	P	P	3	29	P	3	41	31	3	7	P	3
07	3	7	P	3	29	23	3	P	7	3	31	13	3	P	P	3	P	11	3	P
09	7	3	47	P	3	13	P	3	53	P	3	P	P	3	7	11	3	P	13	3
11	P	P	3	P	P	3	7	P	3	41	P	3	13	7	3	P	23	3	37	P
13	3	P	P	3	19	7	3	P	29	3	23	11	3	P	P	3	P	47	3	7
17	P	29	3	7	P	3	P	11	3	P	7	3	P	31	3	P	P	3	11	P
19	3	13	7	3	41	11	3	P	P	3	P	P	3	P	13	3	7	P	3	P
21	43	3	P	11	3	P	P	3	7	23	3	P	P	3	11	7	3	61	P	3
23	7	11	3	23	P	3	43	7	3	37	P	3	11	P	3	13	P	3	P	P
27	P	3	17	13	3	7	37	3	11	P	3	53	7	3	23	P	3	P	43	3
29	P	P	3	17	7	3	11	P	3	29	13	3	P	P	3	P	19	3	7	P
31	3	P	23	3	11	P	3	P	19	3	7	31	3	P	47	3	P	7	3	P
33	19	3	7	P	3	17	P	3	P	7	3	13	53	3	P	P	3	P	P	3
37	3	P	P	3	P	43	3	7	P	3	P	P	3	47	7	3	P	37	3	31
39	P	3	P	P	3	P	7	3	17	P	3	43	41	3	19	P	3	P	11	3
41	13	P	3	P	P	3	19	P	3	17	P	3	7	13	3	P	11	3	23	7
43	3	P	P	3	7	P	3	13	P	3	17	7	3	P	11	3	P	19	3	P
47	23	19	3	P	P	3	P	41	3	7	11	3	17	P	3	P	7	3	P	P
49	3	7	13	3	31	P	3	P	7	3	P	47	3	17	P	3	41	23	3	11
51	7	3	P	P	3	P	11	3	P	13	3	23	P	3	7	53	3	11	P	3
53	P	P	3	13	11	3	7	P	3	P	43	3	P	7	3	11	13	3	P	59
57	11	3	37	P	3	P	P	3	P	P	3	7	P	3	P	P	3	13	7	3
59	29	17	3	7	P	3	P	31	3	11	7	3	P	P	3	P	P	3	17	37
61	3	P	7	3	23	13	3	11	P	3	P	29	3	P	P	3	7	P	3	17
63	P	3	31	17	3	11	P	3	7	P	3	P	13	3	P	7	3	53	P	3
67	3	11	P	3	P	17	3	P	47	3	P	P	3	7	P	3	19	P	3	P
69	P	3	P	23	3	7	17	3	19	P	3	P	7	3	P	43	3	P	53	3
71	19	13	3	P	7	3	P	17	3	P	37	3	P	P	3	P	P	3	7	11
73	3	41	P	3	P	31	3	47	13	3	7	19	3	P	23	3	P	7	3	29
77	31	7	3	P	P	3	P	P	3	13	17	3	29	11	3	7	P	3	P	41
79	3	P	43	3	37	P	3	7	P	3	P	11	3	31	7	3	13	P	3	23
81	P	3	P	P	3	29	7	3	43	11	3	P	17	3	59	P	3	19	P	3
83	P	37	3	P	13	3	P	11	3	19	P	3	7	17	3	P	29	3	11	7
87	P	3	P	7	3	13	P	3	P	29	3	P	19	3	11	17	3	7	13	3
89	P	11	3	P	19	3	P	P	3	7	P	3	11	P	3	37	7	3	P	P
91	3	7	29	3	47	P	3	P	7	3	11	P	3	P	P	3	P	17	3	13
93	7	3	P	P	3	P	P	3	11	41	3	31	37	3	7	P	3	P	17	3
97	3	13	P	3	11	7	3	P	P	3	19	23	3	43	13	3	P	P	3	7
99	P	3	11	P	3	23	P	3	13	P	3	7	P	3	P	59	3	29	7	3

	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59
01	p	3	p	11	3	7	43	3	p	13	3	p	7	3	11	p	3	p	p	3
03	p	11	3	13	7	3	p	p	3	p	p	3	11	p	3	p	13	3	7	p
07	p	3	7	59	3	p	17	3	11	7	3	p	41	3	p	p	3	13	p	3
09	19	7	3	31	p	3	11	17	3	p	p	3	p	p	3	7	71	3	37	19
11	3	p	p	3	11	13	3	7	17	3	p	19	3	47	7	3	31	p	3	23
13	p	3	11	19	3	p	7	3	p	17	3	p	13	3	p	37	3	29	p	3
17	3	23	p	3	7	p	3	53	p	3	29	7	3	13	p	3	41	p	3	61
19	p	3	p	7	3	p	31	3	61	p	3	p	17	3	p	p	3	7	11	3
21	p	13	3	29	p	3	p	p	3	7	p	3	23	17	3	p	7	3	p	31
23	3	7	41	3	p	p	3	p	7	3	p	47	3	p	11	3	p	59	3	p
27	p	p	3	p	19	3	7	29	3	13	11	3	p	7	3	p	17	3	p	p
29	3	p	p	3	41	7	3	p	11	3	47	23	3	73	61	3	13	17	3	7
31	29	3	p	61	3	23	11	3	p	p	3	7	p	3	p	p	3	11	7	3
33	37	p	3	7	11	3	41	p	3	p	7	3	p	p	3	11	43	3	19	17
37	11	3	19	p	3	13	p	3	7	p	3	11	p	3	p	7	3	p	13	3
39	7	p	3	p	23	3	p	7	3	11	p	3	13	19	3	29	p	3	p	p
41	3	41	p	3	p	19	3	11	47	3	71	53	3	7	p	3	p	p	3	13
43	13	3	p	43	3	7	p	3	29	p	3	37	7	3	p	23	3	p	p	3
47	3	11	31	3	p	p	3	47	37	3	7	p	3	p	13	3	p	7	3	19
49	p	3	7	p	3	p	p	3	13	7	3	19	29	3	p	31	3	p	p	3
51	p	7	3	19	p	3	p	p	3	p	p	3	59	p	3	7	p	3	p	11
53	3	p	p	3	61	29	3	7	23	3	31	p	3	53	7	3	p	11	3	p
57	p	p	3	p	p	3	p	67	3	p	13	3	7	11	3	p	p	3	p	7
59	3	p	p	3	7	47	3	p	43	3	p	7	3	23	53	3	p	13	3	59
61	31	3	p	7	3	p	59	3	p	11	3	13	p	3	43	67	3	7	p	3
63	17	23	3	p	p	3	p	11	3	7	61	3	19	31	3	p	7	3	11	67
67	7	3	17	11	3	p	13	3	31	p	3	p	23	3	7	19	3	73	p	3
69	13	11	3	17	41	3	7	19	3	p	37	3	11	7	3	p	p	3	p	47
71	3	43	p	3	17	7	3	13	p	3	11	p	3	41	p	3	53	29	3	7
73	p	3	p	p	3	17	p	3	11	p	3	7	p	3	13	p	3	23	7	3
77	3	p	7	3	11	23	3	17	p	3	p	31	3	19	p	3	7	53	3	43
79	p	3	11	29	3	19	p	3	7	13	3	p	p	3	p	7	3	p	p	3
81	7	37	3	13	p	3	31	7	3	17	p	3	p	p	3	p	13	3	p	p
83	3	47	p	3	p	p	3	p	19	3	13	71	3	7	p	3	p	p	3	31
87	61	53	3	41	7	3	43	p	3	p	p	3	17	p	3	37	11	3	7	p
89	3	59	p	3	67	13	3	p	p	3	7	p	3	17	11	3	p	7	3	53
91	p	3	7	p	3	p	p	3	67	7	3	29	11	3	17	p	3	p	43	3
93	p	7	3	23	p	3	13	p	3	p	11	3	67	p	3	7	p	3	71	13
97	17	3	p	p	3	p	7	3	59	19	3	p	p	3	23	29	3	11	p	3
99	p	13	3	53	11	3	37	p	3	p	p	3	7	p	3	11	41	3	17	7

	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
01	17	P	3	P	37	3	7	P	3	67	P	3	19	7	3	13	11	3	29	P
03	3	17	P	3	19	7	3	P	P	3	47	P	3	67	11	3	P	P	3	7
07	P	31	3	7	43	3	P	19	3	P	7	3	P	P	3	P	P	3	37	P
09	3	41	7	3	13	23	3	P	11	3	43	P	3	P	31	3	7	13	3	11
11	P	3	P	P	3	17	11	3	7	P	3	13	P	3	P	7	3	11	73	3
13	7	P	3	59	11	3	17	7	3	31	P	3	P	71	3	11	23	3	13	41
17	11	3	P	P	3	7	13	3	17	P	3	11	7	3	P	P	3	P	P	3
19	13	29	3	71	7	3	P	P	3	11	P	3	P	13	3	73	19	3	7	P
21	3	P	P	3	P	P	3	11	19	3	7	P	3	P	41	3	P	7	3	89
23	19	3	7	P	3	11	37	3	P	7	3	17	31	3	13	P	3	P	P	3
27	3	11	13	3	P	61	3	7	P	3	P	P	3	17	7	3	29	P	3	P
29	P	3	P	P	3	P	7	3	P	13	3	P	P	3	17	P	3	59	P	3
31	37	P	3	13	59	3	19	53	3	29	79	3	7	P	3	17	13	3	41	7
33	3	P	23	3	7	47	3	P	P	3	13	7	3	P	P	3	17	11	3	P
37	P	17	3	P	41	3	P	P	3	7	31	3	P	11	3	P	7	3	17	P
39	3	7	17	3	47	13	3	23	7	3	P	11	3	41	43	3	P	71	3	17
41	7	3	79	17	3	31	29	3	P	11	3	37	13	3	7	P	3	P	P	3
43	P	P	3	P	17	3	7	11	3	53	P	3	P	7	3	19	P	3	11	13
47	P	3	P	11	3	P	17	3	41	P	3	7	P	3	11	P	3	61	7	3
49	23	11	3	7	P	3	61	17	3	P	7	3	11	P	3	P	P	3	47	P
51	3	P	7	3	P	P	3	43	13	3	11	P	3	P	P	3	7	23	3	P
53	P	3	13	P	3	P	P	3	7	17	3	23	P	3	29	7	3	P	P	3
57	3	47	P	3	11	79	3	29	P	3	P	17	3	7	P	3	13	P	3	73
59	73	3	11	P	3	7	P	3	19	P	3	P	7	3	P	P	3	P	29	3
61	11	61	3	P	7	3	P	P	3	P	23	3	53	17	3	P	47	3	7	19
63	3	P	P	3	23	P	3	P	P	3	7	13	3	37	17	3	79	7	3	P
67	P	7	3	P	29	3	59	67	3	P	37	3	13	53	3	7	11	3	P	31
69	3	31	P	3	P	P	3	7	P	3	P	67	3	P	7	3	P	17	3	13
71	13	3	P	23	3	P	7	3	P	P	3	71	11	3	31	67	3	19	17	3
73	P	P	3	P	P	3	P	13	3	19	11	3	7	73	3	P	P	3	P	7
77	59	3	P	7	3	P	11	3	13	P	3	P	19	3	P	P	3	7	P	3
79	P	37	3	P	11	3	P	P	3	7	P	3	29	47	3	11	7	3	P	79
81	3	7	11	3	P	P	3	P	7	3	73	43	3	11	P	3	P	31	3	23
83	7	3	61	13	3	29	41	3	P	P	3	11	P	3	7	P	3	43	P	3
87	3	23	P	3	13	7	3	11	71	3	19	P	3	83	P	3	P	13	3	7
89	P	3	19	P	3	11	P	3	83	29	3	7	37	3	P	P	3	P	7	3
91	P	41	3	7	P	3	P	P	3	P	7	3	23	19	3	P	P	3	13	61
93	3	11	7	3	43	19	3	P	61	3	41	P	3	P	59	3	7	P	3	P
97	7	P	3	P	73	3	37	7	3	P	47	3	P	13	3	71	43	3	53	11
99	3	P	P	3	67	P	3	13	P	3	31	23	3	7	P	3	P	11	3	19

	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
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07	3	11	29	3	7	47	3	P	P	3	P	7	3	41	23	3	13	17	3	P
09	P	3	P	7	3	67	P	3	23	59	3	P	P	3	97	37	3	7	17	3
11	P	P	3	P	13	3	79	31	3	7	P	3	19	P	3	P	7	3	P	11
13	3	7	43	3	47	P	3	P	7	3	P	13	3	67	P	3	P	11	3	23
17	P	P	3	P	19	3	7	23	3	37	71	3	13	7	3	31	59	3	P	47
19	3	23	P	3	P	7	3	P	P	3	29	11	3	P	P	3	P	P	3	7
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23	71	P	3	7	P	3	P	11	3	P	7	3	23	P	3	89	P	3	11	P
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39	P	3	7	31	3	P	53	3	P	7	3	13	P	3	P	P	3	P	P	3
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47	13	P	3	17	P	3	P	P	3	23	83	3	7	13	3	P	11	3	43	7
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53	P	31	3	P	79	3	17	P	3	7	11	3	19	47	3	41	7	3	59	37
57	7	3	23	61	3	43	11	3	17	13	3	P	P	3	7	19	3	11	P	3
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71	7	P	3	11	43	3	13	7	3	P	47	3	73	P	3	17	19	3	P	13
73	3	11	P	3	37	P	3	31	19	3	43	P	3	7	P	3	17	29	3	P
77	41	13	3	P	7	3	P	67	3	47	29	3	P	P	3	61	P	3	7	11
79	3	P	17	3	61	23	3	P	13	3	7	67	3	83	P	3	P	7	3	17
81	P	3	7	17	3	P	P	3	83	7	3	P	P	3	19	11	3	P	41	3
83	59	7	3	83	17	3	19	P	3	13	31	3	P	11	3	7	23	3	P	67
87	P	3	P	P	3	31	7	3	P	11	3	P	19	3	53	P	3	P	P	3
89	P	19	3	P	13	3	P	11	3	89	61	3	7	41	3	43	P	3	11	7
91	3	P	P	3	7	11	3	59	17	3	P	7	3	P	P	3	11	P	[illegible]	97
93	P	3	P	7	3	13	P	3	P	17	3	29	P	3	11	53	3	7	13	3
97	3	7	P	3	29	P	3	19	7	3	11	17	3	P	P	3	P	97	[illegible]	13
99	7	3	43	37	3	P	P	3	11	P	3	P	17	3	7	29	3	4	9	3

THE TABLE OF

	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119
01	73	3	101	P	3	P	P	3	7	11	3	17	23	3	13	7	3	P	P	3
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13	17	3	7	P	3	P	P	3	13	7	3	P	P	3	101	29	3	13	P	3
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39	3	61	P	3	7	P	3	P	37	3	13	7	3	P	89	3	23	11	3	53
41	P	3	P	7	3	P	P	3	P	P	3	17	P	3	P	11	3	7	P	3
43	P	P	3	P	23	3	47	I	3	7	P	3	17	11	3	29	7	3	109	73
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71	P	3	7	89	3	13	P	3	61	7	3	P	23	3	19	41	3	47	11	3
73	P	7	3	P	P	3	19	53	3	P	17	3	13	43	3	7	11	3	P	89
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81	3	13	P	3	7	23	3	P	11	3	103	7	3	P	13	3	P	P	3	11
83	43	3	71	7	3	I	11	3	13	P	3	P	37	3	97	17	3	7	P	3
87	3	7	11	3	P	41	3	19	7	3	23	P	3	11	P	3	P	17	3	71
89	7	3	P	13	3	P	P	3	P	31	3	11	97	3	7	107	3	P	17	3
91	107	73	3	P	P	3	7	I	3	11	13	3	P	7	3	P	P	3	29	17
93	3	89	19	3	13	7	3	11	P	3	P	79	3	59	103	3	P	13	3	7
97	P	P	3	7	P	3	P	67	3	41	7	3	P	P	3	P	P	3	13	P
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03	11	3	7	p	3	p	17	3	113	7	3	11	23	3	73	37	3	41	p	3
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09	p	3	13	41	3	11	7	3	59	17	3	29	67	3	19	13	3	23	p	3
11	p	103	3	11	p	3	19	47	3	13	17	3	7	61	3	p	67	3	97	7
13	3	11	61	3	7	23	3	p	p	3	p	7	3	p	p	3	13	19	3	p
17	107	19	3	103	13	3	47	p	3	7	p	3	p	17	3	59	7	3	p	11
19	3	7	59	3	p	p	3	41	7	3	23	13	3	p	17	3	p	11	3	p
21	7	3	p	p	3	13	p	3	p	43	3	p	31	3	7	11	3	79	13	3
23	37	29	3	p	p	3	7	p	3	p	83	3	13	7	3	19	17	3	p	p
27	13	3	41	p	3	73	p	3	p	11	3	7	p	3	p	p	3	p	7	3
29	p	71	3	7	47	3	p	11	3	p	7	3	97	p	3	53	p	3	11	17
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33	p	3	43	11	3	p	p	3	7	109	3	37	p	3	11	7	3	p	71	3
37	3	67	23	3	p	p	3	p	37	3	11	p	3	7	43	3	19	p	3	p
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43	3	p	p	3	11	p	3	23	p	3	7	19	3	67	p	3	p	7	3	107
47	11	7	3	p	p	3	97	p	3	p	41	3	79	103	3	7	p	3	13	37
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51	p	3	p	113	3	p	7	3	p	p	3	109	101	3	p	p	3	19	11	3
53	13	p	3	31	97	3	p	p	3	19	p	3	7	13	3	103	11	3	83	7
57	p	3	53	7	3	p	p	3	83	p	3	23	11	3	13	47	3	7	101	3
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73	3	p	7	3	41	13	3	11	73	3	p	p	3	p	p	3	7	p	3	p
77	7	p	3	11	31	3	13	7	3	17	p	3	p	p	3	37	61	3	p	13
79	3	11	109	3	p	61	3	p	p	3	17	43	3	7	23	3	p	31	3	19
81	p	3	p	73	3	7	53	3	23	71	3	17	7	3	113	p	3	43	p	3
83	p	43	3	19	7	3	p	p	3	p	p	3	17	p	3	p	p	3	7	11
87	p	3	7	p	3	29	19	3	p	7	3	p	p	3	17	11	3	p	p	3
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91	3	23	31	3	43	p	3	7	p	3	p	11	3	p	7	3	13	p	3	p
93	17	3	p	37	3	p	7	3	53	11	3	p	41	3	p	31	3	17	23	3
97	3	p	17	3	7	11	3	p	p	3	31	7	3	89	p	3	11	p	3	17
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09	7	89	3	47	61	3	17	7	3	37	73	3	P	19	3	P	P	3	11	P
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57	P	107	3	11	7	3	P	13	3	31	37	3	P	17	3	97	P	3	7	P
59	3	11	71	3	109	29	3	P	23	3	7	P	3	P	13	3	P	7	3	P
61	P	3	7	P	3	P	P	3	13	7	3	131	41	3	19	17	3	P	53	3
63	P	7	3	P	101	3	19	P	3	P	113	3	61	97	3	3	17	3	P	11
67	P	3	P	13	3	P	7	3	101	19	3	P	31	3	P	11	3	109	17	3
69	P	19	3	P	43	3	79	41	3	71	13	3	7	11	3	P	P	3	107	7
71	3	103	53	3	7	73	3	31	P	3	43	7	3	29	P	3	41	13	3	P
73	P	3	P	7	3	P	P	3	47	11	3	13	23	3	101	P	3	7	61	3
77	3	7	41	3	P	11	3	19	7	3	P	89	3	P	P	3	11	29	3	P
79	7	3	73	11	3	59	13	3	P	P	3	41	37	3	7	P	3	23	19	3
81	13	11	3	P	P	3	7	97	3	P	19	3	11	7	3	P	P	3	P	41
83	3	P	19	3	53	7	3	13	P	3	11	P	3	P	P	3	P	P	3	7
87	P	P	3	7	P	3	11	P	3	P	7	3	59	P	3	43	23	3	31	P
89	3	P	7	3	11	53	3	103	P	3	23	P	3	P	P	3	7	P	3	P
91	P	3	11	37	3	47	P	3	7	13	3	P	P	3	P	7	3	P	P	3
93	7	P	3	13	P	3	P	7	3	P	P	3	P	P	3	73	13	3	29	19
97	P	3	43	19	3	7	59	3	61	23	3	29	7	3	P	P	3	13	11	3
99	17	97	3	23	7	3	P	107	3	89	P	3	P	127	3	P	11	3	7	41

	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199
01	47	23	3	P	P	3	11	P	3	41	P	3	7	P	3	P	17	3	P	7
03	3	43	109	3	7	P	3	59	P	7	31	7	3	97	P	3	P	17	3	13
07	11	19	3	P	79	3	23	13	3	3	83	3	P	43	3	P	7	3	29	17
09	3	7	131	3	41	83	3	53	7	7	P	97	3	P	13	3	P	P	3	43
11	7	3	P	P	3	107	37	3	13	P	3	29	P	3	7	109	3	23	11	3
13	P	59	3	P	P	3	7	P	3	P	P	3	P	7	3	13	11	3	P	P
17	43	3	P	13	3	P	P	3	31	P	3	7	11	3	P	29	3	P	7	3
19	37	P	3	7	113	3	43	P	3	P	7	3	P	P	3	131	23	3	P	P
21	3	P	7	3	13	P	3	97	11	3	23	P	3	139	P	3	7	13	3	11
23	67	3	P	73	3	P	11	3	7	127	3	13	47	3	P	7	3	11	43	3
27	3	P	11	3	P	97	3	61	67	3	53	31	3	7	P	3	19	P	3	P
29	11	3	P	P	3	7	13	3	19	23	3	11	7	3	P	59	3	109	79	3
31	13	P	3	23	7	3	31	P	3	11	P	3	P	13	3	P	67	3	7	19
33	3	P	P	3	P	43	3	11	37	3	7	19	3	P	P	3	29	7	3	31
37	17	7	3	11	103	3	P	41	3	29	P	3	P	61	3	7	73	3	83	P
39	3	11	13	3	P	P	3	7	P	3	79	P	3	83	7	3	41	P	3	127
41	P	3	17	P	3	P	7	3	83	13	3	P	71	3	P	P	3	19	P	3
43	P	P	3	13	P	3	103	P	3	19	137	3	7	23	3	P	13	3	P	7
47	P	3	71	7	3	17	29	3	47	P	3	41	19	3	P	11	3	7	89	3
49	P	P	3	59	19	3	17	P	3	7	43	3	P	11	3	113	7	3	23	P
51	3	7	P	3	P	13	3	17	7	3	P	11	3	37	53	3	43	P	3	71
53	7	3	P	P	3	P	23	3	17	11	3	107	13	3	7	P	3	P	P	3
57	3	67	P	3	P	7	3	P	109	3	17	P	3	13	P	3	11	23	3	7
59	P	3	19	11	3	67	47	3	P	P	3	7	P	3	11	P	3	P	7	3
61	P	11	3	7	P	3	P	73	3	67	7	3	11	19	3	31	P	3	P	P
63	3	41	7	3	37	19	3	29	13	3	11	P	3	17	P	3	7	P	3	P
67	7	37	3	P	59	3	11	7	3	13	23	3	P	107	3	17	71	3	P	41
69	3	P	P	3	11	31	3	137	P	3	P	29	3	7	P	3	13	53	3	19
71	17	3	11	P	3	7	P	3	113	61	3	19	7	3	P	P	3	17	31	3
73	11	17	3	19	7	3	71	P	3	P	P	3	P	P	3	23	103	3	7	P
77	P	3	7	17	3	13	19	3	43	7	3	127	37	3	P	P	3	P	11	3
79	101	7	3	P	17	3	P	89	3	P	P	3	13	P	3	7	11	3	103	P
81	3	P	101	3	P	17	3	7	79	3	P	P	3	P	7	3	P	131	3	13
83	13	3	47	31	3	P	7	3	23	41	3	P	11	3	P	P	3	73	59	3
87	3	13	P	3	7	P	3	P	11	3	P	7	3	P	13	3	P	47	3	11
89	P	3	P	7	3	29	11	3	13	17	3	31	P	3	P	19	3	7	P	3
91	79	P	3	53	11	3	P	19	3	7	17	3	101	P	3	11	7	3	P	P
93	3	7	11	3	P	P	3	P	7	3	61	17	3	11	101	3	47	P	3	P
97	P	31	3	P	53	3	7	P	3	11	13	3	23	7	3	P	P	3	101	P
99	3	P	29	3	13	7	3	11	P	3	71	73	3	19	17	3	P	13	3	7

	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219
01	3	P	P	3	23	13	3	127	11	3	P	P	3	7	P	3	P	P	3	11
03	83	3	89	79	3	7	11	3	71	P	3	47	7	3	17	P	3	11	P	3
07	3	P	11	3	P	P	3	P	P	3	7	P	3	11	P	3	17	7	3	19
09	11	3	7	23	3	P	37	3	P	7	3	11	127	3	79	137	3	17	113	3
11	P	7	3	19	P	3	P	139	3	11	P	3	P	101	3	7	P	3	17	P
13	3	P	17	3	137	73	3	7	13	3	P	43	3	P	7	3	P	P	3	17
17	37	P	3	1	17	3	53	P	3	13	P	3	7	P	3	P	P	3	P	7
19	3	11	P	3	7	17	3	P	109	3	P	7	3	P	P	3	13	37	3	23
21	P	3	73	7	3	P	17	3	47	P	3	P	P	3	31	P	3	7	P	3
23	P	P	3	P	13	3	41	17	3	7	P	3	19	P	3	P	7	3	139	11
27	7	3	113	P	3	13	P	3	59	17	3	37	P	3	7	11	3	P	13	3
29	P	P	3	29	31	3	7	19	3	P	17	3	13	7	3	P	43	3	83	P
31	3	41	P	3	P	7	3	P	37	3	P	11	3	83	29	3	97	31	3	7
33	13	3	P	P	3	P	47	3	83	11	3	7	17	3	P	61	3	103	7	3
37	3	13	7	3	107	11	3	89	67	3	109	23	3	19	13	3	7	P	3	P
39	29	3	37	11	3	19	P	3	7	P	3	P	67	3	11	7	3	P	P	3
41	7	11	3	P	P	3	P	7	3	43	53	3	11	P	3	13	17	3	P	37
43	3	P	31	3	P	P	3	P	19	3	11	P	3	7	41	3	23	17	3	P
47	P	P	3	P	7	3	11	P	3	P	13	3	P	P	3	29	P	3	7	17
49	3	P	P	3	11	P	3	P	P	3	7	P	3	37	89	3	P	7	3	47
51	P	3	7	47	3	P	107	3	29	7	3	13	79	3	19	23	3	P	P	3
53	11	7	3	P	113	3	19	P	3	23	37	3	53	131	3	7	59	3	13	29
57	31	3	47	P	3	61	7	3	P	19	3	P	29	3	43	P	3	P	11	3
59	13	19	3	P	41	3	73	P	3	P	P	3	7	13	3	P	11	3	P	7
61	3	P	P	3	7	29	3	13	23	3	P	7	3	41	11	3	P	4[illegible]	3	P
63	P	3	23	7	3	P	P	3	31	P	3	P	11	3	13	P	3	7	P	3
67	3	7	13	3	97	131	3	19	7	3	P	61	3	23	P	3	47	P	3	11
69	7	3	P	P	3	67	11	3	41	13	3	P	P	3	7	P	3	11	19	3
71	P	23	3	13	11	3	7	P	3	67	19	3	89	7	3	11	13	3	P	127
73	3	P	11	3	59	7	3	P	P	3	13	31	3	11	109	3	P	P	3	7
77	17	P	3	7	P	3	23	79	3	11	7	3	P	P	3	P	53	3	131	P
79	3	17	7	3	P	13	3	11	P	3	107	P	3	P	47	3	7	29	3	31
81	43	3	17	89	3	11	P	3	7	P	3	59	13	3	P	7	3	23	P	3
83	7	P	3	11	P	3	13	7	3	3	29	3	P	P	3	113	P	3	79	13
87	53	3	P	19	3	7	137	3	P	31	3	P	7	3	P	P	3	P	43	3
89	P	13	3	P	7	3	17	P	3	139	P	3	61	73	3	P	23	3	7	11
91	3	61	103	3	31	59	3	17	13	3	7	P	3	P	P	3	109	7	3	P
93	71	3	7	P	3	P	P	3	17	7	3	P	107	3	P	11	3	19	P	3
97	3	19	P	3	103	43	3	7	P	3	17	11	3	P	7	3	13	71	3	P
99	101	3	53	P	3	P	7	3	P	11	3	17	19	3	P	P	3	P	61	3

(b)

	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
01	7	3	149	29	3	P	97	3	151	P	3	13	P	3	7	71	3	137	P	3
03	P	23	3	P	43	3	7	73	3	37	P	3	3	7	3	19	P	3	13	11
07	59	3	53	P	3	71	13	3	P	P	3	7	23	3	89	11	3	151	7	3
09	13	P	3	7	P	3	23	P	3	31	7	3	P	11	3	P	P	3	29	P
11	3	P	7	3	73	P	3	13	P	3	P	11	3	P	41	3	7	131	3	P
13	P	3	97	53	3	47	P	3	7	11	3	29	139	3	13	7	3	23	P	3
17	3	17	13	3	29	11	3	P	P	3	P	P	3	7	P	3	11	37	3	P
19	97	3	17	11	3	7	P	3	19	13	3	61	7	3	11	29	3	P	P	3
21	19	11	3	13	7	3	P	P	3	P	P	3	11	P	3	43	13	3	7	19
23	3	P	71	3	17	101	3	31	29	3	7	19	3	83	59	3	P	7	3	47
27	P	7	3	83	41	3	11	P	3	101	P	3	P	P	3	7	P	3	P	71
29	3	P	P	3	11	13	3	7	37	3	P	101	3	41	7	3	P	61	3	P
31	P	3	11	137	3	P	7	3	17	23	3	P	13	3	P	P	3	19	P	3
33	11	P	3	23	P	3	13	127	3	17	31	3	7	P	3	101	P	3	P	7
37	P	3	37	7	3	31	P	3	41	P	3	17	19	3	23	P	3	7	11	3
39	P	13	3	89	19	3	P	P	3	7	P	3	17	P	3	P	7	3	31	37
41	3	7	23	3	P	P	3	P	7	3	P	73	3	17	11	3	47	P	3	89
43	7	3	13	P	3	P	P	3	53	P	3	P	11	3	7	13	3	P	113	3
47	3	P	P	3	P	7	3	23	11	3	19	79	3	37	P	3	13	P	3	7
49	17	3	19	P	3	P	11	3	73	53	3	7	67	3	131	P	3	11	7	3
51	P	17	3	7	11	3	P	P	3	59	7	3	P	19	3	11	67	3	17	43
53	3	P	7	3	P	19	3	61	P	3	P	13	3	11	47	3	7	P	3	17
57	7	P	3	79	17	3	139	7	3	11	P	3	13	P	3	P	41	3	P	P
59	3	P	P	3	37	17	3	11	P	3	P	P	3	7	P	3	59	23	3	13
61	13	3	113	59	3	7	17	3	P	P	3	19	7	3	29	P	3	P	107	3
63	P	37	3	11	7	3	131	13	3	P	P	3	43	61	3	P	P	3	7	31
67	P	3	7	P	3	P	19	3	13	7	3	P	53	3	31	P	3	P	29	3
69	29	7	3	P	P	3	P	P	3	103	17	3	P	P	3	7	P	3	P	11
71	3	P	P	3	23	P	3	7	P	3	P	17	3	P	7	3	P	11	3	P
73	P	3	P	13	3	P	7	3	89	P	3	P	17	3	P	11	3	P	P	3
77	3	67	P	3	7	107	3	P	P	3	47	7	3	97	17	3	P	13	3	P
79	P	3	P	7	3	67	P	3	137	11	3	13	P	3	53	17	3	7	P	3
81	71	41	3	P	P	3	37	11	3	7	P	3	31	193	3	P	7	3	11	P
83	3	7	P	3	P	11	3	P	7	3	41	97	3	67	23	3	11	17	3	29
87	13	11	3	61	113	3	7	P	3	127	P	3	11	7	3	103	P	3	P	17
89	3	P	31	3	43	7	3	13	47	3	11	P	3	19	83	3	P	P	3	7
91	P	3	P	P	3	19	P	3	11	83	3	7	P	3	13	31	3	37	7	3
93	P	P	3	7	83	3	11	23	3	P	7	3	P	149	3	P	19	3	P	P
97	19	3	11	P	3	59	P	3	7	13	3	P	P	3	P	7	3	53	23	3
99	7	79	3	13	149	3	P	7	3	109	P	3	23	P	3	P	13	3	P	103

	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259
01	P	7	3	19	13	3	73	17	3	37	23	3	11	P	3	7	P	3	[illegible]	59
03	3	P	P	3	23	107	3	7	17	3	11	13	3	P	7	3	P	P	[illegible]	P
07	P	P	3	109	P	3	11	31	3	P	17	3	7	P	3	23	29	3	131	7
09	3	P	43	3	7	P	3	P	P	3	89	7	3	P	P	3	P	47	3	13
11	3	3	11	7	3	127	P	3	43	29	3	P	17	3	P	97	3	7	5	3
13	11	P	3	41	P	3	151	13	3	7	P	3	19	17	3	31	7	3	83	P
17	7	3	61	P	3	P	103	3	13	P	3	P	151	3	7	17	3	P	11	3
19	P	89	3	83	P	3	7	19	3	P	127	3	P	7	3	13	11	3	P	P
21	3	P	53	3	P	7	3	59	P	3	131	P	3	P	11	3	P	17	3	7
23	P	3	P	13	3	137	P	3	103	P	3	7	11	3	P	P	3	29	7	3
27	3	23	7	3	13	P	3	79	11	3	29	P	3	19	47	3	7	13	3	11
29	P	3	P	P	3	19	11	3	7	97	3	13	P	3	59	7	3	11	23	3
31	7	59	3	29	11	3	P	7	3	107	P	3	23	73	3	11	19	3	13	P
33	3	P	11	3	53	P	3	P	19	3	P	41	3	7	29	3	P	P	3	P
37	13	P	3	P	7	3	71	29	3	11	P	3	P	13	3	P	31	3	7	37
39	3	101	P	3	P	53	3	11	59	3	7	23	3	P	P	3	P	7	3	P
41	29	3	7	101	3	11	41	3	P	7	3	31	43	3	13	P	3	P	P	3
43	P	7	3	11	P	3	19	109	3	P	79	3	P	P	3	7	P	3	43	P
47	139	3	P	97	3	P	7	3	P	13	3	P	P	3	P	59	3	P	P	3
49	P	19	3	13	23	3	157	P	3	61	37	3	7	P	3	29	13	3	P	7
51	3	P	P	3	7	P	3	53	P	3	13	7	3	101	31	3	113	11	3	P
53	67	3	79	7	3	43	89	3	29	P	3	P	P	3	P	11	3	7	103	3
57	3	7	127	3	37	13	3	19	7	3	P	11	3	P	P	3	P	43	3	101
59	7	3	17	P	3	41	P	3	P	11	3	139	13	3	7	61	3	P	19	3
61	P	37	3	17	61	3	7	11	3	109	19	3	P	7	3	P	67	3	11	13
63	3	73	19	3	17	7	3	P	23	3	71	P	3	13	P	3	11	P	3	7
67	41	11	3	7	43	3	17	P	3	P	7	3	11	P	3	37	P	3	P	23
69	3	P	7	3	P	79	3	17	13	3	11	P	3	23	P	3	7	73	3	P
71	P	3	13	P	3	P	P	3	7	P	3	P	37	3	P	7	3	P	41	3
73	7	23	3	P	P	3	11	7	3	13	P	3	127	P	3	107	P	3	23	19
77	P	3	11	19	3	7	P	3	P	P	3	17	7	3	73	P	3	149	113	3
79	11	P	3	P	7	3	23	71	3	P	31	3	17	41	3	P	P	3	7	83
81	3	P	P	3	P	47	3	P	139	3	7	13	3	17	83	3	61	7	3	P
83	P	3	7	37	3	13	P	3	149	7	3	P	131	3	17	P	3	19	11	3
87	3	19	149	3	47	23	3	7	41	3	P	89	3	53	7	3	17	107	3	13
89	13	3	107	29	3	67	7	3	P	P	3	P	11	3	71	P	3	17	P	3
91	P	17	3	P	19	3	P	13	3	67	11	3	7	P	3	157	23	3	17	7
93	3	13	17	3	7	P	3	P	11	3	13	7	3	67	13	3	P	P	3	11
97	P	P	3	31	11	3	P	137	3	7	P	3	41	109	3	11	7	3	19	P
99	3	7	11	3	P	17	3	P	7	3	19	113	3	11	43	3	31	P	3	P

	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279
01	3	43	7	3	17	P	3	P	P	3	13	41	3	23	11	3	7	P	3	P
03	P	3	P	29	3	17	37	3	7	P	3	P	11	3	67	7	3	13	P	3
07	3	P	73	3	P	13	3	17	11	3	113	P	3	7	P	3	19	103	3	11
09	31	3	P	P	3	7	11	3	17	71	3	P	7	3	P	P	3	11	P	3
11	19	P	3	83	7	3	13	P	3	17	P	3	P	31	3	11	P	3	7	13
13	3	P	11	3	61	P	3	P	P	3	7	19	3	11	79	3	53	7	3	103
17	P	7	3	P	P	3	43	P	3	11	P	3	17	59	3	7	P	3	P	P
19	3	P	157	3	29	23	3	7	13	3	41	47	3	17	7	3	71	53	3	P
21	P	3	13	P	3	11	7	3	P	P	3	37	163	3	17	13	3	19	43	3
23	53	151	3	11	P	3	79	P	3	13	61	3	7	89	3	17	23	3	P	7
27	17	3	P	7	3	41	P	3	139	P	3	P	19	3	P	P	3	7	P	3
29	P	17	3	113	13	3	31	P	3	7	151	3	73	P	3	P	7	3	17	11
31	3	7	17	3	P	43	3	P	7	3	P	13	3	151	P	3	P	11	3	17
33	7	3	37	17	3	13	P	3	P	23	3	43	31	3	7	11	3	P	13	3
37	3	59	P	3	P	7	3	P	47	3	19	11	3	P	P	3	29	P	3	7
39	13	3	19	P	3	P	17	3	P	11	3	7	P	3	23	P	3	P	7	3
41	P	P	3	7	137	3	P	11	3	29	7	3	P	19	3	P	131	3	11	P
43	3	13	7	3	31	11	3	47	17	3	P	P	3	37	13	3	7	P	3	P
47	7	11	3	P	53	3	P	7	3	P	17	3	11	23	3	13	P	3	P	P
49	3	79	P	3	P	139	3	23	P	3	11	17	3	7	P	3	43	P	3	19
51	109	3	P	13	3	7	29	3	11	P	3	19	7	3	97	P	3	P	P	3
53	P	P	3	19	7	3	11	31	3	P	13	3	P	17	3	59	P	3	7	P
57	71	3	7	P	3	P	19	3	107	7	3	13	97	3	P	17	3	41	89	3
59	11	7	3	43	P	3	53	P	3	P	P	3	P	109	3	7	17	3	13	73
61	3	P	P	3	47	P	3	7	P	3	P	157	3	P	7	3	139	17	3	P
63	67	3	P	41	3	101	7	3	P	59	3	23	137	3	29	43	3	P	11	3
67	3	137	P	3	7	31	3	13	67	3	P	7	3	P	11	3	73	P	3	P
69	131	3	109	7	3	163	P	3	97	149	3	101	11	3	13	19	3	7	29	3
71	29	P	3	P	103	3	149	19	3	7	11	3	P	101	3	79	7	3	47	83
73	3	7	13	3	23	P	3	41	7	3	P	29	3	31	83	3	P	P	3	11
77	89	P	3	13	11	3	7	P	3	53	P	3	P	7	3	11	13	3	61	101
79	3	47	11	3	P	7	3	61	P	3	13	P	3	11	P	3	89	P	3	7
81	11	3	41	23	3	19	P	3	P	P	3	7	P	3	P	P	3	13	7	3
83	P	P	3	7	71	3	P	P	3	11	7	3	P	139	3	P	19	3	P	P
87	19	3	97	P	3	11	P	3	7	P	3	31	13	3	P	7	3	37	79	3
89	7	P	3	11	P	3	13	7	3	137	103	3	29	61	3	47	P	3	167	13
91	3	11	61	3	59	P	3	73	P	3	P	P	3	7	37	3	P	P	3	23
93	97	3	P	P	3	7	P	3	P	P	3	71	7	3	19	41	3	P	P	3
97	3	17	P	3	P	P	3	127	13	3	7	P	3	P	31	3	P	7	3	P
99	P	3	7	P	3	67	P	3	37	7	3	59	P	3	107	11	3	P	23	3

INCOMPOSITS.

	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299
01	P	3	P	7	3	11	37	3	83	P	3	P	P	3	P	P	3	7	17	3
03	41	157	3	11	P	3	P	P	3	7	13	3	19	P	3	163	7	3	P	17
07	7	3	67	P	3	29	P	3	P	137	3	13	P	3	7	19	3	61	41	3
09	37	P	3	P	P	3	7	19	3	P	P	3	P	7	3	23	29	3	13	11
11	3	P	P	3	P	7	3	P	47	3	67	43	3	P	P	3	P	11	3	7
13	109	3	89	23	3	P	13	3	P	29	3	7	131	3	67	11	3	43	7	3
17	3	31	7	3	157	P	3	13	P	3	P	11	3	19	23	3	7	P	3	P
19	P	3	P	P	3	19	P	3	7	11	3	37	61	3	13	7	3	113	P	3
21	7	61	3	127	97	3	P	7	3	P	P	3	P	109	3	53	19	3	11	P
23	3	P	13	3	43	11	3	P	19	3	P	P	3	7	P	3	11	P	3	23
27	P	11	3	13	7	3	P	23	3	P	P	3	11	P	3	P	13	3	7	P
29	3	23	P	3	P	47	3	P	127	3	7	P	3	139	P	3	P	7	3	173
31	P	3	7	41	3	103	P	3	11	7	3	P	P	3	19	P	3	13	23	3
33	17	7	3	29	P	3	11	59	3	P	P	3	23	P	3	7	P	3	P	37
37	23	3	11	43	3	P	7	3	P	19	3	P	13	3	P	P	3	131	P	3
39	11	19	3	17	P	3	13	29	3	43	71	3	7	P	3	109	107	3	53	7
41	3	107	31	3	7	P	3	41	151	3	113	7	3	13	59	3	P	P	3	79
43	29	3	61	7	3	17	P	3	P	103	3	151	P	3	P	31	3	7	11	3
47	3	7	47	3	P	P	3	17	7	3	31	P	3	P	11	3	23	151	3	P
49	7	3	13	P	3	P	P	3	17	P	3	103	11	3	7	13	3	71	19	3
51	P	P	3	P	23	3	7	P	3	13	11	3	P	7	3	29	149	3	P	61
53	3	47	19	3	37	7	3	P	11	3	17	P	3	149	P	3	13	P	3	7
57	P	37	3	7	11	3	P	149	3	23	7	3	17	31	3	11	47	3	73	29
59	3	29	7	3	149	P	3	P	P	3	P	13	3	11	89	3	7	P	3	P
61	11	3	59	79	3	13	P	3	7	P	3	11	29	3	17	7	3	P	13	3
63	7	P	3	113	P	3	P	7	3	11	P	3	13	P	3	17	P	3	P	19
67	13	3	23	19	3	7	109	3	P	83	3	P	7	3	79	P	3	17	P	3
69	P	17	3	11	7	3	P	13	3	59	41	3	P	43	3	P	P	3	7	23
71	3	11	17	3	71	P	3	P	P	3	7	31	3	23	13	3	P	7	3	17
73	67	3	7	17	3	P	53	3	13	7	3	P	73	3	P	P	3	19	P	3
77	3	19	P	3	P	17	3	7	67	3	P	163	3	29	7	3	59	11	3	31
79	43	3	P	13	3	P	7	3	P	P	3	P	19	3	41	11	3	97	P	3
81	P	P	3	101	19	3	23	17	3	73	13	3	7	11	3	P	67	3	P	7
83	3	P	P	3	7	101	3	107	17	3	127	7	3	P	P	3	P	13	3	P
87	P	71	3	P	61	3	P	11	3	7	17	3	P	P	3	P	7	3	11	157
89	3	7	P	3	31	11	3	P	7	3	19	17	3	P	37	3	11	P	3	P
91	7	3	19	11	3	P	13	3	167	53	3	P	17	3	7	127	3	31	71	3
93	13	11	3	P	P	3	7	P	3	79	47	3	11	7	3	101	23	3	167	89
97	P	3	P	73	3	P	P	3	11	107	3	7	P	3	13	17	3	83	7	3
99	P	163	3	7	P	3	11	31	3	47	7	3	83	P	3	P	17	3	29	131

	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319
01	19	31	3	157	7	3	71	11	3	13	29	3	41	113	3	17	P	3	7	19
03	3	P	P	3	P	11	3	P	P	3	7	19	3	23	31	3	11	7	3	61
07	37	7	3	P	13	3	127	P	3	31	101	3	11	P	3	7	P	3	17	P
09	3	P	17	3	47	P	3	7	P	3	11	13	3	131	7	3	73	37	3	17
11	P	3	P	17	3	13	7	3	11	P	3	53	23	3	101	P	3	19	13	3
13	P	P	3	P	17	3	11	P	3	19	P	3	7	173	3	P	101	3	29	7
17	13	3	11	7	3	P	17	3	P	43	3	29	19	3	89	P	3	7	P	3
19	11	P	3	P	19	3	67	13	3	7	P	3	P	P	3	43	7	3	47	59
21	3	7	47	3	29	23	3	31	7	3	67	P	3	P	13	3	103	P	3	137
23	7	3	P	P	3	131	113	3	13	17	3	P	P	3	7	29	3	P	11	3
27	3	47	167	3	P	7	3	P	29	3	19	17	3	P	11	3	P	P	3	7
29	P	3	19	13	3	P	109	3	P	157	3	7	11	3	53	41	3	P	7	3
31	59	29	3	7	P	3	P	79	3	P	7	3	P	17	3	P	47	3	139	37
33	3	P	7	3	13	19	3	73	11	3	P	163	3	P	17	3	7	13	3	11
37	7	P	3	23	11	3	P	7	3	P	41	3	P	P	3	11	17	3	13	109
39	3	P	11	3	61	P	3	59	P	3	P	P	3	7	149	3	29	17	3	19
41	11	3	P	P	3	7	13	3	P	P	3	11	7	3	23	P	3	P	17	3
43	13	43	3	19	7	3	P	71	3	11	37	3	157	13	3	P	P	3	7	17
47	P	3	7	P	3	11	19	3	109	7	3	P	P	3	13	P	3	53	P	3
49	151	7	3	11	P	3	P	97	3	P	61	3	P	23	3	7	P	3	P	43
51	3	11	13	3	37	137	3	7	P	3	P	P	3	107	7	3	31	P	3	89
53	41	3	P	127	3	P	7	3	P	13	3	P	P	3	71	139	3	113	53	3
57	3	53	79	3	7	P	3	P	59	3	13	7	3	P	83	3	P	11	3	P
59	P	3	P	7	3	P	23	3	P	83	3	P	P	3	163	11	3	7	P	3
61	23	P	3	97	83	3	P	19	3	7	89	3	43	11	3	37	7	3	151	31
63	3	7	53	3	41	13	3	P	7	3	P	11	3	79	73	3	P	23	3	P
67	107	71	3	P	P	3	7	11	3	173	47	3	P	7	3	P	P	3	11	13
69	3	P	P	3	P	7	3	29	P	3	P	71	3	13	P	3	11	P	3	7
71	P	3	P	11	3	19	P	3	P	P	3	7	P	3	11	131	3	P	7	3
73	17	11	3	7	31	3	37	P	3	47	7	3	11	137	3	P	19	3	P	P
77	19	3	13	37	3	P	P	3	7	P	3	P	P	3	P	7	3	43	127	3
79	7	103	3	17	29	3	11	7	3	13	P	3	31	P	3	23	79	3	71	113
81	3	P	107	3	11	53	3	P	P	3	P	P	3	7	P	3	13	61	3	P
83	67	3	11	23	3	7	61	3	89	P	3	P	7	3	19	P	3	37	P	3
87	3	P	31	3	43	73	3	17	67	3	7	13	3	P	23	3	P	7	3	29
89	P	3	7	P	3	13	P	3	17	7	3	P	67	3	P	31	3	83	11	3
91	P	7	3	P	P	3	47	41	3	17	P	3	13	P	3	7	11	3	P	P
93	3	109	P	3	P	P	3	7	P	3	17	P	3	P	7	3	41	P	3	13
97	P	P	3	113	P	3	P	13	3	139	11	3	7	P	3	19	29	3	167	7
99	3	13	41	3	7	37	3	19	11	3	137	7	3	17	13	3	[illegible]	P	3	11

	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339
01	3	47	13	3	P	7	3	53	P	3	61	79	3	P	127	3	P	67	3	7
03	P	3	P	P	3	P	P	3	P	13	3	7	P	3	P	P	3	P	7	3
07	3	97	7	3	23	P	3	P	53	3	13	P	3	19	11	3	7	37	3	41
09	P	3	31	P	3	19	P	3	7	P	3	113	11	3	47	7	3	13	P	3
11	7	163	3	79	P	3	P	7	3	P	11	3	P	P	3	23	19	3	P	P
13	3	17	P	3	P	13	3	P	11	3	P	P	3	7	P	3	P	P	3	11
17	101	P	3	17	7	3	13	P	3	P	137	3	59	P	3	11	P	3	7	13
19	3	P	11	3	17	31	3	P	37	3	7	P	3	11	23	3	P	7	3	107
21	11	3	7	P	3	17	P	3	23	7	3	11	139	3	19	P	3	P	31	3
23	31	7	3	P	P	3	17	43	3	11	P	3	P	47	3	7	P	3	149	P
27	P	3	13	P	3	11	7	3	17	19	3	157	149	3	P	13	3	29	P	3
29	P	19	3	11	P	3	67	23	3	13	P	3	7	P	3	P	P	3	P	7
31	3	11	167	3	7	P	3	71	P	3	17	7	3	P	101	3	13	89	3	P
33	103	3	P	7	3	P	P	3	P	P	3	17	167	3	67	P	3	7	23	3
37	3	7	P	3	163	P	3	19	7	3	P	13	3	17	29	3	P	11	3	P
39	7	3	103	73	3	13	127	3	P	P	3	31	43	3	7	11	3	P	13	3
41	179	P	3	P	P	3	7	29	3	P	19	3	13	7	3	17	P	3	43	P
43	3	P	19	3	P	7	3	137	P	3	173	11	3	P	53	3	17	41	3	7
47	73	17	3	7	71	3	P	11	3	47	7	3	P	P	3	P	P	3	11	83
49	3	13	7	3	37	11	3	P	107	3	P	P	3	P	13	3	7	P	3	17
51	P	3	P	11	3	43	103	3	7	83	3	P	41	3	11	7	3	P	P	3
53	7	11	3	P	17	3	P	7	3	31	P	3	11	P	3	13	73	3	97	19
57	P	3	P	13	3	7	17	3	11	P	3	71	7	3	P	23	3	P	P	3
59	P	P	3	P	7	3	11	17	3	23	13	3	97	P	3	37	97	3	7	29
61	3	29	P	3	11	P	3	181	17	3	7	P	3	73	P	3	41	7	3	P
63	P	3	7	P	3	P	89	3	59	7	3	13	29	3	109	P	3	19	P	3
67	3	19	41	3	P	29	3	7	23	3	43	17	3	61	7	3	131	P	3	P
69	P	3	23	P	3	P	7	3	P	P	3	41	17	3	P	P	3	P	11	3
71	13	53	3	P	19	3	37	P	3	P	P	3	7	13	3	59	11	3	P	7
73	3	P	59	3	7	P	3	13	71	3	P	7	3	23	11	3	151	P	3	53
77	P	23	3	P	47	3	41	73	3	7	11	3	107	P	3	P	7	3	19	61
79	3	7	13	3	P	P	3	P	7	3	19	P	3	29	P	3	P	17	3	11
81	7	3	19	P	3	31	11	3	131	13	3	P	23	3	7	P	3	11	17	3
83	P	P	3	13	11	3	7	P	3	P	P	3	83	7	3	11	13	3	31	17
87	11	3	83	139	3	P	P	3	P	P	3	7	P	3	P	P	3	13	7	3
89	P	P	3	7	53	3	97	P	3	11	7	3	P	173	3	P	59	3	P	41
91	3	P	7	3	P	13	3	11	31	3	P	P	3	P	107	7	7	P	3	19
93	67	3	43	29	3	11	P	3	7	P	3	19	13	3	P	3	3	47	P	3
97	3	11	71	3	P	37	3	P	67	3	23	89	3	7	19	3	31	P	3	P
99	P	3	P	179	3	7	19	3	167	P	3	P	7	3	139	P	3	73	109	3

	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359
01	11	3	23	P	3	P	7	3	13	17	3	11	P	3	P	131	3	19	P	3
03	37	67	3	P	P	3	P	P	3	11	17	3	7	43	3	13	P	3	P	7
07	31	3	79	7	3	11	P	3	P	67	3	P	17	3	P	P	3	7	61	3
09	71	23	23	11	19	3	53	61	3	7	13	3	137	17	3	P	7	3	P	149
11	3	7	P	3	13	P	3	103	7	3	157	P	3	P	17	3	149	13	3	P
13	7	3	P	P	3	P	P	3	31	P	3	13	23	3	7	17	3	71	59	3
17	3	109	P	3	127	7	3	149	37	3	19	P	3	P	107	3	P	11	3	7
19	P	3	19	P	3	P	13	3	P	P	3	7	41	3	P	11	3	23	7	3
21	13	149	3	7	P	3	89	P	3	47	7	3	P	11	3	P	179	3	113	17
23	3	P	7	3	29	19	3	13	97	3	P	11	3	P	P	3	7	139	3	P
27	7	P	3	P	173	3	31	7	3	53	P	3	P	P	3	P	23	3	11	37
29	3	P	13	3	P	11	3	P	29	3	23	P	3	7	71	3	11	P	3	19
31	P	3	P	11	3	7	P	3	61	13	3	19	7	3	11	P	3	P	P	3
33	P	11	3	13	7	3	59	47	3	181	53	3	11	89	3	P	13	3	7	P
37	101	3	7	P	3	P	19	3	11	7	3	41	167	3	P	P	3	13	P	3
39	P	7	3	23	P	3	11	P	3	P	37	3	131	P	3	7	157	3	P	83
41	3	P	97	3	11	13	3	7	P	3	67	P	3	59	7	3	29	103	3	127
43	59	3	11	61	3	P	7	3	P	83	3	113	13	3	23	P	3	31	73	3
47	3	P	23	3	7	179	3	P	P	3	101	7	3	13	P	3	43	P	3	103
49	79	3	29	7	3	P	P	3	P	P	3	P	101	3	P	19	3	7	11	3
51	17	13	3	P	47	3	P	19	3	7	P	3	P	23	3	73	7	3	P	P
53	3	7	P	3	131	109	3	23	7	3	P	P	3	P	11	3	101	P	3	157
57	P	P	3	17	P	3	7	P	3	13	11	3	P	7	3	31	181	3	23	41
59	3	P	P	3	17	7	3	P	11	3	P	P	3	19	59	3	13	P	3	7
61	P	3	P	P	3	17	11	3	71	P	3	7	37	3	P	43	3	11	7	3
63	23	127	3	7	11	3	17	P	3	P	7	3	179	P	3	11	19	3	P	P
67	11	3	P	P	3	13	P	3	7	73	3	11	P	3	29	7	3	47	13	3
69	7	47	3	P	P	3	37	7	3	11	P	3	13	113	3	P	53	3	P	P
71	3	P	43	3	P	181	3	11	P	3	17	P	3	7	79	3	P	P	3	13
73	13	3	P	37	3	7	P	3	43	41	3	17	7	3	19	P	3	83	29	3
77	3	11	151	3	23	71	3	83	P	3	7	29	3	17	13	3	P	7	3	P
79	53	3	7	31	3	151	P	3	13	7	3	127	P	3	17	47	3	37	P	3
81	173	7	3	P	29	3	79	P	3	P	P	3	P	P	3	7	31	3	53	11
83	3	P	P	3	P	P	3	7	P	3	P	151	3	41	7	3	17	11	3	P
87	89	17	3	137	P	3	P	43	3	59	13	3	7	11	3	19	127	3	17	7
89	7	179	17	3	7	P	3	19	139	3	3	7	3	43	23	3	89	13	3	17
91	73	3	53	7	3	P	113	3	23	11	3	13	P	3	P	P	3	7	19	3
93	103	31	3	163	17	3	P	11	3	7	19	3	29	P	3	P	7	3	11	P
97	7	3	P	11	3	29	13	3	P	79	3	61	47	3	7	P	3	P	P	3
99	13	11	3	41	P	3	7	17	3	31	P	3	11	7	3	97	29	3	P	P

	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379
01	7	13	3	31	89	3	17	7	3	P	163	3	P	11	3	P	19	3	103	151
03	3	79	41	3	59	173	3	17	13	3	P	11	3	7	113	3	31	37	3	29
07	P	P	3	P	7	3	P	11	3	13	23	3	29	P	3	P	P	3	7	P
09	3	P	P	3	23	11	3	P	P	3	7	43	3	P	P	3	11	7	3	167
11	P	3	7	11	3	29	31	3	131	7	3	17	127	3	11	P	3	43	P	3
13	P	7	3	P	13	3	19	P	3	P	P	3	11	P	3	7	29	3	P	31
17	P	3	P	23	3	13	7	3	11	19	3	P	P	3	17	P	3	P	13	3
19	181	19	3	P	79	3	11	73	3	P	P	3	7	67	3	17	P	3	59	7
21	3	41	29	3	7	59	3	P	P	3	P	7	3	P	23	3	17	67	3	13
23	13	3	11	7	3	P	53	3	23	P	3	P	P	3	P	157	3	7	109	3
27	3	7	17	3	73	P	3	19	7	3	61	137	3	163	13	3	191	31	3	17
29	7	3	P	17	3	P	P	3	13	P	3	107	59	3	7	P	3	29	11	3
31	137	P	3	47	17	3	7	23	3	P	19	3	31	7	3	13	11	3	P	83
33	3	23	19	3	P	7	3	109	P	3	29	71	3	37	11	3	P	97	3	7
37	P	P	3	7	83	3	P	17	3	43	7	3	23	P	3	P	61	3	157	59
39	3	71	7	3	13	61	3	P	11	3	P	P	3	P	29	3	7	13	3	11
41	23	3	P	P	3	P	11	3	7	17	3	13	167	3	P	7	3	11	79	3
43	7	47	3	P	11	3	P	7	3	P	17	3	P	107	3	11	P	3	13	19
47	11	3	67	19	3	7	13	3	P	P	3	11	7	3	P	P	3	P	P	3
49	13	37	3	163	7	3	67	P	3	11	P	3	193	13	3	P	P	3	7	137
51	3	P	P	3	P	P	3	11	43	3	7	97	3	41	17	3	23	7	3	P
53	31	3	7	P	3	11	P	3	137	7	3	53	P	3	13	17	3	19	P	3
57	3	11	13	3	P	139	3	7	P	3	P	73	3	P	7	3	P	17	3	P
59	107	3	101	103	3	P	7	3	29	13	3	P	19	3	47	23	3	61	17	3
61	P	P	3	13	19	3	61	P	3	23	P	3	7	P	3	P	13	3	P	7
63	3	29	P	3	7	P	3	97	191	3	13	7	3	P	P	3	P	11	3	P
67	P	59	3	41	P	3	37	P	3	7	101	3	83	11	3	P	7	3	19	P
69	3	7	P	3	P	13	3	83	7	3	19	11	3	P	89	3	139	179	3	43
71	7	3	19	37	3	P	P	3	P	11	3	P	13	3	7	P	3	107	P	3
73	P	61	3	P	P	3	7	11	3	P	131	3	P	7	3	P	101	3	11	13
77	43	3	P	11	3	79	P	3	P	103	3	7	P	3	11	53	3	37	7	3
79	109	11	3	7	P	3	43	P	3	P	7	3	11	P	3	P	41	3	P	163
81	3	97	7	3	191	157	3	P	13	3	11	P	3	29	37	3	7	P	3	19
83	P	3	13	P	3	P	P	3	7	31	3	19	23	3	P	13	3	P	43	3
87	3	P	131	3	11	P	3	P	P	3	P	41	3	7	19	3	13	29	3	P
89	151	3	11	P	3	7	19	3	37	47	3	P	7	3	P	P	3	23	P	7
91	11	P	3	151	7	3	P	P	3	71	29	3	89	139	3	P	P	3	7	P
93	3	17	P	3	P	23	3	P	79	3	7	13	3	61	P	3	P	7	3	P
97	P	7	3	17	P	3	P	31	3	P	P	3	13	P	3	7	11	3	P	P
99	3	53	P	3	17	P	3	7	P	3	23	P	3	149	7	3	P	P	3	23

	380	381	382	383	384	385	386	387	388	389
01	3	7	P	3	11	P	3	13	7	3
03	7	3	11	P	3	139	P	3	P	P
07	3	53	13	3	193	7	3	P	151	3
09	191	3	19	29	3	97	P	3	197	13
11	P	23	3	7	71	3	P	P	3	167
13	3	P	7	3	107	19	3	P	37	3
17	7	47	3	P	41	3	23	7	3	P
19	3	P	P	3	103	13	3	31	11	3
21	193	3	37	P	3	7	11	3	P	P
23	47	67	3	19	7	3	13	P	3	P
27	11	3	7	P	3	59	19	3	41	7
29	17	7	3	P	83	3	P	P	3	11
31	3	17	P	3	P	53	3	7	13	3
33	73	3	13	P	3	11	7	3	P	P
37	3	11	P	3	7	89	3	P	71	3
39	P	3	P	7	3	17	P	3	P	23
41	109	43	3	23	13	3	17	19	3	7
43	3	7	167	3	37	P	3	17	7	3
47	P	37	3	31	P	3	7	P	3	17
49	3	P	23	3	P	7	3	P	53	3
51	13	3	29	P	3	19	P	3	P	11
53	P	P	3	7	P	3	P	11	3	P
57	19	3	67	11	3	P	29	3	7	163
59	7	11	3	89	P	3	67	7	3	P
61	3	31	P	3	P	P	3	83	P	3
63	17	3	83	13	3	7	23	3	11	47
67	3	P	17	3	11	P	3	P	P	3
69	P	3	7	17	3	P	P	3	47	7
71	11	7	3	P	17	3	P	137	3	P
73	3	59	P	3	79	17	3	7	P	3
77	13	P	3	P	109	3	P	17	3	P
79	3	73	101	3	7	173	3	13	17	3
81	113	3	P	7	3	41	47	3	59	17
83	P	P	3	131	29	3	101	P	3	7
87	7	3	P	23	3	47	11	3	37	13
89	41	P	3	13	11	3	7	79	3	127
91	3	181	11	3	61	7	3	P	P	3
93	11	3	149	P	3	P	P	3	19	P
97	3	P	7	3	137	13	3	11	97	3
99	31	3	P	19	3	11	P	3	7	59

	390	391	392	393	394	395	396	397	398	399
01	43	61	3	P	31	3	199	29	3	P
03	3	P	197	3	7	P	3	P	53	3
07	19	P	3	23	157	3	P	59	3	7
09	3	7	P	3	P	P	3	P	7	3
11	7	3	113	19	3	P	11	3	41	107
13	13	P	3	P	11	3	7	151	3	167
17	11	3	P	P	3	43	173	3	29	179
19	P	P	3	7	P	3	P	P	3	11
21	3	19	7	3	79	P	3	11	P	3
23	P	3	61	P	3	11	P	3	7	13
27	3	11	P	3	89	29	3	P	P	3
29	31	3	P	67	3	7	23	3	P	P
31	23	109	3	37	7	3	P	67	3	73
33	3	P	P	3	47	13	3	P	61	3
37	103	7	3	139	113	3	13	79	3	P
39	3	P	P	3	P	19	3	7	P	3
41	P	3	P	P	3	P	7	3	P	11
43	P	13	3	P	P	3	29	11	3	59
47	P	3	13	7	3	71	41	3	P	43
49	17	11	3	19	103	3	31	P	3	7
51	3	7	P	3	P	P	3	127	7	3
53	7	3	17	23	3	37	19	3	11	P
57	3	P	37	3	11	7	3	83	P	3
59	139	3	11	P	3	13	P	3	23	31
61	11	P	3	7	P	3	17	P	3	89
63	3	P	7	3	19	P	3	17	P	3
67	7	53	3	P	61	3	P	7	3	17
69	3	13	107	3	29	P	3	P	P	3
71	89	3	173	P	3	7	P	3	13	P
73	41	43	3	P	7	3	97	31	3	71
77	23	3	7	13	3	19	11	3	P	7
79	P	7	3	53	11	3	P	P	3	P
81	3	P	11	3	13	P	3	7	19	3
83	11	3	163	P	3	23	7	3	P	P
87	3	149	17	3	7	31	3	11	P	3
89	P	3	101	7	3	11	13	3	113	P
91	13	P	3	11	17	3	19	P	3	7
93	3	7	P	3	73	17	3	13	7	3
97	P	19	3	P	127	3	7	17	3	23
99	3	P	13	3	P	7	3	P	17	3

	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419
01	13	3	7	191	3	101	11	3	P	7	3	23	P	3	19	47	3	11	P	3
03	109	7	3	41	11	3	19	13	3	P	131	3	P	103	3	7	P	3	17	P
07	11	3	31	17	3	P	7	3	13	19	3	11	89	3	47	P	3	179	97	3
09	P	19	3	173	17	3	P	P	3	11	23	3	7	101	3	13	P	3	P	7
11	3	P	79	3	7	17	3	11	37	3	P	7	3	109	P	3	P	53	3	P
13	P	3	P	7	3	11	17	3	P	163	3	P	P	3	P	P	3	7	P	3
17	3	7	131	3	13	31	3	19	7	3	P	P	3	79	83	3	P	13	3	167
19	7	3	37	23	3	P	151	3	P	17	3	13	47	3	7	P	3	P	19	3
21	31	53	3	61	83	3	7	43	3	151	17	3	P	7	3	P	P	3	13	11
23	3	P	19	3	P	7	3	193	P	3	P	17	3	31	23	3	107	11	3	7
27	13	P	3	7	P	3	P	139	3	P	7	3	P	11	3	131	P	3	151	P
29	3	P	7	3	P	P	3	13	P	3	89	11	3	37	17	3	7	P	3	23
31	P	3	P	31	3	P	41	3	7	11	3	P	P	3	13	7	3	29	59	3
33	7	67	3	53	P	3	179	7	3	P	37	3	P	P	3	41	17	3	11	19
37	P	3	P	11	3	7	P	3	97	13	3	31	7	3	11	73	3	P	17	3
39	P	11	3	13	7	3	P	P	3	P	P	3	11	67	3	P	13	3	7	17
41	3	137	P	3	37	71	3	131	P	3	7	P	3	P	29	3	P	7	3	P
43	23	3	7	P	3	P	97	3	11	7	3	P	P	3	P	P	3	13	P	3
47	3	19	167	3	11	13	3	7	P	3	P	23	3	173	7	3	P	109	3	P
49	19	3	11	157	3	23	7	3	P	P	3	P	13	3	181	P	3	83	P	3
51	11	P	3	P	19	3	13	P	3	31	P	3	7	P	3	37	P	3	P	7
53	3	P	P	3	7	107	3	83	P	3	61	7	3	13	P	3	23	43	3	P
57	41	13	3	P	23	3	109	53	3	7	P	3	P	P	3	29	7	3	19	P
59	3	7	127	3	P	P	3	3	7	3	19	79	3	59	11	3	P	P	3	P
61	7	3	13	P	3	47	73	3	29	P	3	P	11	3	7	13	3	P	41	3
63	P	P	3	181	43	3	7	P	3	13	11	3	P	7	3	89	61	3	P	29
67	103	3	67	37	3	113	11	3	P	71	3	7	29	3	P	197	3	11	7	3
69	17	P	3	7	11	3	67	59	3	53	7	3	P	41	3	11	P	3	149	P
71	3	17	7	3	P	29	3	P	23	3	67	13	3	11	113	3	7	P	3	19
73	11	3	17	47	3	13	89	3	7	P	3	11	149	3	67	7	3	37	13	3
77	3	P	13	3	17	P	3	11	41	3	P	P	3	7	19	3	71	P	3	13
79	13	3	47	149	3	7	19	3	P	43	3	P	7	3	P	P	3	41	P	3
81	149	23	3	11	7	3	17	13	3	107	P	3	P	P	3	41	P	3	7	P
83	3	11	P	3	P	P	3	17	P	3	7	P	3	29	13	3	73	7	3	P
87	P	7	3	P	P	3	23	P	3	17	181	3	19	P	3	7	P	3	P	11
89	3	P	P	3	19	37	3	7	31	3	17	P	3	P	7	3	47	11	3	199
91	47	3	43	13	3	P	7	3	103	179	3	17	157	3	P	11	3	23	163	3
93	P	P	3	31	P	3	P	19	3	P	13	3	7	11	3	P	173	3	P	7
97	101	3	59	7	3	P	P	3	P	11	3	13	61	3	17	P	3	7	P	3
99	P	61	3	71	P	3	P	11	3	7	73	3	P	P	3	17	7	3	11	P

	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439
01	97	P	3	7	109	3	13	P	3	P	7	3	P	19	3	41	59	3	P	11
03	3	71	7	3	P	19	3	P	23	3	P	P	3	13	P	3	7	11	3	43
07	7	13	3	P	P	3	137	7	3	107	29	3	P	11	3	139	P	3	71	23
09	3	17	P	3	P	P	3	P	13	3	41	11	3	7	83	3	P	109	3	19
11	43	3	13	29	3	7	P	3	31	11	3	19	7	3	P	13	3	P	193	3
13	P	23	3	17	7	3	43	11	3	13	P	3	79	P	3	53	P	3	7	P
17	P	3	7	11	3	17	19	3	47	7	3	P	23	3	11	P	3	P	43	3
19	P	7	3	101	13	3	17	P	3	167	P	3	11	P	3	7	53	3	29	37
21	3	73	P	3	59	101	3	7	P	3	11	13	3	P	7	3	181	P	3	167
23	P	3	P	P	3	13	7	3	11	P	3	29	P	3	173	71	3	23	13	3
27	3	103	P	3	7	23	3	P	113	3	17	7	3	37	P	3	P	73	3	13
29	13	3	11	7	3	71	47	3	P	P	3	17	139	3	137	19	3	7	41	3
31	11	P	3	P	151	3	89	13	3	7	37	3	17	P	3	101	7	3	53	197
33	3	7	157	3	P	P	3	151	7	3	23	P	3	17	13	3	P	101	3	P
37	127	29	3	P	P	3	7	P	3	P	P	3	P	7	3	13	11	3	59	53
39	3	P	P	3	31	7	3	79	P	3	193	179	3	19	11	3	17	191	3	7
41	17	3	53	13	3	19	P	3	P	23	3	7	11	3	P	P	3	17	7	3
43	P	17	3	7	P	3	P	P	3	P	7	3	83	89	3	P	19	3	17	P
47	19	3	83	17	3	157	11	3	7	67	3	13	59	3	23	7	3	11	163	3
49	7	113	3	P	11	3	P	7	3	29	P	3	61	67	3	11	P	3	13	71
51	3	61	11	3	P	17	3	P	73	3	P	P	3	7	P	3	P	67	3	P
53	11	3	29	41	3	7	13	3	P	P	3	11	7	3	19	97	3	P	P	3
57	3	P	P	3	P	P	3	11	17	3	7	103	3	191	P	3	149	7	3	113
59	137	3	7	P	3	11	29	3	P	7	3	P	181	3	13	43	3	P	61	3
61	P	7	3	11	P	3	37	61	3	P	17	3	P	131	3	7	P	3	23	P
63	3	11	13	3	P	31	3	7	P	3	P	17	3	103	7	3	47	107	3	P
67	23	149	3	13	P	3	P	P	3	P	P	3	7	17	3	19	13	3	P	7
69	3	P	43	3	7	P	3	19	163	3	13	7	3	31	17	3	P	11	3	P
71	P	3	41	7	3	P	71	3	43	97	3	23	P	3	29	11	3	7	19	3
73	P	181	3	P	P	3	139	P	3	7	19	3	109	11	3	P	7	3	73	P
77	7	3	67	31	3	P	P	3	53	11	3	P	13	3	7	P	3	P	17	3
79	29	P	3	P	107	3	7	11	3	P	23	3	113	7	3	P	31	3	11	13
81	3	P	P	3	23	7	3	179	137	3	67	29	3	13	P	3	11	P	3	7
83	P	3	P	11	3	97	P	3	19	53	3	7	P	3	11	41	3	P	7	3
87	3	P	7	3	P	37	3	P	13	3	11	19	3	43	P	3	7	P	3	P
89	P	3	13	19	3	P	P	3	7	P	3	P	73	3	157	7	3	P	P	3
91	7	31	3	P	P	3	11	7	3	13	41	3	P	P	3	P	P	3	P	P
93	3	P	P	3	11	191	3	P	59	3	P	47	3	7	23	3	13	P	3	29
97	11	P	3	P	7	3	P	P	3	19	71	3	29	P	3	P	37	3	7	P
99	3	19	P	3	P	41	3	127	P	3	7	13	3	P	P	3	89	7	3	23

	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459
01	3	P	P	3	7	P	3	P	71	3	11	7	3	89	83	3	31	23	3	197
03	79	3	P	7	3	191	13	3	11	83	3	23	17	3	P	P	3	7	163	3
07	3	7	P	3	11	P	3	13	7	3	P	43	3	P	17	3	59	P	3	29
09	7	3	11	59	3	47	31	3	P	P	3	79	53	3	7	17	3	43	19	3
11	11	P	3	73	89	3	7	P	3	97	19	3	29	7	3	71	17	3	61	31
13	3	31	13	3	23	7	3	61	41	3	P	197	3	113	P	3	P	17	3	7
17	P	157	3	7	P	3	P	97	3	P	7	3	103	P	3	23	11	3	P	17
19	3	P	7	3	43	P	3	197	P	3	13	P	3	P	11	3	7	131	3	47
21	P	3	P	23	3	211	P	3	7	29	3	P	11	3	53	7	3	13	P	3
23	7	P	3	127	31	3	P	7	3	167	11	3	41	61	3	P	43	3	P	19
27	P	3	47	19	3	7	11	3	23	P	3	P	7	3	P	53	3	11	P	3
29	P	P	3	97	7	3	13	P	3	179	37	3	31	P	3	11	103	3	7	13
31	3	P	11	3	157	P	3	41	127	3	7	P	3	11	181	3	P	7	3	23
33	11	3	7	43	3	P	P	3	107	7	3	11	P	3	P	P	3	19	P	3
37	3	19	31	3	37	P	3	7	13	3	29	P	3	P	7	3	47	P	3	71
39	47	3	13	101	3	11	7	3	P	P	3	P	19	3	P	13	3	53	23	3
41	P	37	3	11	19	3	P	P	3	13	73	3	7	P	3	P	P	3	P	7
43	3	11	151	3	7	P	3	101	P	3	31	7	3	P	29	3	13	149	3	P
47	17	131	3	61	13	3	P	29	3	7	107	3	P	137	3	37	7	3	19	11
49	3	7	P	3	P	P	3	73	7	3	19	13	3	101	47	3	191	11	3	P
51	7	3	17	P	3	13	P	3	P	79	3	163	37	3	7	11	3	P	13	3
53	P	67	3	17	P	3	7	P	3	P	P	3	13	7	3	P	71	3	P	P
57	13	3	P	P	3	17	P	3	31	11	3	7	167	3	131	P	3	P	7	3
59	P	P	3	7	23	3	11	11	3	P	7	3	P	67	3	29	P	3	11	P
61	3	13	7	3	173	11	3	17	113	3	P	P	3	P	13	3	7	67	3	19
63	139	3	P	11	3	P	59	3	7	P	3	19	P	3	11	7	3	P	P	3
67	3	29	P	3	53	41	3	89	P	3	11	31	3	7	19	3	P	P	3	43
69	127	3	P	13	3	7	19	3	11	193	3	17	7	3	41	P	3	37	P	3
71	P	P	3	P	7	3	11	P	3	P	13	3	17	59	3	199	109	3	7	P
73	3	163	P	3	11	29	3	P	23	3	7	199	3	17	37	3	P	7	3	31
77	11	7	3	199	79	3	43	P	3	41	P	3	19	P	3	7	P	3	13	23
79	3	P	P	3	19	P	3	7	P	3	61	P	3	23	7	3	17	P	3	P
8	17	3	P	P	3	109	7	3	37	31	3	P	P	3	P	19	3	17	11	3
83	13	17	3	P	P	3	P	19	3	P	P	3	7	13	3	79	11	3	17	7
87	P	3	67	7	3	P	P	3	P	P	3	73	11	3	13	P	3	7	P	3
89	P	P	3	P	17	3	23	P	3	7	11	3	P	P	3	P	7	3	109	P
91	3	7	13	3	P	17	3	47	7	3	67	P	3	19	P	3	P	29	3	11
93	7	3	P	103	3	19	11	3	P	13	3	43	P	3	7	127	3	11	P	3
97	3	193	11	3	P	7	3	P	17	3	13	P	3	11	P	3	P	41	3	7
99	11	3	31	29	3	103	I	3	59	17	3	7	97	3	173	P	3	13	7	3

	460	461	462	463	464	465	476	467	468	469	470	471	472	473	474	475	476	477	478	479
01	157	3	47	P	3	7	P	3	17	P	3	19	11	3	107	P	3	P	13	3
03	179	P	3	19	7	3	29	P	3	17	11	3	13	P	3	67	181	3	7	P
07	13	3	7	P	3	P	11	3	P	7	3	17	P	3	P	P	3	11	P	3
09	139	7	3	P	11	3	127	13	3	61	29	3	17	P	3	7	P	3	P	23
11	3	13	11	3	P	P	3	7	P	3	53	P	3	11	7	3	47	P	3	P
13	11	3	37	29	3	193	7	3	13	43	3	11	31	3	17	P	3	P	137	3
17	3	107	113	3	7	181	3	11	P	3	P	7	3	P	P	3	17	P	3	P
19	17	3	P	7	3	11	P	3	P	P	3	P	23	3	P	19	3	7	P	3
21	P	17	3	11	61	3	23	19	3	7	13	3	P	179	3	P	7	3	17	173
23	3	7	17	3	13	P	3	P	7	3	59	P	3	37	47	3	P	13	3	17
27	P	193	3	P	17	3	7	P	3	167	31	3	83	7	3	P	97	3	13	11
29	3	163	P	3	29	7	3	83	P	3	131	P	3	19	43	3	P	11	3	7
31	191	3	83	107	3	19	13	3	P	71	3	7	73	3	P	11	3	59	7	3
33	13	P	3	7	59	3	P	17	3	P	7	3	149	11	3	P	19	3	31	P
37	19	3	P	P	3	173	149	3	7	11	3	P	P	3	13	7	3	P	P	3
39	7	29	3	149	P	3	P	7	3	73	17	3	97	P	3	137	P	3	11	P
41	3	P	13	3	P	11	3	43	31	3	P	17	3	7	P	3	11	P	3	191
43	41	3	131	11	3	7	P	3	139	13	3	P	7	3	11	P	3	P	P	3
47	3	P	103	3	P	89	3	P	79	3	7	P	3	113	17	3	29	7	3	P
49	P	3	7	P	3	P	P	3	11	7	3	P	37	3	23	17	3	13	59	3
51	P	7	3	P	P	3	11	P	3	29	P	3	P	P	3	7	17	3	109	P
53	3	P	23	3	11	13	3	7	P	3	211	61	3	P	7	3	P	17	3	79
57	11	101	3	151	3	3	13	P	3	P	P	3	7	23	3	19	P	3	P	7
59	3	31	167	3	7	P	3	19	47	3	P	7	3	13	P	3	P	163	3	199
61	P	3	P	7	3	101	29	3	P	151	3	P	167	3	31	199	3	7	11	3
63	73	13	3	71	97	3	P	101	3	7	19	3	151	P	3	P	7	3	23	P
67	7	3	13	199	3	P	23	3	P	67	3	101	11	3	7	13	3	37	151	3
69	23	137	3	89	31	3	7	P	3	13	11	3	P	7	3	P	73	3	P	P
71	3	P	P	3	P	7	3	P	11	3	103	43	3	127	37	3	13	23	3	7
73	P	3	P	79	3	P	11	3	19	107	3	7	41	3	29	113	3	11	7	3
77	3	61	7	3	P	47	3	29	P	3	179	13	3	11	197	7	7	P	3	P
79	11	3	P	19	3	13	P	3	7	109	3	11	P	3	79	3	3	P	13	3
81	7	P	3	P	53	3	P	7	3	11	23	3	13	P	3	P	P	3	P	P
83	3	P	31	3	23	37	3	11	173	3	197	29	3	7	103	3	41	71	3	13
87	17	P	3	11	7	3	P	13	3	19	P	3	P	P	3	23	43	3	7	47
89	7	11	41	3	P	P	3	71	P	3	7	P	3	P	13	3	103	7	3	37
91	P	3	7	23	3	P	P	3	13	7	3	41	19	3	P	P	3	P	83	3
93	P	7	3	17	19	3	53	73	3	P	P	3	P	83	3	7	37	3	47	11
97	31	3	67	13	3	17	7	3	23	P	3	109	P	3	P	11	3	P	211	3
99	P	73	3	P	P	3	17	53	3	43	13	3	7	11	3	P	P	3	19	7

	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499
01	23	103	3	11	29	3	53	31	3	79	19	3	P	7	3	59	193	3	P	139
03	3	11	19	3	97	7	3	113	37	3	P	P	3	47	127	3	P	23	3	7
07	61	73	3	7	P	3	13	53	3	P	7	3	P	P	3	31	113	3	P	11
09	3	P	7	3	P	179	3	67	P	3	P	P	3	13	P	3	7	11	3	29
11	41	3	37	P	3	139	P	3	7	59	3	67	P	3	P	7	3	P	P	3
13	7	13	3	P	P	3	173	7	3	41	23	3	29	11	3	67	P	3	109	19
17	P	3	13	19	3	7	61	3	P	11	3	P	7	3	P	13	3	83	31	3
19	31	P	3	211	7	3	P	11	3	13	P	3	83	149	3	23	29	3	7	P
21	3	P	P	3	41	11	3	83	P	3	7	P	3	31	73	3	11	7	3	P
23	P	3	7	11	3	P	P	3	P	7	3	P	P	3	11	P	3	19	P	3
27	3	17	29	3	79	P	3	7	157	3	11	13	3	107	7	3	P	P	3	P
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31	43	P	3	17	19	3	11	P	3	167	P	3	7	P	3	P	31	3	P	7
33	3	127	139	3	7	P	3	P	47	3	P	7	3	P	P	3	P	41	3	13
37	11	37	3	P	P	3	17	13	3	7	P	3	53	103	3	P	7	3	19	P
39	3	7	P	3	59	P	3	17	7	3	19	P	3	P	13	3	P	P	3	P
41	7	3	19	P	3	P	127	3	13	109	3	157	41	3	7	107	3	P	11	3
43	107	31	3	29	193	3	7	79	3	17	P	3	23	7	3	13	11	3	P	P
47	23	3	P	13	3	43	P	3	P	P	3	7	11	3	197	P	3	P	7	3
49	P	89	3	7	P	3	P	29	3	31	7	3	17	61	3	P	131	3	79	199
51	3	179	7	3	13	47	3	P	11	3	181	23	3	17	P	3	7	13	3	11
53	29	3	73	P	3	23	11	3	7	P	3	13	P	3	17	7	3	11	P	3
57	3	P	11	3	47	59	3	P	P	3	P	P	3	7	19	3	17	P	3	P
59	11	3	P	37	3	7	13	3	P	173	3	11	7	3	P	P	3	17	73	3
61	13	17	3	137	7	3	P	P	3	11	71	3	P	13	3	29	53	3	7	47
63	3	P	17	3	P	P	3	11	131	3	7	211	3	P	P	3	P	7	3	17
67	71	7	3	11	17	3	41	P	3	23	139	3	19	P	3	7	P	3	47	29
69	3	11	13	3	19	17	3	7	P	3	P	P	3	P	7	3	P	157	3	107
71	53	3	P	P	3	P	7	3	P	13	3	P	29	3	61	19	3	71	P	3
73	P	67	3	13	P	3	P	17	3	P	31	3	7	97	3	89	13	3	53	7
77	131	3	23	7	3	31	P	3	37	17	3	P	P	3	P	11	3	7	P	3
79	P	P	3	101	P	3	P	P	3	7	17	3	P	11	3	43	7	3	31	23
81	3	7	P	3	P	13	3	P	7	3	P	11	3	19	P	3	P	67	3	151
83	7	3	53	P	3	19	89	3	P	11	3	137	13	3	7	179	3	P	83	3
87	3	P	109	3	P	7	3	P	19	3	191	101	3	13	17	3	11	P	3	7
89	19	3	43	11	3	P	181	3	P	P	3	7	23	3	11	17	3	P	7	3
91	P	11	3	7	P	3	23	97	3	P	7	3	11	P	3	101	17	3	P	P
93	3	P	7	3	71	P	3	59	13	3	11	P	3	P	43	3	7	17	3	P
97	7	P	3	P	P	3	11	7	3	13	29	3	P	47	3	P	P	3	41	17
99	3	157	P	3	11	23	3	P	107	3	37	P	3	7	P	3	13	19	3	P

	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519
01	3	P	17	3	13	11	3	7	37	3	P	137	3	29	7	3	11	13	3	17
03	31	3	61	11	3	P	7	3	101	109	3	13	P	3	11	P	3	149	P	3
07	3	89	P	3	7	17	3	P	23	3	11	7	3	P	P	3	P	29	3	P
09	43	3	23	7	3	53	13	3	11	P	3	P	41	3	101	19	3	7	103	3
11	13	P	3	P	P	3	11	17	3	7	29	3	83	13	3	P	7	3	197	23
13	3	7	149	3	11	P	3	13	7	3	139	79	3	23	P	3	P	P	3	P
17	11	23	3	67	P	3	7	41	3	59	17	3	P	7	3	P	71	3	P	193
19	3	P	13	3	127	7	3	67	89	3	163	17	3	19	P	3	41	P	3	7
21	P	3	P	P	3	19	223	3	P	13	3	7	17	3	P	P	3	P	7	3
23	P	P	3	7	P	3	23	P	3	P	7	3	181	17	3	67	11	3	29	137
27	19	3	P	59	3	P	P	3	7	127	3	29	11	3	P	7	3	13	P	3
29	7	P	3	P	211	3	197	7	3	P	11	3	P	P	3	227	17	3	P	P
31	3	P	P	3	29	13	3	97	11	3	P	P	3	7	P	3	P	17	3	11
33	P	3	191	P	3	7	11	3	P	31	3	P	7	3	19	29	3	11	17	3
37	3	181	11	3	31	97	3	113	29	3	7	P	3	11	P	3	P	7	3	167
39	11	3	7	71	3	P	79	3	P	7	3	11	P	3	P	P	3	31	P	3
41	163	7	3	P	P	3	89	P	3	11	43	3	P	P	3	7	113	3	47	P
43	3	41	47	3	73	P	3	7	13	3	P	199	3	P	7	3	43	59	3	127
47	P	P	3	11	61	3	P	31	3	13	P	3	7	P	3	19	P	3	139	7
49	3	11	109	3	7	P	3	19	P	3	71	7	3	P	P	3	13	P	3	P
51	P	3	31	7	3	P	P	3	211	P	3	P	53	3	23	P	3	7	19	3
53	P	P	3	43	13	3	37	P	3	7	19	3	107	89	3	31	7	3	P	11
57	7	3	29	37	3	13	179	3	P	P	3	P	P	3	7	11	3	73	13	3
59	113	P	3	P	P	3	7	193	3	131	P	3	13	7	3	47	P	3	P	223
61	3	103	P	3	P	7	3	23	181	3	P	11	3	P	P	3	19	191	3	7
63	13	3	P	P	3	59	29	3	19	11	3	7	P	3	53	P	3	37	7	3
67	3	13	7	3	109	11	3	P	P	3	223	19	3	31	13	3	7	P	3	157
69	P	3	17	11	3	61	23	3	7	P	3	P	167	3	11	7	3	P	P	3
71	7	11	3	17	41	3	P	7	3	P	P	3	11	47	3	13	163	3	P	P
73	3	131	P	3	17	103	3	P	P	3	11	73	3	7	P	3	P	23	3	P
77	P	P	3	P	7	3	11	P	3	19	13	3	47	83	3	P	31	3	7	P
79	3	19	127	3	11	37	3	17	83	3	7	61	3	191	P	3	P	7	3	59
81	61	3	7	83	3	P	59	3	17	7	3	13	19	3	P	P	3	53	29	3
83	11	7	3	P	19	3	P	43	3	17	23	3	P	P	3	7	P	3	13	227
87	P	3	P	P	3	P	7	3	151	67	3	17	P	3	P	79	3	P	11	3
89	13	31	3	41	29	3	173	P	3	P	47	3	7	13	3	23	11	3	19	7
91	3	53	P	3	7	P	3	13	P	3	19	7	3	17	11	3	P	67	3	P
93	P	3	19	7	3	P	163	3	P	P	3	P	11	3	13	P	3	7	P	3
97	3	7	13	3	P	19	3	79	7	3	37	P	3	103	23	3	17	P	3	11
99	7	3	179	101	3	P	11	3	23	13	3	P	43	3	7	P	3	11	P	3

	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539
01	149	3	P	P	3	P	23	3	7	P	3	P	P	3	P	7	3	83	11	3
03	7	P	3	193	13	3	41	7	3	P	P	3	83	151	3	P	11	3	173	19
07	131	3	17	19	3	7	31	3	P	191	3	23	7	3	P	P	3	43	13	3
09	P	107	3	17	7	3	P	P	3	157	11	3	13	P	3	73	P	3	7	31
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13	13	3	7	P	3	17	11	3	P	7	3	P	127	3	31	59	3	11	P	3
17	3	13	11	3	23	P	3	7	P	3	P	P	3	11	7	3	P	P	3	P
19	11	3	79	113	3	29	7	3	13	P	3	11	19	3	P	109	3	P	P	3
21	P	P	3	P	19	3	101	P	3	11	37	3	7	71	3	13	29	3	107	7
23	3	47	P	3	7	53	3	11	101	3	17	7	3	P	41	3	P	31	3	P
27	P	P	3	11	103	3	P	P	3	7	13	3	17	P	3	P	7	3	19	P
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31	7	3	19	43	3	131	P	3	23	41	3	13	P	3	7	199	3	P	P	3
33	61	37	3	59	P	3	7	P	3	43	181	3	P	7	3	17	P	3	13	11
37	17	3	P	199	3	107	13	3	P	P	3	7	139	3	P	11	3	17	7	3
39	13	17	3	7	41	3	P	23	3	167	7	3	P	11	3	37	P	3	17	P
41	3	23	7	3	229	P	3	13	53	3	29	11	3	41	P	3	7	61	3	13
43	71	3	89	17	3	P	61	3	7	11	3	19	37	3	13	7	3	223	23	3
47	3	P	13	3	179	11	3	P	43	3	P	P	3	7	19	3	11	71	3	73
49	23	3	P	11	3	7	17	3	41	13	3	P	7	3	11	P	3	59	P	3
51	P	11	3	13	7	3	37	17	3	P	P	3	11	31	3	P	13	3	7	P
53	3	P	P	3	P	P	3	71	17	3	7	23	3	P	P	3	P	7	3	163
57	P	7	3	41	P	3	11	P	3	P	17	3	19	229	3	7	P	3	P	79
59	3	43	P	3	11	13	3	7	P	3	97	17	3	P	7	3	23	P	3	P
61	79	3	11	P	3	P	7	3	P	211	3	P	13	7	193	19	3	37	P	3
63	11	P	3	P	23	3	13	19	3	P	47	3	7	17	3	29	103	3	61	7
67	P	3	P	7	3	P	P	3	29	P	3	79	P	3	127	17	3	7	11	3
69	P	13	3	P	71	3	31	P	3	7	P	3	P	83	3	P	7	3	103	29
71	3	7	167	3	137	P	3	113	7	3	73	P	3	19	11	3	191	19	3	31
73	7	3	13	83	3	19	P	3	37	P	3	P	11	3	7	13	3	P	17	3
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79	19	3	23	P	3	P	11	3	P	31	3	7	P	3	P	131	3	11	7	3
81	P	P	3	7	11	3	139	47	3	P	7	3	P	P	3	11	P	3	P	23
83	3	P	7	3	31	P	3	P	P	3	109	13	3	11	79	3	7	P	3	37
87	7	23	3	P	73	3	19	7	3	11	P	3	13	197	3	41	37	3	P	P
89	3	P	P	3	P	43	3	11	P	3	P	P	3	7	89	3	53	19	3	13
91	13	3	P	P	3	7	P	3	227	19	3	43	7	3	149	P	3	3	P	3
93	113	19	3	11	7	3	23	13	3	197	P	3	137	107	3	P	P	3	7	P
97	59	3	7	151	3	149	P	3	13	7	3	P	223	3	61	P	3	23	P	3
99	53	7	3	61	47	3	151	37	3	P	29	3	P	67	3	7	P	3	P	11

	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559
01	P	P	3	13	P	3	P	19	3	7	1	3	P	17	3	P	7	3	41	P
03	3	7	67	3	P	P	3	11	7	3	13	P	3	29	17	3	P	53	3	P
07	53	61	3	11	41	7	7	227	3	P	67	3	P	7	3	47	17	3	P	37
09	3	11	151	3	P	3	3	P	23	3	P	P	3	19	67	3	3	17	3	7
11	P	3	23	P	3	19	97	3	59	43	3	7	13	3	P	P	3	P	7	3
13	P	53	3	7	P	3	13	P	3	89	7	3	P	P	3	43	19	3	P	11
17	19	3	P	29	3	P	P	3	7	P	3	P	P	3	151	7	3	P	P	3
19	7	13	3	P	P	3	193	7	3	P	37	3	P	11	3	59	P	3	P	199
21	3	P	59	3	P	P	3	P	13	3	P	11	3	7	157	3	P	P	3	P
23	89	3	13	P	3	7	P	3	73	11	3	199	7	3	19	13	3	103	P	3
27	3	113	211	3	37	11	3	P	109	3	7	P	3	61	43	3	11	7	3	P
29	97	3	7	11	3	31	P	3	P	7	3	29	P	3	11	P	3	23	P	3
31	71	7	3	P	13	3	P	229	3	163	113	3	11	P	3	7	P	3	31	P
33	3	P	193	3	29	23	3	7	P	3	11	13	3	P	7	3	P	P	3	P
37	P	43	3	67	P	3	11	127	3	137	47	3	7	P	3	19	23	3	P	7
39	3	P	73	3	7	P	3	19	29	3	23	7	3	P	P	3	P	139	3	13
41	13	3	11	7	3	P	101	3	173	P	3	67	37	3	P	P	3	7	19	3
43	11	29	3	31	P	3	53	13	3	7	19	3	P	P	3	67	7	3	P	43
47	7	3	17	P	3	P	P	3	13	23	3	P	101	3	7	P	3	107	11	3
49	P	173	3	17	71	3	7	53	3	P	P	3	P	7	3	13	11	3	P	P
51	3	P	P	3	17	7	3	P	P	3	P	131	3	P	11	3	19	197	3	7
53	191	3	227	13	3	17	31	3	19	179	3	7	11	3	23	73	3	127	7	3
57	3	31	7	3	13	89	3	17	11	3	P	19	3	197	P	3	7	13	3	11
59	P	3	29	19	3	P	11	3	7	P	3	13	P	3	31	7	3	11	83	3
61	7	41	3	P	11	3	47	7	3	17	P	3	73	23	3	11	P	3	13	107
63	3	P	11	3	107	P	3	23	83	3	17	P	3	7	37	3	P	P	3	191
67	13	P	3	P	7	3	P	P	3	11	53	3	17	13	3	181	P	3	7	P
69	3	19	P	3	P	197	3	11	P	3	7	43	3	17	P	3	179	7	3	97
71	139	3	7	P	3	11	23	3	37	7	3	P	19	3	13	61	3	43	P	3
73	23	7	3	11	19	3	P	P	3	P	P	3	31	P	3	7	P	3	59	223
77	17	3	P	P	3	P	7	3	P	13	3	23	167	3	29	149	3	17	71	3
79	41	17	3	13	157	3	P	P	3	P	P	3	7	79	3	P	13	3	17	7
81	3	P	17	3	7	P	3	29	P	3	13	7	3	P	109	3	P	11	3	17
83	P	3	19	7	3	P	149	3	71	P	3	139	59	3	113	11	3	7	29	3
87	3	7	P	3	23	13	3	P	7	3	31	11	3	97	P	3	233	P	3	P
89	7	3	233	137	3	71	17	3	131	11	3	229	13	3	7	P	3	47	P	3
91	P	47	3	109	29	3	7	11	3	127	89	3	P	7	3	23	P	3	11	13
93	3	P	P	3	P	7	3	157	17	3	37	97	3	13	211	3	11	P	3	7
97	47	11	3	7	P	3	83	37	3	43	7	3	11	31	3	53	P	3	P	P
99	3	83	7	3	P	71	3	P	13	3	11	17	3	P	19	3	7	P	3	29

	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579
01	3	P	43	3	P	P	3	P	79	3	7	11	3	P	61	3	P	7	3	P
03	P	3	7	13	3	P	23	3	43	7	3	17	P	3	137	P	3	19	P	3
07	3	19	P	3	13	11	3	7	P	3	109	P	3	17	7	3	11	13	3	79
09	P	3	P	11	3	P	7	3	P	P	3	13	19	3	11	131	3	P	P	3
11	79	11	3	P	19	3	P	P	3	P	47	3	7	223	3	17	53	3	13	7
13	3	P	67	3	7	31	3	P	P	3	11	7	3	37	P	3	17	P	3	29
17	13	17	3	199	P	3	11	43	3	7	23	3	29	13	3	113	7	3	17	P
19	3	7	17	3	11	P	3	13	7	3	19	P	3	31	67	3	157	P	3	17
21	7	3	11	17	3	29	41	3	P	P	3	239	P	3	7	97	3	197	67	3
23	11	P	3	157	17	3	7	131	3	P	127	3	P	7	3	23	29	3	53	P
27	179	3	59	23	3	P	17	3	P	13	3	7	89	3	P	P	3	P	7	3
29	43	37	3	7	73	3	P	17	3	P	7	3	151	P	3	P	11	3	P	53
31	3	P	7	3	P	P	3	P	17	3	13	P	3	P	11	3	7	P	3	19
33	137	3	53	P	3	P	P	3	7	17	3	19	11	3	79	7	3	13	151	3
37	3	73	P	3	P	13	3	P	11	3	P	17	3	7	19	3	P	P	3	11
39	P	3	P	53	3	7	11	3	113	97	3	P	7	3	71	163	3	11	P	3
41	P	31	3	103	7	3	13	23	3	P	P	3	P	17	3	11	P	3	7	13
43	3	23	11	3	P	P	3	179	P	3	7	P	3	11	17	3	59	7	3	P
47	41	7	3	29	47	3	37	P	3	11	P	3	19	P	3	7	17	3	P	P
49	3	P	P	3	19	193	3	7	13	3	89	P	3	P	7	3	P	17	3	167
51	23	3	13	37	3	11	7	3	139	P	3	67	P	3	73	13	3	P	17	3
53	P	233	3	11	P	3	181	19	3	13	59	3	7	83	3	67	P	3	P	7
57	29	3	101	7	3	23	53	3	P	P	3	61	31	3	P	P	3	7	47	3
59	61	89	3	P	13	3	P	211	3	7	P	3	P	41	3	P	7	3	P	11
61	3	7	127	3	131	163	3	31	7	3	43	13	3	19	37	3	23	11	3	149
63	7	3	P	157	3	13	P	3	101	P	3	P	173	3	7	11	3	47	13	3
67	3	P	P	3	P	7	3	P	19	3	149	11	3	P	P	3	P	61	3	7
69	13	3	P	P	3	P	61	3	29	11	3	7	P	3	101	23	3	41	7	3
71	47	P	3	7	149	3	P	11	3	23	7	3	P	103	3	P	101	3	11	29
73	3	13	7	3	P	11	3	P	P	3	P	P	3	P	13	3	7	P	3	P
77	7	11	3	P	P	3	19	7	3	227	P	3	11	181	3	13	137	3	31	P
79	3	P	167	3	P	29	3	P	23	3	11	P	3	7	229	3	P	19	3	37
81	P	3	23	13	3	7	P	3	11	19	3	211	7	3	47	71	3	P	P	3
83	17	19	3	P	7	3	11	P	3	P	13	3	P	P	3	89	37	3	7	23
87	P	3	7	113	3	71	P	3	163	7	3	13	P	3	P	P	3	P	107	3
89	11	7	3	17	P	3	83	109	3	P	P	3	59	P	3	7	P	3	13	103
91	3	83	181	3	17	P	3	7	P	3	37	P	3	29	7	3	31	P	3	P
93	P	3	41	P	3	17	7	3	P	P	3	P	23	3	P	P	3	P	11	3
97	3	P	19	3	7	P	3	13	P	3	P	7	3	P	11	3	P	29	3	59
99	P	3	P	7	3	P	31	3	17	P	3	47	11	3	13	239	3	7	P	3

	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599
01	31	3	11	137	3	19	P	3	127	P	3	7	53	3	191	13	3	227	7	7
03	11	97	3	7	P	3	P	47	3	13	7	3	73	31	3	157	19	3	79	37
07	19	3	P	199	3	41	103	3	7	P	3	P	P	3	P	7	3	P	11	3
09	7	P	3	P	13	3	29	7	3	P	P	3	P	127	3	P	11	3	P	137
11	3	P	P	3	P	P	3	P	23	3	P	13	3	7	11	3	P	29	3	181
13	P	3	23	P	3	7	P	3	103	P	3	P	7	3	19	P	3	211	13	3
17	3	89	P	3	P	163	3	71	11	3	7	31	3	23	P	3	P	7	3	11
19	13	3	7	29	3	139	11	3	131	7	3	P	P	3	P	53	3	11	41	3
21	17	7	3	P	11	3	31	13	3	P	P	3	P	137	3	7	P	3	163	P
23	3	11	11	3	37	43	3	7	59	3	P	P	3	11	7	3	109	P	3	31
27	P	37	3	17	P	3	23	P	3	11	67	3	7	41	3	13	P	3	29	7
29	3	P	P	3	7	107	3	11	89	3	P	7	3	79	67	3	P	P	3	P
31	P	3	P	7	3	11	P	3	P	31	3	29	61	3	103	59	3	7	19	3
33	131	61	3	11	71	3	17	P	3	7	13	3	P	P	3	37	7	3	P	73
37	7	3	P	P	3	P	191	3	17	P	3	13	37	3	7	29	3	31	53	3
39	127	47	3	227	P	3	7	151	3	17	43	3	P	7	3	P	23	3	13	11
41	3	53	139	3	P	7	3	P	29	3	17	P	3	P	P	3	19	11	3	7
43	P	3	P	41	3	P	13	3	19	P	3	7	P	3	P	11	3	P	7	3
47	3	P	7	3	211	127	3	13	83	3	137	11	3	17	P	3	7	P	3	151
49	P	3	31	19	3	P	223	3	7	11	3	P	179	3	13	7	3	149	97	3
51	7	P	3	23	P	3	89	7	3	167	P	3	193	P	3	17	P	3	11	P
53	3	P	13	3	P	11	3	41	229	3	P	149	3	7	P	3	11	P	3	167
57	P	11	3	13	7	3	P	P	3	19	73	3	11	P	3	P	13	3	7	P
59	3	19	17	3	53	31	3	67	71	3	7	P	3	P	37	3	P	7	3	17
61	P	3	7	17	3	157	P	3	11	7	3	67	19	3	97	P	3	13	31	3
63	31	7	3	P	17	3	11	P	3	P	P	3	P	23	3	7	P	3	P	61
67	P	3	11	P	3	P	7	3	37	P	3	P	13	3	P	P	3	59	131	3
69	11	P	3	P	59	3	13	17	3	109	P	3	7	P	3	71	P	3	19	7
71	3	P	P	3	7	37	3	P	17	3	19	7	3	13	P	3	P	P	3	P
73	P	3	19	7	3	P	23	3	113	17	3	47	P	3	P	41	3	7	11	3
77	3	7	101	3	P	19	3	53	7	3	P	17	3	P	11	3	83	23	3	37
79	7	3	13	P	3	P	P	3	97	P	3	23	11	3	7	13	3	P	P	3
81	241	71	3	79	P	3	7	43	3	13	11	3	P	7	3	P	37	3	233	P
83	3	83	167	3	233	7	3	29	11	3	P	P	3	43	17	3	13	191	3	7
87	29	31	3	7	11	3	P	P	3	61	7	3	101	P	3	11	17	3	P	223
89	3	P	7	3	23	41	3	P	P	3	37	13	3	11	19	3	7	17	3	239
91	11	3	71	P	3	13	19	3	7	P	3	11	211	3	41	7	3	P	13	3
93	7	P	3	P	29	3	P	7	3	11	P	3	13	P	3	23	P	3	101	17
97	13	3	97	23	3	7	79	3	P	P	3	P	7	3	P	61	3	P	89	3
99	P	P	3	11	7	3	P	13	3	41	113	3	19	P	3	107	P	3	7	P

	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619
01	29	P	3	47	11	3	P	101	3	P	P	3	7	59	3	11	229	3	23	7
03	3	P	11	3	7	17	3	P	41	3	53	7	3	11	P	3	P	P	3	103
07	23	P	3	13	29	3	P	17	3	7	P	3	97	101	3	P	7	3	19	31
09	3	7	P	3	193	P	3	11	7	3	13	53	3	37	P	3	P	23	3	P
11	7	3	19	41	3	11	P	3	P	17	3	23	P	3	7	P	3	13	113	3
13	P	47	3	11	P	3	7	109	3	P	17	3	41	7	3	137	P	3	P	101
17	P	3	P	P	3	73	P	3	61	P	3	7	13	3	P	227	3	P	7	3
19	47	179	3	7	31	3	13	P	3	P	7	3	29	17	3	P	43	3	P	11
21	3	59	7	3	23	P	3	41	P	3	139	P	3	13	17	3	7	11	3	19
23	193	3	P	179	3	29	P	3	7	P	3	19	P	3	239	7	3	P	211	3
27	3	P	229	3	P	P	3	P	13	3	P	11	3	7	19	3	P	17	3	P
29	P	3	13	23	3	7	19	3	59	11	3	P	7	3	47	13	3	P	17	3
31	173	157	3	P	7	3	P	11	3	13	P	3	P	P	3	37	P	3	7	17
33	3	P	29	3	23	11	3	P	127	3	7	113	3	P	23	3	11	7	3	P
37	P	7	3	P	13	3	P	P	3	P	67	3	11	83	3	7	P	3	P	241
39	3	P	59	3	19	P	3	7	83	3	11	13	3	P	7	3	53	107	3	23
41	P	3	107	83	3	13	7	3	11	149	3	P	47	3	P	19	3	29	13	3
43	97	137	3	P	P	3	11	19	3	P	P	3	7	P	3	P	P	3	P	7
47	13	3	11	7	3	191	P	3	71	59	3	47	73	3	43	P	3	7	23	3
49	11	P	3	29	P	3	P	13	3	7	41	3	23	31	3	61	7	3	127	P
51	3	7	P	3	61	151	3	79	7	3	P	P	3	19	13	3	P	P	3	41
53	7	3	89	P	3	19	131	3	13	P	3	P	P	3	7	P	3	37	11	3
57	3	43	P	3	P	7	3	P	19	3	P	23	3	P	11	3	P	P	3	7
59	19	3	P	13	3	23	P	3	P	47	3	7	11	3	41	P	3	151	7	3
61	17	P	3	7	103	3	P	P	3	P	7	3	P	43	3	P	197	3	P	P
63	3	17	7	3	13	71	3	P	11	3	227	31	3	P	P	3	7	13	3	11
67	7	P	3	17	11	3	19	7	3	41	79	3	197	109	3	11	P	3	13	P
69	3	P	11	3	17	37	3	67	P	3	173	P	3	7	P	3	83	19	3	31
71	11	3	P	73	3	7	13	3	29	19	3	11	7	3	P	23	3	223	P	3
73	13	19	3	P	7	3	17	P	3	11	157	3	71	13	3	67	P	3	7	29
77	P	3	7	173	3	11	47	3	17	7	3	131	29	3	13	139	3	163	43	3
79	63	7	3	11	197	3	P	P	3	17	103	3	233	P	3	7	37	3	P	P
81	3	11	13	3	31	29	3	7	23	3	17	193	3	P	7	3	P	P	3	P
83	P	3	23	P	3	47	7	3	107	13	3	17	P	3	P	P	3	31	19	3
87	3	139	19	3	7	43	3	89	P	3	13	7	3	17	P	3	P	11	3	P
89	P	3	P	7	3	P	P	3	P	71	3	43	167	3	17	11	3	7	199	3
91	P	23	3	131	241	3	137	31	3	7	P	3	P	11	3	17	7	3	59	P
93	3	7	7	3	P	13	3	P	7	3	199	11	3	29	P	3	17	61	3	47
97	19	17	3	P	P	3	7	11	3	181	107	3	P	7	3	31	103	3	11	13
99	3	37	17	3	101	7	3	63	P	3	P	19	3	13	89	3	11	29	3	7

	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639
01	3	13	P	3	P	P	3	P	P	3	251	89	3	7	13	3	P	11	3	P
03	P	3	17	P	3	7	P	3	13	P	3	P	7	3	19	11	3	P	P	3
07	3	173	P	3	17	P	3	73	181	3	7	11	3	29	163	3	P	7	3	P
09	59	3	7	13	3	17	137	3	107	7	3	223	31	3	P	41	3	P	P	3
11	3	7	3	P	139	3	17	11	3	53	13	3	P	P	3	7	P	3	11	79
13	P	179	P	3	13	11	3	7	23	3	61	P	3	P	7	3	11	13	3	P
17	3	11	3	101	P	3	P	59	3	17	29	3	7	P	3	19	P	3	13	7
19	P	P	P	3	7	101	3	19	P	3	11	7	3	23	P	3	113	P	3	41
21	109	3	43	7	3	103	13	3	11	P	3	17	191	3	P	P	3	7	19	3
23	13	23	3	P	P	3	11	P	3	7	19	3	17	13	3	139	7	3	P	97
27	7	3	11	P	3	31	P	3	P	P	3	P	23	3	7	P	3	P	83	3
29	11	P	3	157	163	3	7	149	3	P	P	3	53	7	3	17	P	3	29	P
31	3	P	13	3	149	7	3	P	83	3	P	P	3	P	137	3	17	101	3	7
33	17	3	P	83	3	P	P	3	19	13	3	7	37	3	229	P	3	17	7	3
37	3	P	7	3	29	23	3	43	31	3	13	19	3	P	11	3	7	P	3	17
39	P	3	109	17	3	P	P	3	7	P	3	103	11	3	P	7	3	13	71	3
41	7	P	3	31	17	3	37	7	3	113	11	3	P	97	3	P	23	3	P	43
43	3	P	67	3	41	13	3	P	11	3	23	233	3	7	P	3	31	P	3	11
47	P	29	3	P	7	3	13	17	3	19	67	3	P	P	3	11	P	3	7	13
49	3	19	11	3	197	P	3	131	17	3	7	P	3	11	67	3	P	7	3	P
51	11	3	7	P	3	71	31	3	P	7	3	11	19	3	107	103	3	37	67	3
53	P	7	3	23	19	3	P	P	3	11	17	3	43	P	3	7	53	3	P	31
57	P	3	13	127	3	11	7	3	239	157	3	137	17	3	23	13	3	103	P	3
59	229	61	3	11	P	3	P	97	3	13	P	3	7	17	3	P	P	3	19	7
61	3	11	23	3	7	73	3	P	P	3	19	7	3	P	17	3	13	P	3	167
63	53	3	19	7	3	P	223	3	37	79	3	83	41	3	P	17	3	7	P	3
67	3	7	71	3	P	19	3	23	7	3	P	13	3	P	P	3	P	11	3	47
69	7	3	73	47	3	13	29	3	P	P	3	181	151	3	7	11	3	43	13	3
71	P	P	3	97	179	3	7	41	3	P	59	3	13	7	3	151	P	3	23	17
73	3	79	P	3	P	7	3	P	P	3	P	11	3	127	P	3	41	P	3	7
77	23	97	3	7	P	3	233	11	3	71	7	3	P	P	3	P	37	3	11	P
79	3	13	7	3	43	11	3	67	227	3	P	P	3	61	13	3	7	23	3	137
81	P	3	61	11	3	P	19	3	7	P	3	23	P	3	11	7	3	P	127	3
83	7	11	3	P	P	3	P	7	3	P	199	3	11	241	3	13	43	3	191	109
87	47	3	199	13	3	7	P	3	11	P	3	179	7	3	P	P	3	227	29	3
89	29	P	3	89	7	3	11	37	3	P	13	3	19	P	3	P	P	3	7	61
91	3	P	167	3	11	P	3	P	61	3	7	29	3	P	173	3	P	7	3	89
93	31	3	7	43	3	53	71	3	109	7	3	13	167	3	P	19	3	P	181	3
97	3	37	P	3	P	P	3	7	P	3	P	P	3	P	7	3	P	131	3	P
99	P	3	P	23	3	59	7	3	31	73	3	P	P	3	P	P	3	[illegible]	11	3

	640	541	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	569
01	7	3	19	P	3	53	P	3	11	P	3	P	113	3	7	17	3	P	29	3
03	29	13	3	P	P	3	7	89	3	41	P	3	P	7	3	31	17	3	23	59
07	P	3	11	107	3	251	23	3	229	47	3	7	197	3	P	13	3	P	7	3
09	11	P	3	7	29	3	P	P	3	13	7	3	61	P	3	109	P	3	P	17
11	3	61	7	3	41	31	3	163	P	3	P	P	3	241	149	3	7	23	3	19
13	P	3	157	73	3	P	P	3	7	139	3	19	P	3	P	7	3	P	11	3
17	3	97	P	3	37	149	3	P	P	3	79	13	3	7	11	3	P	P	3	29
19	P	3	149	P	3	7	19	3	53	P	3	P	7	3	P	P	3	P	13	3
21	73	37	3	131	7	3	P	61	3	P	11	3	13	83	3	P	211	3	7	P
23	3	P	P	3	23	113	3	59	11	3	7	P	3	P	P	3	137	7	3	11
27	43	7	3	P	11	3	P	13	3	P	P	3	19	P	3	7	29	3	P	P
29	3	13	11	3	19	173	3	7	241	3	P	P	3	11	7	3	P	P	3	P
31	11	3	P	23	3	47	7	3	13	29	3	11	37	3	59	19	3	P	P	3
33	P	59	3	P	P	3	P	19	3	11	P	3	7	79	3	13	P	3	43	7
37	P	3	61	7	3	11	109	3	23	P	3	53	89	3	P	P	3	7	P	3
39	17	31	3	11	P	3	37	41	3	7	13	3	P	223	3	P	7	3	P	233
41	3	7	227	3	13	233	3	101	7	3	193	P	3	19	31	3	41	13	3	23
43	7	3	17	37	3	19	127	3	61	101	3	13	53	3	7	P	3	29	P	3
47	3	23	41	3	17	7	3	P	19	3	29	P	3	101	P	3	P	11	3	7
49	19	3	47	229	3	17	13	3	P	107	3	7	71	3	P	11	3	37	7	3
51	13	P	3	7	P	3	17	73	3	P	7	3	23	11	3	P	P	3	P	P
53	3	P	7	3	P	P	3	13	P	3	P	11	3	P	29	3	7	47	3	101
57	7	P	3	139	43	3	19	7	3	17	67	3	P	P	3	P	P	3	11	P
59	3	83	13	3	73	11	3	31	79	3	17	23	3	7	67	3	11	19	3	17
61	29	3	179	11	3	7	P	3	37	13	3	17	7	3	11	53	3	P	67	3
63	P	11	3	13	7	3	P	P	3	167	P	3	11	163	3	P	13	3	7	P
67	P	3	7	191	3	P	P	3	11	7	3	P	P	3	17	173	3	13	P	3
69	79	7	3	59	23	3	11	239	3	P	31	3	P	131	3	7	97	3	199	41
71	3	P	P	3	11	13	3	7	P	3	P	P	3	P	7	3	17	89	3	37
73	17	3	11	P	3	31	7	3	29	23	3	P	13	3	233	23	3	17	19	3
77	3	29	17	3	7	P	3	211	P	3	59	7	3	13	41	3	P	P	3	17
79	139	3	P	7	3	P	P	3	P	181	3	P	29	3	3	P	3	7	11	3
81	P	13	3	P	17	3	71	P	3	7	151	3	97	P	3	P	7	3	P	P
83	3	7	P	3	P	17	3	P	7	3	37	P	3	151	11	3	19	157	3	P
87	19	P	3	31	59	3	7	17	3	13	11	3	P	7	3	P	P	3	41	19
89	3	P	53	3	P	7	3	67	11	3	P	19	3	23	43	3	13	P	3	7
91	P	3	239	19	3	P	11	3	P	17	3	7	109	3	79	107	3	11	7	3
93	107	23	3	7	11	3	3	P	3	103	7	3	P	P	3	11	179	3	131	P
97	11	3	113	71	3	13	31	3	7	P	3	11	17	3	P	7	3	19	13	3
99	7	43	3	P	13	3	23	7	3	11	P	3	13	17	3	P	P	3	P	31

	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679
01	13	7	3	P	23	3	P	P	3	149	11	3	17	13	3	7	P	3	1	P
03	3	P	239	3	P	73	3	7	11	3	P	P	3	19	7	3	67	79	3	11
07	149	P	3	61	11	3	43	41	3	23	37	3	7	P	3	11	P	3	P	7
09	3	P	11	3	7	P	3	19	P	3	113	7	3	16	P	3	17	P	3	59
11	11	3	73	7	3	227	59	3	71	13	3	11	P	3	P	P	3	7	19	3
13	251	17	3	13	P	3	29	P	3	7	19	3	P	83	3	181	7	3	17	113
17	7	3	23	17	3	11	P	3	109	61	3	41	P	3	7	107	3	13	73	3
19	107	37	3	11	17	3	7	137	3	P	29	3	P	7	3	251	P	3	P	23
21	3	11	P	3	127	7	3	P	P	3	P	P	3	23	P	3	19	241	3	7
23	103	3	47	29	3	P	17	3	19	P	3	7	13	3	191	P	3	P	7	3
27	3	89	7	3	181	71	3	53	17	3	97	19	3	13	P	3	7	11	3	P
29	P	3	103	19	3	P	P	3	7	17	3	P	23	3	P	7	3	89	P	3
31	7	13	3	113	P	3	23	7	3	P	17	3	P	11	3	P	P	3	29	P
33	3	41	107	3	31	P	3	P	13	3	P	11	3	7	P	3	47	P	3	P
37	P	P	3	P	7	3	37	11	3	13	43	3	71	17	3	P	239	3	7	41
39	3	19	19	3	29	11	3	P	89	3	7	P	3	P	17	3	11	7	3	P
41	P	3	7	11	3	P	103	3	P	7	3	P	19	3	111	17	3	P	1,9	3
43	211	7	3	P	13	3	P	31	3	P	P	3	11	P	3	7	17	3	P	P
47	P	3	31	P	3	13	7	3	11	P	3	83	P	3	P	P	3	37	13	3
49	257	29	3	43	P	3	11	P	3	P	P	3	7	P	3	31	61	3	19	7
51	3	83	97	3	7	61	3	1	P	2	19	7	3	47	37	3	P	P	3	13
53	13	3	11	7	3	P	P	3	P	23	3	P	109	3	P	43	3	7	P	3
57	3	7	59	3	P	19	3	241	7	3	P	P	3	193	13	3	29	P	3	P
59	7	3	173	P	3	101	191	3	13	P	3	239	103	3	7	P	3	P	11	3
61	31	1	3	P	41	3	7	191	3	29	P	3	P	7	3	13	11	3	79	P
63	3	109	23	3	P	7	3	P	P	3	199	47	3	31	11	3	71	P	3	7
67	1	127	3	7	P	3	163	179	3	167	7	3	137	23	3	P	157	3	P	P
69	3	P	7	3	13	P	3	23	11	3	47	P	3	P	19	3	7	13	3	11
71	P	3	P	31	3	P	11	3	7	193	3	13	P	3	109	7	3	11	67	3
73	7	P	3	P	11	3	61	7	3	P	P	3	P	89	3	11	31	3	13	101
77	11	3	191	P	3	7	13	3	P	P	3	11	7	3	P	P	3	P	103	3
79	13	P	3	41	7	3	131	43	3	11	P	3	19	13	3	P	P	3	7	P
81	3	17	79	3	19	139	3	11	47	3	7	P	3	43	P	3	53	7	3	157
83	P	3	7	P	3	11	P	3	P	7	3	23	61	3	13	19	3	P	P	3
87	3	11	13	3	17	P	3	7	211	3	73	P	3	79	7	3	113	53	3	P
89	P	3	151	197	3	17	7	3	P	13	3	P	P	3	P	P	3	P	29	3
91	29	P	3	13	P	3	17	P	3	31	23	3	7	P	3	257	13	3	P	7
93	3	37	P	3	7	P	3	17	151	3	13	7	3	19	P	3	139	11	3	1
97	157	53	3	67	29	3	P	P	3	7	229	3	173	11	3	23	7	3	43	97
99	3	7	167	3	P	13	3	67	7	3	17	11	3	P	P	3	P	151	3	53

	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699
01	3	11	7	3	73	P	3	23	107	3	P	43	3	37	P	3	7	47	3	13
03	13	3	241	167	3	61	31	3	7	P	3	19	P	3	P	7	3	43	29	3
07	3	13	P	3	67	P	3	127	83	3	151	29	3	7	13	3	47	11	3	53
09	47	3	P	83	3	7	19	3	53	P	3	P	7	3	31	11	3	P	P	3
11	23	P	3	P	7	3	P	P	3	137	P	3	67	11	3	13	151	3	7	P
13	3	P	P	3	37	131	3	P	P	3	7	11	3	P	41	3	67	7	3	151
17	17	7	3	53	31	3	59	11	3	P	13	3	19	P	3	7	43	3	11	139
19	3	17	P	3	13	11	3	7	P	3	P	P	3	103	7	3	11	13	3	29
21	251	3	17	11	3	P	7	3	P	41	3	13	P	3	11	19	3	113	P	3
23	P	11	3	17	53	3	163	19	3	157	23	3	7	181	3	37	P	3	13	7
27	59	3	P	7	3	17	13	3	11	P	3	P	37	3	P	251	3	7	P	3
29	13	193	3	P	41	3	11	P	3	7	P	3	107	13	3	23	7	3	P	P
31	3	7	31	3	11	P	3	13	7	3	P	73	3	19	P	3	179	103	3	P
33	7	3	11	23	3	19	P	3	17	29	3	257	P	3	7	31	3	137	P	3
37	3	61	13	3	P	7	3	P	19	3	17	47	3	P	23	3	83	P	3	7
39	19	3	P	37	3	P	P	3	23	13	3	7	P	3	P	P	3	P	7	3
41	P	P	3	7	89	3	83	53	3	71	7	3	17	P	3	197	11	3	211	P
43	3	83	7	3	P	P	3	P	43	3	13	P	3	17	11	3	7	97	3	23
47	7	P	3	41	P	3	19	7	3	P	11	3	P	31	3	17	257	3	P	113
49	3	23	139	3	P	13	3	P	11	3	29	P	3	7	37	3	17	19	3	11
51	17	3	131	P	3	7	11	3	31	19	3	P	7	3	199	157	3	11	23	3
53	P	17	3	29	7	3	13	197	3	53	199	3	23	223	3	11	P	3	7	13
57	11	3	7	17	3	179	71	3	37	7	3	11	P	3	P	P	3	79	P	3
59	P	7	3	197	17	3	P	29	3	11	53	3	P	43	3	7	41	3	P	P
61	3	P	P	3	223	17	3	7	13	3	P	23	3	139	7	3	P	P	3	43
63	29	3	13	137	3	11	7	3	P	P	3	P	P	3	P	13	3	P	19	3
67	3	11	19	3	7	P	3	P	17	3	P	7	3	71	P	3	13	P	3	31
69	43	3	233	7	3	191	P	3	61	17	3	263	113	3	127	73	3	7	109	3
71	P	P	3	P	13	3	43	P	3	7	17	3	53	P	3	29	7	3	107	11
73	3	7	67	3	P	47	3	97	7	3	P	13	3	173	P	3	19	11	3	167
77	19	79	3	101	P	3	7	P	3	23	67	3	13	7	3	41	P	3	P	19
79	3	29	P	3	31	7	3	109	P	3	37	11	3	P	17	3	59	P	3	7
81	13	3	P	19	3	P	173	3	P	11	3	7	29	3	P	17	3	31	7	3
83	103	41	3	7	P	3	P	11	3	101	7	3	79	P	3	149	17	3	11	47
87	P	3	23	11	3	107	P	3	7	149	3	43	193	3	11	7	3	19	17	3
89	7	11	3	P	P	3	149	7	3	19	59	3	11	P	3	13	227	3	47	17
91	3	19	47	3	P	113	3	P	P	3	11	P	3	7	P	3	P	101	3	P
93	149	3	31	13	3	7	73	3	11	P	3	P	7	3	P	P	3	71	37	3
97	3	47	163	3	11	P	3	89	P	3	7	P	3	29	P	3	P	7	3	P
99	P	3	7	P	3	181	P	3	P	7	3	13	23	3	P	79	3	223	P	3

	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719
01	P	3	P	7	3	P	17	3	101	P	3	97	13	3	11	127	3	7	19	3
03	P	11	3	229	23	3	13	17	3	7	19	3	11	113	3	P	7	3	59	13
07	7	3	P	167	3	P	P	3	11	17	3	211	31	3	7	23	3	P	P	3
09	P	13	3	P	181	3	7	P	3	23	17	3	P	7	3	43	101	3	P	P
11	3	P	61	3	11	7	3	31	13	3	P	17	3	29	P	3	19	P	3	7
13	53	3	11	167	3	107	241	3	19	P	3	3	17	3	P	13	3	P	7	3
17	3	P	7	3	67	151	3	P	23	3	47	19	3	P	17	3	7	29	3	P
19	P	3	23	19	3	97	P	3	7	P	3	P	229	3	P	7	3	P	11	3
21	7	P	3	P	13	3	P	7	3	P	29	3	67	73	3	37	11	3	P	23
23	3	P	P	3	P	109	3	197	P	3	P	13	3	7	11	3	67	17	3	71
27	239	23	3	P	7	3	P	107	3	19	11	3	13	P	3	P	41	3	7	17
29	3	19	P	3	P	P	3	P	11	3	7	P	3	P	P	3	83	7	3	11
31	13	3	7	53	3	251	11	3	193	7	3	83	19	3	61	233	3	11	109	3
33	59	7	3	61	11	3	23	13	3	89	251	3	P	P	3	7	P	3	29	P
37	11	3	P	37	3	P	7	3	13	P	3	11	P	3	P	P	3	23	P	3
39	P	P	3	31	P	3	P	127	3	11	P	3	7	P	3	13	71	3	19	7
41	3	P	P	3	7	23	3	11	P	3	19	7	3	P	199	3	31	P	3	P
43	89	3	19	7	3	11	41	3	P	61	3	P	191	3	P	29	3	7	P	3
47	3	7	199	3	13	19	3	263	7	3	23	P	3	P	37	3	P	13	3	P
49	7	3	P	103	3	P	31	3	P	P	3	13	P	3	7	P	3	157	P	3
51	P	29	3	P	P	3	7	139	3	P	227	3	43	7	3	P	137	3	13	11
53	3	31	163	3	47	7	3	P	P	3	41	P	3	P	P	3	79	11	3	7
57	13	P	3	7	P	3	P	173	3	P	7	3	P	11	3	163	131	3	181	47
59	3	17	7	3	P	37	3	13	59	3	P	11	3	P	19	3	7	73	3	227
61	P	3	17	71	3	41	19	3	7	11	3	P	P	3	13	7	3	P	P	3
63	7	P	3	17	31	3	P	7	3	29	179	3	P	P	3	P	P	3	11	P
67	P	3	29	11	3	7	P	3	P	13	3	P	7	3	11	59	3	43	P	3
69	41	11	3	13	7	3	17	P	3	P	P	3	11	23	3	P	13	3	7	79
71	3	47	P	3	19	P	3	17	131	3	7	P	3	149	P	3	P	7	3	P
73	79	3	7	P	3	P	29	3	11	7	3	103	263	3	P	19	3	13	41	3
77	3	P	31	3	11	13	3	7	P	3	17	109	3	137	7	3	229	P	3	167
79	P	3	11	P	3	163	7	3	P	P	3	17	13	3	P	31	3	179	P	3
81	11	P	3	P	P	3	13	37	3	167	P	3	7	41	3	47	43	3	P	7
83	3	P	67	3	7	P	3	P	73	3	31	7	3	13	P	3	97	23	3	167
87	109	13	3	59	P	3	P	71	3	7	67	3	P	P	3	17	7	3	P	P
89	3	7	P	3	P	P	3	29	7	3	P	257	3	P	11	3	17	P	3	193
91	7	3	13	43	3	73	223	3	P	P	3	P	11	3	7	13	3	17	29	3
93	29	17	3	P	157	3	7	P	3	13	11	3	P	7	3	P	P	3	17	P
97	191	3	P	17	3	227	11	3	31	P	3	7	83	3	19	P	3	11	7	3
99	P	P	3	7	11	3	19	83	3	P	7	3	37	P	3	11	P	3	P	P

	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739
01	89	P	3	17	7	3	97	P	3	P	37	[illegible]	71	23	3	31	11	3	7	67
03	3	P	103	3	17	P	3	23	47	3	7	41	3	P	11	3	89	7	3	263
07	13	7	3	P	61	3	17	P	3	P	11	3	19	13	3	7	P	3	23	P
09	3	P	163	3	19	31	3	7	11	3	P	29	3	P	7	3	P	P	3	11
11	107	3	P	167	3	59	7	3	17	P	3	113	179	3	13	19	3	11	31	3
13	23	37	3	P	11	3	P	19	3	17	P	3	7	167	3	11	P	3	223	7
17	11	3	257	7	3	127	P	3	P	13	3	11	211	3	P	P	3	7	97	3
19	P	41	3	13	139	3	101	P	3	7	P	3	17	157	3	37	7	3	P	193
21	3	7	P	3	P	47	3	11	7	3	13	P	3	17	P	3	83	P	3	29
23	7	3	P	31	3	11	P	3	P	P	3	83	37	3	7	P	3	13	P	3
27	3	11	P	3	23	7	3	P	19	3	103	P	3	P	101	3	17	P	3	7
29	17	3	P	151	3	29	59	3	67	233	3	7	13	3	97	P	3	17	7	3
31	P	17	3	7	P	3	13	257	3	P	7	3	67	P	3	23	29	3	17	11
33	3	53	7	3	113	P	3	P	173	3	199	P	3	13	P	3	7	11	3	17
37	7	13	3	P	17	3	19	7	3	P	P	3	P	11	3	151	P	3	47	107
39	3	P	29	3	107	17	3	P	13	3	P	11	3	7	23	3	211	19	3	P
41	61	3	13	P	3	7	17	3	23	11	3	P	7	3	271	13	3	37	41	3
43	P	19	3	73	7	3	P	11	3	13	P	3	P	71	3	251	P	3	7	P
47	P	3	7	11	3	P	P	3	97	7	3	193	89	3	11	P	3	29	P	3
49	109	7	3	71	13	3	P	23	3	P	17	3	11	41	3	7	47	3	P	73
51	3	23	P	3	53	P	3	7	263	3	7	13	3	P	7	3	P	P	3	P
53	P	3	P	P	3	13	7	3	11	P	3	191	17	3	P	P	3	131	13	3
57	3	59	19	7	7	73	3	31	41	3	43	7	3	109	17	3	73	P	3	13
59	13	3	11	3	3	P	113	3	P	P	3	149	P	3	P	17	3	7	P	3
61	11	P	3	269	P	3	P	13	3	7	P	3	61	P	3	P	7	3	233	P
63	3	7	127	3	233	149	3	P	7	3	P	23	3	P	13	3	19	17	3	37
67	19	P	3	P	P	3	7	P	3	131	31	3	41	7	3	13	11	3	P	17
69	3	P	P	3	P	7	3	53	P	3	89	19	3	P	11	3	23	71	3	7
71	97	3	P	13	3	31	P	3	P	43	3	7	11	3	P	P	3	P	7	3
73	P	P	3	7	23	3	P	61	3	P	7	3	47	239	3	29	P	3	31	P
77	P	3	P	157	3	P	11	3	7	P	3	13	P	3	P	7	3	11	P	3
79	7	89	3	P	11	3	P	7	3	19	P	3	127	P	3	11	P	3	13	29
81	3	19	11	P	P	181	3	73	31	3	107	P	3	7	179	3	P	89	3	167
83	11	3	41	3	3	7	13	3	P	59	3	11	7	3	P	P	3	P	P	3
87	3	37	P	3	173	29	3	11	23	3	7	163	3	P	43	3	31	7	3	241
89	P	3	7	191	3	11	P	3	P	7	3	P	83	3	13	P	3	113	37	3
91	P	7	3	11	71	3	157	83	3	47	P	3	P	79	3	7	59	3	19	23
93	3	11	13	3	P	229	3	7	P	3	19	5[illegible]	3	23	7	3	P	109	3	61
97	17	23	3	13	P	3	139	P	3	P	67	3	7	19	3	P	13	3	P	7
99	3	17	197	3	7	19	3	43	269	3	13	[illegible]	3	29	67	3	P	11	3	P

	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759
01	3	P	P	3	47	7	3	11	131	3	179	13	3	257	P	3	19	17	3	7
03	43	3	P	67	3	11	61	3	19	P	3	7	157	3	P	P	3	P	7	3
07	3	11	7	3	37	P	3	P	239	3	107	19	3	P	P	3	7	P	3	13
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13	3	13	47	3	P	269	3	P	79	3	P	31	3	7	13	3	83	11	3	P
17	P	137	3	P	7	3	29	P	3	19	P	3	P	11	3	13	P	3	7	89
19	3	19	P	3	P	43	3	P	23	3	7	11	3	109	53	3	P	7	3	31
21	P	3	7	13	3	P	71	3	P	7	3	43	19	3	199	P	3	P	P	3
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27	P	3	199	11	3	P	7	3	P	31	3	13	P	3	11	P	3	41	191	3
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31	3	P	P	3	7	P	3	P	P	3	11	7	3	71	P	3	53	P	3	P
33	101	3	19	7	3	73	13	3	11	P	3	P	23	3	241	P	3	7	P	3
37	3	7	61	3	11	19	3	13	7	3	P	227	3	P	P	3	43	53	3	P
39	7	3	11	79	3	131	101	3	67	137	3	29	P	3	7	P	3	23	181	3
41	11	151	3	17	P	3	7	31	3	P	P	3	67	7	3	P	P	3	149	P
43	3	P	13	3	17	7	3	41	P	3	101	163	3	59	37	3	67	P	3	7
47	P	53	3	7	109	3	17	P	3	149	7	3	47	P	3	31	11	3	73	173
49	3	P	7	3	P	127	3	17	29	3	13	P	3	151	11	3	7	211	3	53
51	P	3	41	149	3	P	19	3	7	241	3	223	11	3	197	7	3	13	101	3
53	7	29	3	P	P	3	P	7	3	17	11	3	P	P	3	P	151	3	P	151
57	103	3	P	P	3	7	11	3	P	23	3	17	7	3	61	P	3	11	31	3
59	31	P	3	23	7	3	13	P	3	P	47	3	17	179	3	11	P	3	7	13
61	3	P	11	3	19	P	3	P	P	3	7	P	3	11	59	3	29	7	3	37
63	11	3	7	P	3	173	197	3	43	7	3	11	73	3	17	19	3	239	107	3
67	3	P	23	3	113	P	3	7	13	3	271	P	3	P	7	3	17	P	3	P
69	17	3	13	31	3	11	7	3	P	61	3	P	P	3	163	13	3	17	P	3
71	P	17	3	11	P	3	89	P	3	13	41	3	7	23	3	P	31	3	17	7
73	3	11	17	3	7	P	3	23	P	3	37	7	3	19	71	3	13	P	3	17
77	P	P	3	P	13	3	53	37	3	7	193	3	P	P	3	P	7	3	23	11
79	3	7	P	3	71	17	3	P	7	3	P	13	3	43	P	3	P	11	3	P
81	7	3	59	P	3	13	17	3	103	97	3	P	83	3	7	11	3	P	13	3
83	23	31	3	P	211	3	7	17	3	167	P	3	13	7	3	P	P	3	P	P
87	13	3	P	73	3	P	P	3	P	11	3	7	79	3	19	131	3	P	7	3
89	43	P	3	7	P	3	19	11	3	31	7	3	P	P	3	269	P	3	11	P
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93	P	3	P	11	3	97	113	3	7	19	3	P	17	3	11	7	3	P	29	3
97	3	P	P	3	23	P	3	P	P	3	11	29	3	7	17	3	59	P	3	P
99	P	3	191	13	3	7	P	3	11	37	3	139	7	3	103	17	3	229	71	3

	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779
01	P	3	181	41	3	113	7	3	P	11	3	P	P	3	17	19	3	13	P	3
03	P	P	3	P	P	3	P	11	3	53	P	3	7	23	3	17	71	3	11	7
07	17	3	P	7	3	P	P	3	89	P	3	83	13	3	11	179	3	7	29	3
09	29	11	3	137	109	3	13	79	3	7	53	3	11	97	3	P	7	3	17	13
11	3	7	17	3	43	P	3	41	7	3	11	29	3	13	199	3	P	P	3	17
13	7	3	P	17	3	19	23	3	11	P	3	59	P	3	7	P	3	P	P	3
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19	19	3	11	167	3	P	17	3	P	P	3	7	37	3	P	13	3	P	7	3
21	11	163	3	7	P	3	193	17	3	13	7	3	31	167	3	P	P	3	59	67
23	3	P	7	3	P	59	3	73	17	3	P	233	3	P	139	3	7	P	3	29
27	7	269	3	127	13	3	19	7	3	43	17	3	29	53	3	P	11	3	223	149
29	3	P	31	3	23	103	3	277	P	3	P	13	3	7	11	3	149	19	3	P
31	P	3	P	37	3	7	P	3	P	19	3	137	7	3	P	31	3	P	13	3
33	139	19	3	P	7	3	197	P	3	107	11	3	13	17	3	23	29	3	7	P
37	13	3	7	23	3	P	11	3	P	7	3	P	P	3	211	17	3	11	277	3
39	P	7	3	97	11	3	173	13	3	47	41	3	P	P	3	7	17	3	P	59
41	3	13	11	3	P	P	3	7	43	3	P	P	3	11	7	3	P	17	3	41
43	11	3	P	P	3	P	7	3	13	P	3	11	P	3	43	P	3	P	17	3
47	3	P	19	3	7	41	3	11	P	3	P	7	3	P	P	3	P	P	3	23
49	113	3	P	7	3	11	P	3	31	P	3	179	P	3	41	P	3	7	P	3
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53	3	7	P	3	13	37	3	P	7	3	29	P	3	103	73	3	19	13	3	137
57	19	P	3	29	101	3	7	P	3	41	251	3	23	7	3	P	79	3	13	11
59	3	P	P	3	157	7	3	59	151	3	263	19	3	P	29	3	P	11	3	7
61	23	3	P	19	3	P	13	3	101	P	3	7	P	3	71	11	3	P	7	3
63	13	P	3	7	P	3	31	29	3	P	7	3	P	11	3	P	37	3	P	53
67	29	3	53	P	3	23	P	3	7	11	3	P	P	3	13	7	3	19	P	3
69	7	59	3	P	47	3	43	7	3	19	P	3	P	P	3	P	101	3	11	P
71	3	19	13	3	P	11	3	P	P	3	37	P	3	7	P	3	11	83	3	103
73	127	3	89	11	3	7	P	3	P	13	3	229	7	3	11	P	3	P	43	3
77	3	17	83	3	31	73	3	P	59	3	7	71	3	P	P	3	173	7	3	P
79	P	3	7	P	3	P	P	3	11	7	3	113	P	3	P	23	3	13	47	3
81	P	7	3	17	P	3	11	P	3	23	P	3	109	223	3	7	P	3	19	29
83	3	29	P	3	11	13	3	7	P	3	19	79	3	P	7	3	131	P	3	P
87	11	47	3	P	P	3	13	31	3	167	157	3	7	19	3	P	P	3	71	7
89	3	61	P	3	7	19	3	17	23	3	127	7	3	13	P	3	P	107	3	167
91	P	3	23	7	3	191	53	3	17	P	3	P	P	3	P	P	3	7	11	3
93	47	13	3	79	P	3	271	41	3	7	P	3	37	193	3	31	7	3	P	23
97	7	3	13	241	3	P	P	3	131	37	3	17	11	3	7	13	3	P	61	3
99	P	23	3	19	227	3	7	61	3	13	11	3	17	7	3	73	P	3	P	P

	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799
01	7	P	3	P	P	3	83	7	3	P	13	3	P	P	3	107	P	3	P	P
03	3	83	P	3	13	29	3	211	P	3	199	P	3	7	271	3	23	13	3	P
07	P	37	3	P	7	3	P	P	3	19	41	3	103	71	3	43	11	3	7	P
09	3	19	197	3	89	P	3	31	P	3	7	239	3	P	11	3	P	7	3	41
11	181	3	7	P	3	P	13	3	53	7	3	P	11	3	P	23	3	79	P	3
13	13	7	3	71	19	3	127	P	3	23	11	3	113	13	3	7	P	3	P	157
17	P	3	17	P	3	P	7	3	269	53	3	61	37	3	13	131	3	11	P	3
19	61	191	3	17	11	3	29	223	3	P	31	3	7	P	3	11	103	3	19	7
21	3	P	11	3	7	233	3	P	23	3	19	7	3	11	43	3	P	29	3	229
23	11	3	19	7	3	17	P	3	P	13	3	11	227	3	P	281	3	7	P	3
27	3	7	137	3	P	19	3	11	7	3	13	67	3	23	P	3	P	61	3	257
29	7	3	P	29	3	11	61	3	17	1	3	53	P	3	7	67	3	13	P	3
31	P	23	3	11	107	3	7	131	3	17	P	3	P	7	3	P	P	3	97	67
33	3	11	P	3	41	7	3	43	31	3	17	P	3	P	P	3	P	71	3	7
37	73	P	3	7	P	3	13	P	3	193	7	3	17	P	3	P	97	3	29	11
39	3	P	7	3	P	P	3	71	P	3	P	P	3	13	19	3	7	11	3	P
41	P	3	P	P	3	P	19	3	7	P	3	29	P	3	17	7	3	23	P	3
43	7	13	3	157	47	3	P	7	3	89	P	3	109	11	3	17	73	3	P	P
47	17	3	13	P	3	7	31	3	37	11	3	P	7	3	53	13	3	17	P	3
49	P	17	3	47	7	3	P	11	3	13	137	3	19	P	3	P	23	3	7	31
51	3	31	17	3	19	11	3	61	29	3	7	P	3	73	P	3	11	7	3	17
53	89	3	7	11	3	P	P	3	P	7	3	P	41	3	11	19	3	173	47	3
57	3	P	139	3	67	17	3	7	P	3	11	13	3	P	7	3	P	P	3	37
59	P	3	P	127	3	13	7	3	11	23	3	P	P	3	181	P	3	47	13	3
61	251	47	3	23	31	3	11	17	3	281	173	3	7	61	3	P	37	3	P	7
63	3	P	61	3	7	251	3	79	17	3	P	7	3	19	229	3	29	31	3	13
67	11	P	3	P	P	3	97	13	3	7	17	3	31	P	3	251	7	3	P	P
69	3	7	23	3	131	P	3	227	7	3	37	17	3	139	13	3	P	P	3	211
71	7	3	29	109	3	P	151	3	13	157	3	41	17	3	7	47	3	241	11	3
73	101	P	3	181	97	3	7	37	3	151	107	3	P	7	3	13	11	3	P	P
77	163	3	P	13	3	P	29	3	P	P	3	7	11	3	19	17	3	P	7	3
79	1	P	3	7	P	3	19	P	3	P	7	3	P	P	3	P	17	3	23	P
81	3	37	7	3	13	179	3	P	11	3	31	P	3	163	P	3	7	13	3	11
83	113	3	P	103	3	P	11	3	7	19	3	13	P	3	61	7	3	11	17	3
87	3	41	11	3	P	89	3	P	P	3	P	P	3	7	101	3	P	23	3	P
89	11	3	79	43	3	7	13	3	P	P	3	11	7	3	29	P	3	73	P	3
91	13	P	3	277	7	3	P	P	3	11	139	3	37	13	3	19	P	3	7	41
93	3	P	59	3	53	P	3	11	P	3	7	P	3	P	P	3	P	7	3	167
97	29	7	3	11	P	3	P	P	3	197	19	3	179	P	3	7	P	3	109	P
99	3	P	13	3	23	53	3	7	257	3	83	29	3	P	7	3	P	199	3	P

	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819
01	3	7	11	3	37	79	3	P	7	3	P	P	3	11	P	3	13	P	3	P
03	7	3	139	131	3	19	P	3	P	17	3	11	P	3	7	149	3	P	179	3
07	3	P	P	3	P	7	3	11	19	3	59	13	3	P	127	3	79	P	3	7
09	19	3	P	P	3	11	149	3	P	P	3	7	17	3	P	P	3	101	7	3
11	29	P	3	7	191	3	P	43	3	P	7	3	13	17	3	37	P	3	23	101
13	3	11	7	3	97	P	3	P	211	3	P	29	3	31	17	3	7	41	3	13
17	7	113	3	P	29	3	19	7	3	P	P	3	241	233	3	P	17	3	P	11
19	3	13	97	3	137	73	3	53	P	3	P	P	3	7	13	3	P	11	3	P
21	P	3	P	31	3	7	P	3	13	19	3	23	7	3	P	11	3	71	17	3
23	43	19	3	47	7	3	37	89	3	P	P	3	P	11	3	13	31	3	7	17
27	79	3	7	13	3	P	P	3	131	7	3	31	43	3	107	P	3	P	47	3
29	191	7	3	P	P	3	P	11	3	P	13	3	29	167	3	7	P	3	11	P
31	3	227	P	3	13	11	3	7	P	3	P	P	3	P	7	3	11	13	3	P
33	163	3	P	67	3	29	7	3	P	P	3	13	P	3	11	P	3	37	19	3
37	3	127	19	3	7	P	3	P	229	3	11	7	3	163	31	3	P	P	3	P
39	P	3	P	7	3	43	13	3	11	29	3	41	P	3	P	67	3	7	P	3
41	13	P	3	P	257	3	11	263	3	7	P	3	137	13	3	73	7	3	223	67
43	3	7	29	3	11	239	3	13	7	3	P	53	3	P	23	3	19	43	3	P
47	11	P	3	P	P	3	7	P	3	61	P	3	113	7	3	P	P	3	P	19
49	3	P	13	3	P	7	3	P	P	3	P	19	3	P	79	3	P	P	3	7
51	P	3	P	19	3	109	P	3	233	13	3	7	31	3	47	P	3	29	7	3
53	17	P	3	7	43	3	59	23	3	P	7	3	193	P	3	P	11	3	P	P
57	223	3	17	107	3	P	P	3	7	73	3	P	11	3	P	7	3	13	23	3
59	7	71	3	17	61	3	79	7	3	19	11	3	23	P	3	P	37	3	109	41
61	3	19	83	3	17	17	3	P	11	3	103	277	3	7	29	3	127	P	3	11
63	23	3	P	P	3	7	11	3	P	P	3	P	7	3	P	P	3	11	71	3
67	3	P	11	3	67	P	3	17	193	3	7	23	3	11	41	3	P	7	3	P
69	11	3	7	P	3	23	P	3	17	7	3	11	181	3	257	P	3	P	P	3
71	P	7	3	179	P	3	P	37	3	11	P	3	67	P	3	7	P	3	19	P
73	3	P	P	3	P	197	3	7	13	3	17	P	3	P	7	3	23	P	3	P
77	P	P	3	11	23	3	P	P	3	13	P	3	7	19	3	29	P	3	41	7
79	3	11	P	3	7	19	3	P	31	3	89	7	3	17	59	3	13	53	3	73
81	73	3	43	7	3	61	P	3	29	47	3	P	P	3	17	23	3	7	37	3
83	53	181	3	31	13	3	P	P	3	7	P	3	P	97	3	17	7	3	P	11
87	7	3	P	P	3	13	P	3	47	109	3	19	29	3	7	11	3	17	13	3
89	283	17	3	19	P	3	7	P	3	P	131	3	13	7	3	83	P	3	17	163
91	3	P	17	3	P	7	3	173	23	3	83	11	3	199	19	3	151	89	3	7
93	13	3	23	17	3	83	19	3	41	11	3	7	P	3	227	139	3	263	7	3
97	3	13	7	3	101	11	3	43	P	3	P	P	3	23	13	3	7	157	3	167
99	173	3	59	11	3	P	17	3	7	107	3	P	P	3	11	7	3	P	P	3

	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839
01	43	3	7	P	3	17	P	3	31	7	3	P	19	3	P	11	3	P	47	3
03	P	7	3	13	19	3	17	191	3	P	P	3	P	11	3	7	13	3	181	P
07	P	3	P	P	3	P	7	3	17	11	3	41	P	3	P	113	3	13	43	3
09	P	47	3	53	23	3	P	11	3	17	P	3	7	227	3	37	P	3	11	7
11	3	157	229	3	7	11	3	107	P	3	17	7	3	P	239	3	11	97	3	P
13	P	3	19	7	3	109	P	3	P	P	3	17	13	3	11	23	3	7	P	3
17	3	7	P	3	73	19	3	181	7	3	11	P	3	13	P	3	P	P	3	31
19	7	3	P	263	3	179	P	3	11	283	3	43	P	3	7	47	3	P	79	3
21	P	13	3	191	P	3	7	P	3	101	61	3	P	7	3	17	P	3	109	P
23	3	41	P	3	11	7	3	P	13	3	P	103	3	97	P	3	17	29	3	7
27	11	17	3	7	139	3	53	P	3	13	7	3	P	103	3	101	241	3	17	23
29	3	P	7	3	31	P	3	P	113	3	79	97	3	23	19	3	7	101	3	17
31	P	3	P	17	3	P	19	3	7	127	3	59	P	3	P	7	3	31	11	3
33	7	23	3	281	13	3	P	7	3	239	43	3	P	167	3	103	11	3	P	P
37	P	3	P	137	3	7	17	3	P	197	3	P	7	3	P	P	3	P	13	3
39	P	P	3	P	7	3	23	17	3	P	11	3	13	P	3	139	P	3	7	P
41	3	P	P	3	19	59	3	97	11	3	7	71	3	P	181	3	P	7	3	11
43	13	3	7	67	3	197	11	3	37	7	3	29	P	3	P	19	3	11	P	3
47	3	13	11	3	29	23	3	7	P	3	P	17	3	11	7	3	233	83	3	127
49	11	3	233	P	3	P	7	3	13	109	3	11	17	3	P	29	3	89	191	3
51	P	113	3	P	41	3	P	83	3	11	53	3	7	17	3	13	23	3	71	7
53	3	P	83	3	7	31	3	11	29	3	23	7	3	19	17	3	P	61	3	37
57	31	29	3	11	P	3	P	P	3	7	13	3	P	P	3	P	7	3	P	59
59	3	7	43	3	13	P	3	P	7	3	P	137	3	31	P	3	269	13	3	113
61	7	3	P	P	3	P	131	3	41	23	3	13	139	3	7	P	3	P	17	3
63	137	P	3	23	P	3	7	P	3	P	P	3	53	7	3	P	P	3	13	11
67	P	3	P	31	3	P	13	3	173	163	3	7	P	3	19	11	3	211	7	3
69	13	127	3	7	P	3	19	37	3	29	7	3	P	11	3	193	31	3	P	P
71	3	P	7	3	P	P	3	13	79	3	P	11	3	263	P	3	7	19	3	131
73	P	3	29	P	3	71	47	3	7	11	3	31	P	3	13	7	3	P	P	3
77	3	37	13	3	67	11	3	23	179	3	P	P	3	7	P	3	11	P	3	79
79	211	3	P	11	3	7	29	3	67	13	3	223	7	3	11	P	3	199	37	3
81	79	11	3	13	7	3	89	P	3	P	251	3	11	199	3	19	13	3	7	137
83	3	P	107	3	P	269	3	19	P	3	7	193	3	P	31	3	67	7	3	P
87	23	7	3	P	P	3	11	P	3	31	19	3	37	61	3	7	53	3	149	P
89	3	P	19	3	11	13	3	7	P	3	P	41	3	P	7	3	P	23	3	47
91	103	3	11	47	3	P	7	3	P	37	3	23	13	3	29	P	3	P	P	3
93	11	P	3	P	P	3	13	P	3	149	P	3	7	89	3	179	127	3	43	7
97	53	3	17	7	3	151	41	3	19	P	3	271	31	3	P	P	3	7	11	3
99	19	13	3	17	P	3	P	P	3	7	23	3	P	P	3	41	7	3	53	19

	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859
01	167	37	3	7	P	3	11	P	3	59	7	3	P	197	3	13	P	3	239	17
03	3	31	7	3	11	P	3	71	137	3	167	P	3	P	41	3	7	P	3	P
07	7	151	3	P	P	3	19	7	3	197	13	3	139	23	3	37	P	3	53	271
09	3	241	107	3	13	P	3	23	P	3	P	P	3	7	223	3	59	13	3	137
11	P	3	P	57	3	7	211	3	P	19	3	13	7	3	P	233	3	P	11	3
13	29	19	3	P	7	3	191	P	3	P	151	3	P	P	3	P	11	3	7	53
17	P	3	7	P	3	223	13	3	89	7	3	47	11	3	229	P	3	P	P	3
19	13	7	3	P	29	3	37	P	3	P	11	3	31	13	3	7	P	3	P	151
21	3	P	P	3	P	P	3	7	11	3	P	P	3	41	7	3	P	23	3	11
23	73	3	P	37	3	P	7	3	271	163	3	23	P	3	13	P	3	11	19	3
27	3	P	11	3	7	181	3	193	P	3	P	7	3	11	P	3	P	59	3	29
29	11	3	P	7	3	137	P	3	41	13	3	11	P	3	P	31	3	7	P	3
31	17	P	3	13	P	3	P	P	3	7	23	3	29	P	3	P	7	3	P	P
33	3	7	131	3	23	P	3	11	7	3	13	P	3	P	37	3	19	P	3	P
37	19	P	3	11	P	3	7	P	3	157	P	3	P	7	3	23	29	3	P	19
39	3	11	P	3	17	7	3	101	43	3	277	19	3	61	P	3	P	83	3	7
41	31	3	61	19	3	17	53	3	37	29	3	7	13	3	43	113	3	179	7	3
43	229	P	3	7	P	3	13	83	3	173	7	3	P	31	3	131	P	3	P	11
47	P	3	P	P	3	59	47	3	7	P	3	P	P	3	P	7	3	19	P	3
49	7	13	3	P	P	3	P	7	3	17	P	3	163	11	3	P	41	3	293	61
51	3	19	173	3	79	P	3	P	13	3	17	11	3	7	P	3	97	P	3	23
53	P	3	13	67	3	7	P	3	53	11	3	17	7	3	P	13	3	29	P	3
57	3	23	109	3	P	11	3	131	P	3	7	31	3	17	97	3	11	7	3	43
59	P	3	7	11	3	P	P	3	P	7	3	P	P	3	11	67	3	191	23	3
61	P	7	3	29	13	3	31	P	3	P	P	3	11	P	3	7	P	3	19	67
63	3	P	P	3	P	103	3	7	113	3	11	13	3	P	7	3	17	139	3	31
67	P	17	3	239	P	3	11	29	3	P	257	3	7	19	3	41	P	3	17	7
69	3	73	17	3	7	19	3	103	P	3	97	7	3	P	P	3	P	199	3	13
71	13	3	11	7	3	23	227	3	P	31	3	53	71	3	127	P	3	7	43	3
73	11	41	3	139	17	3	P	13	3	7	241	3	269	59	3	83	7	3	79	149
77	7	3	71	P	3	83	17	3	13	P	3	19	53	3	7	P	3	31	11	3
79	83	P	3	19	23	3	7	17	3	P	149	3	107	7	3	13	11	3	157	127
81	3	P	271	3	P	7	3	149	17	3	P	103	3	P	11	3	47	P	3	7
83	47	3	89	13	3	41	19	3	29	17	3	7	11	3	73	23	3	109	7	3
87	3	29	7	3	13	251	3	P	11	3	P	17	3	103	P	3	7	13	3	11
89	P	3	31	P	3	P	11	3	7	37	3	13	17	3	53	7	3	11	P	3
91	7	P	3	P	11	3	P	7	3	P	P	3	19	17	3	11	P	3	13	P
93	3	59	11	3	19	29	3	P	23	3	P	P	3	7	17	3	67	P	3	113
97	13	269	3	37	7	3	P	19	3	11	43	3	P	13	3	P	17	3	7	23
99	3	P	P	3	P	31	3	11	73	3	7	P	3	23	193	3	43	7	3	P

	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879
01	3	29	P	3	7	P	3	277	11	3	19	7	3	67	71	3	17	P	3	11
03	17	3	13	7	3	23	11	3	61	43	3	P	29	3	P	13	3	7	P	3
07	3	7	11	3	71	19	3	31	7	3	167	P	3	11	P	3	13	229	3	17
09	7	3	P	17	3	P	257	3	47	233	3	11	37	3	7	P	3	139	277	3
11	P	P	3	P	13	3	7	P	3	11	P	3	P	7	3	P	79	3	P	P
13	3	P	73	3	P	7	3	11	P	3	P	13	3	P	61	3	P	239	3	7
17	P	P	3	7	103	3	37	17	3	23	7	3	13	P	3	P	41	3	137	P
19	3	11	7	3	89	241	3	P	17	3	173	P	3	29	19	3	7	P	3	13
21	13	3	151	37	3	31	19	3	7	17	3	P	P	3	P	7	3	P	53	3
23	7	71	3	P	P	3	29	7	3	P	17	3	P	P	3	P	P	3	31	11
27	P	3	23	173	3	7	P	3	13	P	3	151	7	3	P	11	3	37	71	3
29	P	43	3	131	7	3	P	P	3	P	29	3	19	11	3	13	P	3	7	23
31	3	P	53	3	19	P	3	43	31	3	7	11	3	23	17	3	P	7	3	P
33	227	3	7	13	3	P	41	3	71	7	3	P	83	3	P	17	3	59	P	3
37	3	P	83	3	13	11	3	7	P	3	P	79	3	P	7	3	11	13	3	47
39	97	3	P	11	3	P	7	3	37	P	3	13	23	3	11	P	3	P	17	3
41	139	11	3	P	P	3	23	127	3	227	P	3	7	167	3	P	P	3	13	7
43	3	P	P	3	7	37	3	P	P	3	11	7	3	19	P	3	P	P	3	P
47	13	277	3	79	137	3	11	223	3	7	61	3	43	13	3	P	7	3	107	31
49	3	7	P	3	11	23	3	13	7	3	P	P	3	113	157	3	P	47	3	37
51	7	3	11	P	3	41	73	3	P	P	3	P	P	3	7	29	3	P	59	3
53	11	101	3	P	P	3	7	P	3	89	263	3	P	7	3	P	23	3	P	281
57	47	3	P	P	3	101	193	3	P	13	3	7	P	3	19	P	3	127	7	3
59	41	29	3	7	31	3	19	101	3	P	7	3	71	P	3	P	11	3	103	P
61	3	P	7	3	P	P	3	53	P	3	13	43	3	199	11	3	7	19	3	P
63	89	3	P	67	3	107	79	3	7	19	3	101	11	3	149	7	3	13	41	3
67	3	199	281	3	P	13	3	P	11	3	83	67	3	7	47	3	29	P	3	11
69	P	3	P	P	3	7	11	3	P	P	3	61	7	3	23	67	3	11	P	3
71	17	P	3	P	7	3	13	P	3	29	P	3	197	41	3	11	P	3	7	13
73	3	17	11	3	43	P	3	19	109	3	7	179	3	11	P	3	73	7	3	P
77	P	7	3	17	P	3	P	107	3	11	19	3	P	23	3	7	43	3	P	P
79	3	P	19	3	17	P	3	7	13	3	31	P	3	59	7	3	P	61	3	97
81	59	3	13	P	3	11	7	3	283	P	3	P	P	3	P	13	3	41	P	3
83	P	P	3	11	197	3	17	P	3	13	P	3	7	P	3	P	P	3	23	7
87	31	3	P	7	3	P	23	3	17	37	3	P	191	3	89	P	3	7	P	3
89	19	79	3	P	13	3	P	59	3	7	73	3	41	31	3	P	7	3	179	11
91	3	7	P	3	P	131	3	229	7	3	17	13	3	281	P	3	P	11	3	P
93	7	3	P	19	3	13	P	3	31	79	3	17	P	3	7	11	3	P	13	3
97	3	P	P	3	67	7	3	29	113	3	251	11	3	17	59	3	P	P	3	7
99	13	3	211	P	3	P	281	3	67	11	3	7	P	3	17	251	3	19	7	3

	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899
01	P	3	193	P	3	7	41	3	P	19	3	P	7	3	13	P	3	271	89	3
03	P	19	3	227	7	3	251	107	3	P	P	3	P	P	3	37	P	3	7	11
07	P	3	7	233	3	67	P	3	P	7	3	P	37	3	29	11	3	109	31	3
09	17	7	3	13	211	3	P	43	3	67	P	3	P	11	3	7	13	3	P	P
11	3	17	P	3	P	61	3	7	P	3	13	11	3	31	7	3	P	283	3	47
13	283	3	17	47	3	P	7	3	P	11	3	P	P	3	P	P	3	13	19	3
17	3	P	19	3	7	11	3	79	P	3	P	7	3	P	P	3	11	73	3	P
19	P	3	47	7	3	17	23	3	P	P	3	P	13	3	11	P	3	7	P	3
21	23	11	3	P	29	3	13	P	3	7	P	3	11	179	3	P	7	3	P	13
23	3	7	P	3	P	P	3	17	7	3	11	P	3	13	223	3	19	23	3	P
27	19	13	3	P	P	3	7	83	3	17	127	3	P	7	3	P	P	3	43	19
29	3	P	83	3	11	7	3	P	13	3	17	19	3	P	37	3	47	53	3	7
31	47	3	11	19	3	223	263	3	211	113	3	7	P	3	P	13	3	61	7	3
33	11	31	3	7	191	3	61	89	3	13	7	3	17	157	3	P	P	3	P	139
37	P	3	P	P	3	29	151	3	7	P	3	P	P	3	17	7	3	19	11	3
39	7	53	3	P	13	3	137	7	3	19	269	3	233	41	3	17	11	3	P	P
41	3	19	P	3	59	37	3	P	73	3	P	13	3	7	11	3	17	43	3	53
43	17	3	79	23	3	7	P	3	P	29	3	97	7	3	P	151	3	17	13	3
47	3	181	17	3	241	P	3	P	11	3	7	239	3	47	23	3	157	7	3	11
49	13	3	7	17	3	73	11	3	23	7	3	59	31	3	P	149	3	11	P	3
51	191	7	3	53	11	3	P	13	3	P	P	3	149	199	3	7	37	3	19	293
53	3	13	11	3	197	17	3	7	P	3	19	P	3	11	7	3	P	P	3	23
57	173	199	3	149	53	3	P	17	3	11	P	3	7	19	3	13	P	3	59	7
59	3	23	P	3	7	19	3	11	17	3	29	7	3	193	P	3	P	P	3	P
61	107	3	P	7	3	11	P	3	P	17	3	163	P	3	137	P	3	7	23	3
63	83	131	3	11	P	3	P	37	3	7	13	3	23	P	3	P	7	3	73	P
67	7	3	61	97	3	31	P	3	P	43	3	13	17	3	7	P	3	P	P	3
69	P	P	3	19	P	3	7	29	3	P	P	3	P	7	3	43	P	3	13	11
71	3	37	103	3	P	7	3	P	181	3	P	23	3	P	17	3	P	11	3	7
73	29	3	41	67	3	23	13	3	P	193	3	7	P	3	131	11	3	107	7	3
77	3	P	7	3	103	101	3	13	31	3	281	11	3	139	P	3	7	17	3	P
79	P	3	43	P	3	283	71	3	7	11	3	257	73	3	13	7	3	P	17	3
81	7	109	3	31	23	3	P	7	3	101	229	3	19	P	3	29	P	3	11	17
83	3	163	13	3	19	11	3	47	P	3	P	101	3	7	43	3	11	P	3	P
87	59	11	3	13	7	3	131	19	3	23	P	3	11	P	3	101	13	3	7	29
89	3	29	P	3	107	P	3	P	103	3	7	P	3	71	109	3	P	7	3	P
91	137	3	7	157	3	P	31	3	11	7	3	79	29	3	P	P	3	13	P	3
93	P	7	3	37	P	3	11	P	3	P	41	3	P	P	3	7	257	3	241	31
97	37	3	11	P	3	19	7	3	P	P	3	191	13	3	31	P	3	P	P	3
99	11	89	3	109	P	3	13	P	3	61	139	3	7	P	3	P	19	3	P	3

	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919
01	P	11	3	73	P	3	7	13	3	P	17	3	11	7	3	37	139	3	P	29
03	3	13	P	3	P	7	3	P	P	3	11	17	3	P	13	3	47	P	3	7
07	P	P	3	7	P	3	11	61	3	P	7	3	223	17	3	13	101	3	P	73
09	3	251	7	3	11	29	3	P	71	3	P	31	3	P	17	3	7	293	3	P
11	P	3	11	13	3	P	19	3	7	P	3	179	197	3	P	7	3	P	P	3
13	7	97	3	P	23	3	31	7	3	229	13	3	53	127	3	P	17	3	P	107
17	P	3	P	37	3	7	P	3	197	P	3	13	7	3	113	23	3	41	11	3
19	P	227	3	181	7	3	P	83	3	23	P	3	19	53	3	71	11	3	7	17
21	3	P	83	3	19	131	3	257	P	3	7	P	3	29	11	3	P	7	3	P
23	P	3	7	41	3	P	13	3	P	7	3	293	11	3	P	19	3	37	P	3
27	3	P	P	3	31	P	3	7	11	3	227	P	3	271	7	3	59	29	3	11
29	197	3	23	59	3	P	7	3	61	79	3	P	P	3	13	P	3	11	229	3
31	P	193	3	103	11	3	P	P	3	P	29	3	7	P	3	11	P	3	131	7
33	3	173	11	3	7	P	3	41	P	3	P	7	3	11	P	3	43	P	3	149
37	179	23	3	13	P	3	233	31	3	7	59	3	P	149	3	239	7	3	P	89
39	3	7	P	3	P	37	3	11	7	3	13	P	3	241	61	3	P	199	3	P
41	7	3	31	61	3	11	P	3	P	211	3	P	23	3	7	P	3	13	P	3
43	127	109	3	11	149	3	7	103	3	199	181	3	P	7	3	31	113	3	29	P
47	53	3	P	167	3	P	P	3	P	P	3	7	13	3	19	43	3	23	7	3
49	17	P	3	7	151	3	13	P	3	103	7	3	P	167	3	83	37	3	53	11
51	3	17	7	3	29	23	3	151	47	3	83	P	3	13	109	3	7	11	3	P
53	P	3	17	P	3	83	269	3	7	19	3	P	P	3	P	7	3	P	31	3
57	3	89	43	3	17	137	3	47	13	3	23	11	3	7	P	3	151	P	3	P
59	P	3	13	P	3	7	P	3	43	11	3	P	7	3	P	13	3	89	97	3
61	113	29	3	109	7	3	17	11	3	13	41	3	263	103	3	19	71	3	7	P
63	3	P	P	3	61	11	3	17	P	3	7	P	3	211	P	3	11	7	3	41
67	P	7	3	23	13	3	71	139	3	17	19	3	11	P	3	7	31	3	P	P
69	3	37	19	3	P	41	3	7	89	3	11	13	3	P	7	3	29	163	3	P
71	P	3	P	P	3	13	7	3	11	P	3	17	107	3	23	P	3	P	13	3
73	P	P	3	P	P	3	11	43	3	29	61	3	7	P	3	P	P	3	P	7
77	13	3	11	7	3	53	P	3	19	P	3	73	97	3	17	P	3	7	79	3
79	11	31	3	P	173	3	P	13	3	7	P	3	37	23	3	17	7	3	139	19
81	3	7	P	3	P	239	3	23	7	3	P	19	3	P	13	3	17	P	3	59
83	7	3	137	19	3	P	29	3	13	37	3	P	P	3	7	P	3	17	11	3
87	3	P	17	3	41	7	3	P	P	3	79	67	3	P	11	3	277	263	3	7
89	P	3	P	13	3	157	23	3	97	P	3	7	11	3	191	67	3	19	7	3
91	23	P	3	7	17	3	89	163	3	19	7	3	P	59	3	P	P	3	43	67
93	3	19	7	3	13	17	3	P	11	3	71	P	3	P	P	3	7	23	3	11
97	7	P	3	P	11	3	P	7	3	P	P	3	P	P	3	11	47	3	13	P
99	3	P	11	3	P	P	3	29	17	3	P	P	3	7	P	3	107	41	3	197

	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939
01	3	31	137	3	P	233	3	3	P	3	P	151	3	13	7	3	P	P	3	P
03	P	3	P	241	3	P	7	7	17	61	3	P	11	3	23	P	3	P	19	3
07	3	P	19	3	7	P	3	P	11	3	17	7	3	P	P	3	P	83	3	11
09	P	3	13	7	3	79	11	3	P	53	3	17	83	3	29	13	3	7	P	3
11	101	P	3	P	11	3	37	83	3	7	281	3	17	23	3	11	7	3	P	P
13	3	7	11	3	P	71	3	23	7	3	47	P	3	11	109	3	13	31	3	P
17	19	251	3	P	13	3	7	P	3	11	191	3	31	7	3	17	179	3	23	19
19	3	P	P	3	P	7	3	11	101	3	167	13	3	P	P	3	17	7	3	7
21	17	3	P	19	3	11	23	3	P	P	3	7	73	3	103	41	3	17	7	3
23	23	17	3	7	29	3	P	P	3	43	7	3	13	P	3	P	251	3	17	P
27	13	3	P	17	3	67	P	3	7	P	3	23	53	3	P	7	3	19	P	3
29	7	181	3	127	17	3	211	7	3	19	41	3	P	P	3	P	P	3	101	11
31	3	13	149	3	P	17	3	47	P	3	31	P	3	7	13	3	109	11	3	29
33	P	3	P	P	3	7	17	3	13	199	3	P	7	3	233	11	3	67	103	3
37	3	199	P	3	23	37	3	P	17	3	7	11	3	P	223	3	P	7	3	P
39	31	3	7	13	3	29	P	3	263	7	3	P	P	3	41	89	3	P	107	3
41	P	7	3	107	97	3	P	11	3	P	13	3	P	31	3	7	29	3	11	P
43	3	P	P	3	13	11	3	7	227	3	19	17	3	269	7	3	11	13	3	37
47	83	11	3	P	193	3	P	163	3	41	P	3	7	17	3	139	37	3	13	7
49	3	43	29	3	7	19	3	137	P	3	11	7	3	277	17	3	71	241	3	P
51	P	3	P	7	3	P	13	3	11	P	3	P	P	3	113	17	3	7	P	3
53	13	P	3	P	59	3	11	P	3	7	P	3	P	13	3	P	7	3	127	47
57	7	3	11	P	3	P	P	3	P	P	3	19	P	3	7	P	3	29	17	3
59	11	157	3	19	P	3	7	23	3	P	P	3	179	7	3	P	73	3	47	17
61	3	23	13	3	P	7	3	P	P	3	29	52	3	89	19	3	229	P	3	7
63	43	3	257	P	3	151	19	3	P	13	3	7	P	3	P	P	3	P	7	3
67	3	37	7	3	P	P	3	P	P	3	13	151	3	73	11	3	7	41	3	P
69	23	3	P	P	3	P	P	3	7	31	3	P	11	3	151	7	3	13	37	3
71	7	61	3	71	89	3	P	7	3	239	11	3	19	P	3	137	47	3	P	P
73	3	P	53	3	19	13	3	163	11	3	163	23	3	7	211	3	283	79	3	11
77	P	P	3	P	7	3	13	19	3	109	P	3	37	P	3	11	113	3	7	13
79	3	P	11	3	P	43	3	P	131	3	7	P	3	11	P	3	23	7	3	P
81	11	3	7	P	3	P	P	3	293	7	3	11	P	3	P	P	3	191	269	3
83	P	7	3	P	23	3	P	31	3	11	P	3	P	P	3	7	P	3	223	P
87	71	3	13	P	3	11	7	3	29	P	3	P	P	3	P	13	3	P	P	3
89	17	P	3	11	P	3	59	P	3	13	P	3	7	47	3	31	19	3	P	7
91	3	11	41	3	7	53	3	P	19	3	127	7	3	61	P	3	13	71	3	193
93	19	3	17	7	3	P	P	3	P	P	3	41	29	3	P	173	3	7	P	3
97	3	7	P	3	17	29	3	71	7	3	P	13	3	59	P	3	43	11	3	P
99	7	3	23	P	3	13	P	3	P	113	3	P	79	3	7	11	3	97	13	3

	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959
01	23	3	P	181	3	11	13	3	7	43	3	P	31	3	P	7	3	P	P	3
03	7	139	3	11	67	3	P	7	3	P	P	3	P	13	3	43	P	3	P	29
07	P	3	P	P	3	7	89	3	113	P	3	P	7	3	13	P	3	P	149	3
09	P	P	3	P	7	3	37	P	3	107	P	3	19	191	3	149	67	3	7	11
11	3	P	13	3	19	29	3	53	P	3	7	P	3	P	73	3	23	7	3	P
13	41	3	7	37	3	P	P	3	59	7	3	227	P	3	P	11	3	P	P	3
17	3	P	71	3	263	47	3	7	53	3	13	11	3	P	7	3	P	P	3	P
19	149	3	P	257	3	31	7	3	P	11	3	73	P	3	P	23	3	13	P	3
21	167	P	3	P	P	3	P	11	3	23	P	3	7	199	3	59	P	3	11	7
23	3	61	59	3	7	11	3	P	P	3	167	7	3	19	37	3	11	P	3	P
27	17	11	3	P	P	3	13	P	3	7	P	3	11	P	3	P	7	3	79	13
29	3	7	P	3	89	P	3	43	7	3	11	251	3	13	P	3	P	29	3	P
31	7	3	17	P	3	P	173	3	11	59	3	P	P	3	7	P	3	P	61	3
33	P	13	3	17	P	3	7	61	3	P	29	3	P	7	3	83	P	3	47	23
37	271	3	11	29	3	17	101	3	P	139	3	7	131	3	19	13	3	P	7	3
39	11	23	3	7	P	3	17	211	3	13	7	3	P	P	3	P	59	3	239	197
41	3	47	7	3	P	P	3	17	P	3	101	89	3	67	P	3	7	19	3	37
43	157	3	73	P	3	P	31	3	7	19	3	P	23	3	P	7	3	67	11	3
47	3	31	79	3	P	P	3	P	P	3	17	13	3	7	11	3	101	P	3	P
49	P	3	307	P	3	7	P	3	P	P	3	17	7	3	31	P	3	23	13	3
51	163	P	3	P	7	3	P	41	3	P	11	3	13	97	3	19	P	3	7	229
53	3	P	P	3	29	23	3	19	11	3	7	P	3	17	53	3	41	7	3	11
57	P	7	3	157	11	3	103	13	3	269	19	3	P	167	3	7	23	3	P	P
59	3	13	11	3	59	P	3	7	29	3	23	43	3	11	7	3	17	31	3	P
61	11	3	P	127	3	P	7	3	13	P	3	11	P	3	P	P	3	17	257	3
63	P	17	3	197	P	3	181	193	3	11	P	3	7	47	3	13	271	3	17	7
67	109	3	107	7	3	11	137	3	19	23	3	59	P	3	P	227	3	7	37	3
69	19	P	3	11	17	3	41	41	3	7	13	3	47	P	3	P	7	3	P	19
71	3	7	31	3	13	17	3	P	7	3	P	19	3	281	P	3	29	13	3	P
73	7	3	P	19	3	P	17	3	P	73	3	13	P	3	7	31	3	P	P	3
77	3	41	23	3	P	7	3	P	17	3	31	P	3	127	307	3	241	11	3	7
79	P	3	29	P	3	271	13	3	79	17	3	7	P	3	P	11	3	19	7	3
81	13	53	3	7	107	3	73	P	3	19	7	3	151	11	3	P	163	3	P	41
83	3	19	7	3	P	P	3	13	239	3	P	11	3	P	P	3	7	P	3	53
87	7	97	3	37	19	3	P	7	3	43	P	3	P	17	3	61	103	3	11	P
89	3	131	13	3	61	11	3	P	P	3	P	P	3	7	17	3	11	P	3	P
91	37	3	P	11	3	7	[illegible]	3	31	13	3	P	7	3	11	17	3	P	P	3
93	23	11	3	[illegible]	7	3	P	P	[illegible]	P	P	3	11	P	3	109	13	3	7	59
97	73	3	7	P	3	P	281	3	11	7	3	23	233	3	29	P	3	13	17	3
99	P	7	3	[illegible]	53	3	11	47	3	P	61	3	157	19	3	7	83	3	41	17

	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979
01	P	17	3	23	1	3	P	11	3	7	P	3	13	P	3	P	7	[illegible]	11	47
03	3	7	17	3	149	11	3	P	7	3	P	P	3	P	257	3	11	41	3	3
07	19	11	3	193	17	3	7	13	3	P	P	3	11	7	3	281	P	3	47	19
09	3	3	23	3	229	7	3	97	131	3	11	19	3	31	13	3	P	199	3	7
11	67	3	P	19	3	103	17	3	11	P	3	7	41	3	29	P	3	1	7	3
13	P	223	3	7	67	3	11	17	3	199	7	3	P	23	3	13	P	3	P	179
17	P	3	11	13	3	P	79	3	7	17	3	P	67	3	61	7	3	19	29	3
19	7	277	3	61	P	3	53	7	3	19	13	3	191	307	3	113	31	3	23	P
21	3	19	P	3	13	263	3	311	P	3	P	17	3	7	37	3	41	13	3	181
23	131	3	P	P	3	7	23	3	P	103	3	13	7	3	P	P	3	79	11	3
27	3	97	41	3	211	P	3	197	P	3	7	P	3	P	11	3	233	7	3	P
29	109	3	7	P	3	83	13	3	37	7	3	23	11	3	P	17	3	P	P	3
31	13	7	3	P	P	3	71	P	3	P	11	3	P	13	3	7	17	3	19	P
33	3	251	P	3	73	37	3	7	11	3	19	137	3	131	7	3	89	17	3	11
37	137	P	3	P	11	3	41	P	3	31	23	3	7	19	3	11	163	3	227	7
39	3	127	11	3	7	19	3	P	179	3	P	7	3	11	139	3	251	43	3	37
41	11	3	157	7	3	29	241	3	113	13	3	11	P	3	P	103	3	7	P	3
43	P	79	3	13	P	3	P	89	3	7	53	3	47	311	3	23	7	3	P	P
47	7	3	109	23	3	11	127	3	P	29	3	19	31	3	7	P	3	13	P	3
49	139	P	3	11	43	3	7	P	3	67	107	3	79	7	3	P	P	3	P	41
51	3	11	29	3	P	7	3	31	P	3	37	P	3	67	19	3	P	239	3	7
53	P	3	101	P	3	P	19	3	23	P	3	7	13	3	P	P	3	67	7	3
57	3	P	7	3	P	P	3	P	P	3	71	P	3	13	41	3	7	11	3	23
59	P	3	P	167	3	223	163	3	7	P	3	P	P	3	P	7	3	29	P	3
61	7	13	3	173	P	3	P	7	3	47	31	3	19	11	3	P	61	3	P	P
63	3	23	P	3	19	61	3	P	13	3	29	11	3	7	P	3	127	59	3	163
67	17	P	3	29	7	3	P	11	3	13	113	3	23	P	3	43	101	3	7	P
69	3	17	P	3	P	11	3	P	157	3	7	P	3	P	29	3	11	7	3	313
71	23	3	7	11	3	269	1	3	73	7	3	P	211	3	11	P	3	P	P	3
73	191	7	3	17	13	3	277	29	3	P	P	3	11	P	3	7	P	3	97	P
77	29	3	43	P	3	13	7	3	11	37	3	P	89	3	107	P	3	P	13	3
79	P	P	3	31	P	3	11	P	3	P	193	3	7	P	3	P	19	3	P	7
81	3	P	P	3	7	P	3	17	19	3	P	7	3	P	43	3	23	277	3	13
83	13	3	11	7	3	59	109	3	47	293	3	157	P	3	71	P	3	7	P	3
87	3	7	73	3	P	P	3	P	7	3	17	P	3	P	3	3	P	1	3	P
89	7	3	P	113	3	P	31	3	13	P	3	17	271	3	7	23	3	P	11	3
91	307	43	3	41	47	3	7	151	3	23	79	3	17	7	3	13	11	3	53	29
93	3	29	P	3	P	7	3	43	P	3	151	83	3	17	11	3	211	19	3	7
97	P	19	3	7	P	3	P	P	3	P	7	3	149	P	3	17	151	[illegible]	223	43
99	3	P	7	3	13	29	3	P	11	3	89	37	3	173	P	3	7	[illegible]	3	11

	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999
01	3	P	283	3	19	13	3	89	P	3	7	113	3	199	P	3	103	7	3	P
03	23	3	7	197	3	137	151	3	29	7	3	P	13	3	107	19	3	179	11	3
07	3	17	P	3	P	P	3	7	P	3	181	23	3	13	7	3	P	[illegible]	3	P
09	P	3	17	37	3	23	7	3	P	P	3	P	11	3	P	151	3	P	P	3
11	P	13	3	17	P	3	31	P	3	P	11	3	7	47	3	191	P	3	151	7
13	3	41	P	3	7	29	3	P	11	3	P	7	3	19	89	3	23	P	3	11
17	P	59	3	P	11	3	17	P	3	7	P	3	47	P	3	11	7	3	P	41
19	3	7	11	3	P	P	3	17	7	3	83	P	3	11	37	3	13	P	3	163
21	7	3	P	P	3	83	P	3	17	31	3	11	313	3	7	23	3	P	173	3
23	83	P	3	P	13	3	7	269	3	11	P	3	P	7	3	P	P	3	P	P
27	61	3	P	P	3	11	P	3	37	P	3	7	67	3	19	P	3	31	7	3
29	167	P	3	7	P	3	19	P	3	P	7	3	13	71	3	P	67	3	P	P
31	3	11	7	3	257	37	3	P	23	3	167	P	3	17	P	3	7	19	3	13
33	13	3	23	107	3	P	53	3	7	19	3	P	P	3	17	7	3	P	P	3
37	3	13	193	3	173	211	3	P	P	3	97	P	3	7	13	3	17	11	3	37
39	17	3	31	29	3	7	P	3	13	P	3	P	7	3	P	11	3	17	P	3
41	P	17	3	43	7	3	P	293	3	163	P	3	P	11	3	13	37	3	7	139
43	3	P	17	3	P	P	3	19	97	3	7	11	3	41	17	3	P	7	3	17
47	P	7	3	P	17	3	23	11	3	P	13	3	61	P	3	7	251	3	11	89
49	3	61	19	3	13	11	3	7	P	3	37	P	3	P	7	3	11	13	3	127
51	71	3	P	11	3	39	7	3	41	53	3	13	P	3	11	P	3	23	31	3
53	31	11	3	59	P	3	47	17	3	P	P	3	7	73	3	113	227	3	13	7
57	P	3	P	7	3	67	13	3	11	17	3	229	P	3	271	29	3	7	61	3
59	13	103	3	41	P	3	11	61	3	7	17	3	P	13	3	P	7	3	P	19
61	3	7	97	3	11	P	3	13	7	3	23	17	3	67	79	3	P	P	3	P
63	7	3	11	19	3	P	P	3	109	P	3	53	17	3	7	P	3	67	37	3
67	3	89	13	3	P	7	3	283	P	3	157	131	3	P	17	3	P	P	3	7
69	281	3	P	P	3	241	P	3	P	13	3	7	53	3	P	17	3	19	7	3
71	101	127	3	7	59	3	79	43	3	19	7	3	37	P	3	P	11	3	P	P
73	3	19	7	3	P	P	3	P	P	3	13	P	3	43	11	3	7	17	3	257
77	7	31	3	P	19	3	101	7	3	29	11	3	P	P	3	P	263	3	P	17
79	3	P	23	3	P	13	3	P	11	3	P	41	3	7	31	3	P	113	3	11
81	P	3	29	131	3	7	11	3	61	P	3	P	7	3	53	P	3	11	P	3
83	43	47	3	37	7	3	13	173	3	31	P	3	101	23	3	11	83	3	7	13
87	11	3	7	P	3	311	29	3	P	7	3	11	43	3	P	53	3	P	59	3
89	47	7	3	P	149	3	P	223	3	11	P	3	P	19	3	7	P	3	23	P
91	3	149	227	3	P	19	3	7	13	3	197	P	3	P	7	3	131	73	3	P
93	233	3	13	61	3	11	7	3	P	P	3	281	31	3	37	13	3	P	191	3
97	3	11	1	3	7	P	3	31	P	3	41	7	3	P	P	3	13	23	3	19
99	26	3	P	7	3	43	229	3	P	P	3	19	109	3	29	137	3	7	283	3

Sumptibus Mosis Pitt Bibliopolæ Londinensis. 1668.